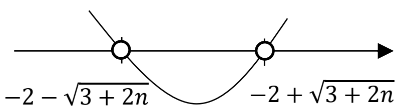
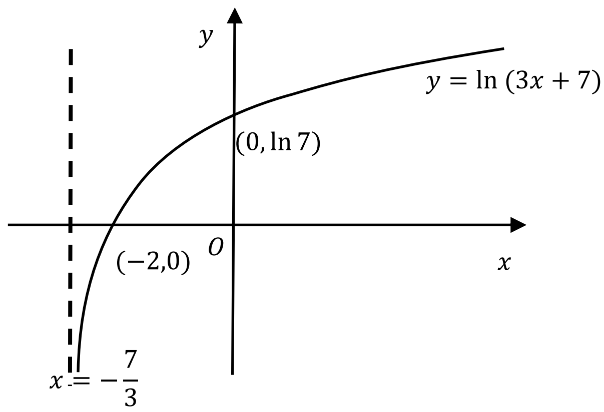
Qn	Solutions
1	For $-x^2+(k+1)x+\left(\frac{k}{2}-\frac{n}{2}\right)<0$, Discriminant < 0 and $a=-1<0$ $b^2-4ac<0$ and $a=-1<0$ $(k+1)^2-4(-1)\left(\frac{k}{2}-\frac{n}{2}\right)<0$ $k^2+2k+1+2k-2n<0$ $k^2+4k+1-2n<0$ Let $k^2+4k+1-2n=0$, then $k=\frac{-4\pm\sqrt{4^2-4(1-2n)}}{2}$ $=-2\pm\sqrt{4-1+2n}$ $=-2\pm\sqrt{3+2n}$  Therefore $-2-\sqrt{3+2n}<k<-2+\sqrt{3+2n}$.

Qn	Solutions
2	Let \$*x*, \$*y* and \$*z* be the cost of a jigsaw puzzle, a spinning top and a Rubik's cube respectively. $4x+2y+5z=264.5$ $3x=4z \Rightarrow 3x-4z=0$ $x+4y=2z+49 \Rightarrow x+4y-2z=49$ Using GC, $x=30, y=16, z=22.5$ The cost of a spinning top is \$16.00

Qn	Solutions
3(a)	$\frac{d}{dx}\left(\frac{2}{\sqrt{1-x^3}}\right)=\frac{d}{dx}2\left(1-x^3\right)^{-\frac{1}{2}}$ $=2\left(-\frac{1}{2}\right)\left(1-x^3\right)^{-\frac{3}{2}}\left(-3x^2\right)$ $=3x^2\left(1-x^3\right)^{-\frac{3}{2}}$
(b)	$\ln\left(\frac{4x}{1-ex^2}\right)=\ln 4x-\ln\left(1-ex^2\right)$ $\frac{d}{dx}\left(\ln 4x-\ln\left(1-ex^2\right)\right)$ $=\frac{4}{4x}+\frac{2ex}{1-ex^2}$ $=\frac{1}{x}+\frac{2ex}{1-ex^2}$
(c)	$\int_1^3 \frac{1}{x}+\frac{1}{x^2}-e^{6-2x}\ dx$ $=\int_1^3 \frac{1}{x}+x^{-2}-e^{6-2x}\ dx$ $=\left[\ln x-\frac{1}{x}+\frac{1}{2}e^{6-2x}\right]_1^3$ $=\left(\ln 3-\frac{1}{3}+\frac{1}{2}e^0\right)-\left(\ln 1-1+\frac{1}{2}e^4\right)$ $=\frac{7}{6}+\ln 3-\frac{1}{2}e^4$

Qn	Solutions
4(a)	
(b)	$y=\ln(3x+7)$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{3x+7}$ At $x=\frac{7}{2}$, $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3}{3\left(\frac{7}{2}\right)+7}=\frac{6}{35}$, $y=\ln\left(3\left(\frac{7}{2}\right)+7\right)=\ln\left(\frac{35}{2}\right)$ Equation of tangent to the curve: $y-\ln\left(\frac{35}{2}\right)=\frac{6}{35}\left(x-\frac{7}{2}\right)$ $35y-35\ln\left(\frac{35}{2}\right)=6x-21$ $35y-6x=35\ln\left(\frac{35}{2}\right)-21$
(c)	Numerical value of area under curve $=\int_{-1}^{1}\ln(3x+7)\,\mathrm{d}x$ $=3.8269$ (4 .d.p)

<table>
<tr><th>Qn</th><th>Solutions</th></tr>
<tr><td>5(a)
(i)</td><td>Let P m be the perimeter of the amusement park and B m be the width of the square.

$2B^2 = 4x^2$
$B = \sqrt{2}x$

$P = 4\sqrt{2}x + 4x + 2y = 10$
$y = 5 - 2\sqrt{2}x - 2x$

$A = 2xy - \left(\sqrt{2}x\right)^2$
$= 2x\left(5 - 2\sqrt{2}x - 2x\right) - 2x^2$
$= 10x - 4\sqrt{2}x^2 - 6x^2$
$= 10x - \left(4\sqrt{2} + 6\right)x^2$</td></tr>
<tr><td>5(a)ii</td><td>$\frac{dA}{dx} = 10 - 8\sqrt{2}x - 12x = 0$
$x = \frac{10}{8\sqrt{2}+12} = \frac{5}{4\sqrt{2}+6}$

<table>
<tr><td>x</td><td>$\left(\frac{5}{4\sqrt{2}+6}\right)^-$
= 0.4288</td><td>$\frac{5}{4\sqrt{2}+6}$</td><td>$\left(\frac{5}{4\sqrt{2}+6}\right)^+$
= 0.4290</td></tr>
<tr><td>Sign of $\frac{dA}{dx}$</td><td>0.0030818</td><td>0</td><td>− 0.001581</td></tr>
<tr><td>slope</td><td>/</td><td>—</td><td>\</td></tr>
</table>

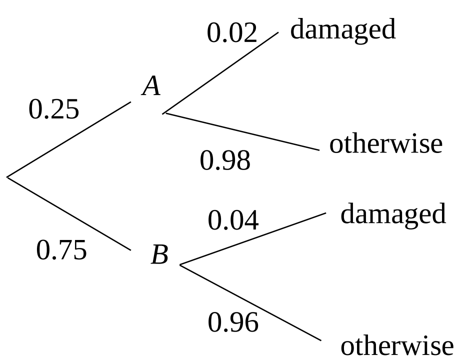
Therefore $x = \frac{5}{4\sqrt{2}+6}$ gives maximum area.</td></tr>
<tr><td>5b(i)</td><td>When $t = 1$, $C = 1.5$</td></tr>
</table>

	$1.5 = 3 - 3e^{-k}$ $3e^{-k} = 1.5$ $e^{-k} = \frac{1}{2}$ $-k = \ln\frac{1}{2}$ $k = -\ln\frac{1}{2} = \ln 2$
(ii)	$C = 3 - 3e^{-\ln 2t}$ $\frac{dC}{dt} = 0.25993 = 0.260$ (3 s.f)
(iii)	$C = 3 - 3e^{-\ln 2t}$ $t \to \infty, C \to 3$ The maintenance cost approaches $3000 in a long run.
(iv)	$\int_0^5 3 - 3e^{-\ln 2t}\ dt = 10.8$ The total maintenance cost over the 5 months is $10800

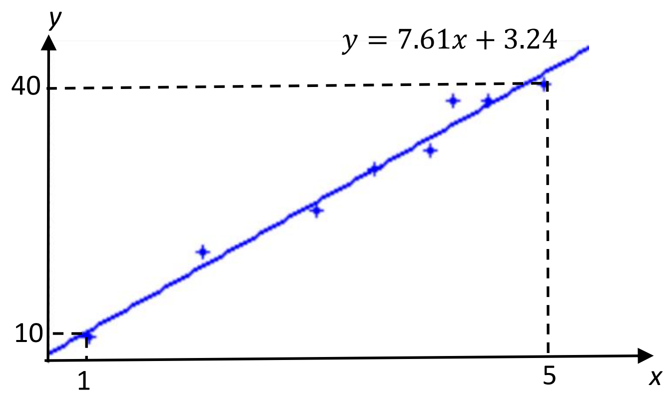
Qn	Solutions
6(a)	Case 1 : 4 girls , 2 boys No. of ways $= {}^8C_4 \times {}^{12}C_2 = 4620$ Case 2 : 5 girls , 1 boy No. of ways $= {}^8C_5 \times {}^{12}C_1 = 672$ Case 3 : 6 girls No. of ways $= {}^8C_6 = 28$ Total no. of ways $= 4620 + 672 + 28 = 5320$
(b)	Total no. of ways $= 3! \times 3! \times 2 = 72$

Qn	Solutions
7(a)	$P(A \cap B) = P(A) - P(A \cap B')$ $= 0.65 - 0.4 = 0.25$

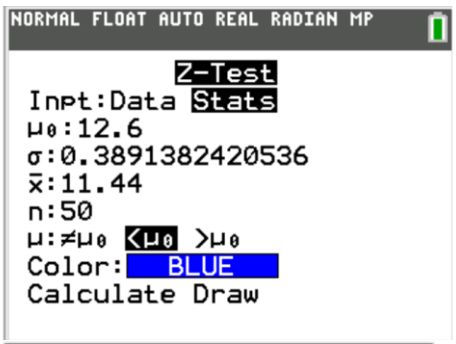
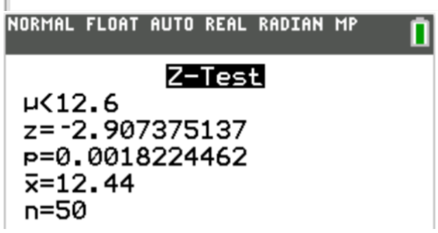
(b)	$P(A \mid B') = \frac{P(A \cap B')}{P(B')} = \frac{8}{9}$ $\frac{0.4}{P(B')} = \frac{8}{9} \Rightarrow P(B') = 0.45$ $P(B) = 0.55$
(c)	$P(A' \cap B') = 1 - [P(A \cap B') + P(B)]$ $= 1 - 0.95 = 0.05$
	$P(A \cap B) = 0.25 \neq P(A) \times P(B) = 0.65 \times 0.55 = 0.3575$ Events *A* and *B* are not independent.

Qn	Solutions
8(a)	
8(b)	P(a LED light is damaged) $= 0.25 \times 0.02 + 0.75 \times 0.04 = 0.035$
8(c)	P(From B | is damaged) $= \frac{\text{P(From B and is damaged)}}{\text{P(is damaged)}}$ $= \frac{0.75 \times 0.04}{0.035} = 0.857$
	Required Probabilty $= (0.035)^2(1 - 0.035) \times 3 = 0.00355$

Qn	Solutions
9(a)	Let X be the random variable denoting the number of electrical components that are faulty out of a box of 20. $X \sim \mathrm{B}\left(20, \frac{p}{100}\right)$ Since $\mathrm{P}(X = 2) = 0.2$, $\binom{20}{2}\left(\frac{p}{100}\right)^2\left(1-\frac{p}{100}\right)^{18} = 0.2$ $190\left(\frac{p}{100}\right)^2\left(1-\frac{p}{100}\right)^{18} = 0.2$ Using GC, since $p < 10$, $\frac{p}{100} = 0.052929$(to 5 s.f.) $\therefore\ p = 5.29$ (to 3 s.f.)
9(b)	Given $p = 5$, $X \sim \mathrm{B}(20, 0.05)$. $\mathrm{P}(X \le 3) = 0.984$ (to 3 s.f.)
9(c)	$\mathrm{P}(X > 3) = 1 - \mathrm{P}(X \le 3)$ $= 0.015902$ (to 5 s.f.) Let Y be the random variable denoting the number of rejected boxes out of 5. $Y \sim \mathrm{B}(5, 0.015902)$ $\mathrm{P}(Y < 2) = \mathrm{P}(Y \le 1)$ $= 0.998$ (to 3 s.f.)
9(d)	$\mathrm{E}(X) = 20 \times 0.05 = 1$ $\mathrm{Var}(X) = 20 \times 0.05 \times 0.95 = 0.95$ Let $\bar{X} = \frac{X_1 + X_2 + \ldots + X_{40}}{40}$, since $n = 40$ is large, by Central Limit Theorem, $\bar{X} \sim \mathrm{N}\left(1, \frac{0.95}{40}\right)$ approximately. $\mathrm{P}(\bar{X} \ge 1) = 0.500$ (to 3 s.f.)

Qn	
10(a)	
(b)	Using GC, product moment correlation coefficient, $r = 0.989$ (3 s.f) There is a strong positive linear correlation between the monthly advertising expenditure and the monthly sales of the bubble tea. As the monthly advertising expenditure increases, the monthly sales of the bubble tea also increases.
(c)	$y = 7.6145x + 3.2356$ $y = 7.61x + 3.24$ (3 s.f.)
(d)	When $x = 3.8$, $y = 7.6145(3.8) + 3.2356$ $=32.1707$ $=32.2$ (3.s.f) The monthly sales of the bubble tea is $32200. Since *r* = 0.989 is close to +1 and *x*= 3.8 is within the data range of *x*, the estimation is reliable.

Qn	
11(a)	Let *X* be the mass of a randomly chosen tyre (in kilograms) Unbiased estimate of population mean, $\bar{x} = \dfrac{-128}{50} + 15 = 12.44$ Unbiased estimate of population variance,

	$s^2 = \frac{1}{49}\left[335.1 - \frac{(-128)^2}{50}\right] = \frac{53}{350} = 0.151\ (3\text{sf})$
(b)	$H_0 : \mu = 12.6$ $H_1 : \mu < 12.6$ Test at 5% significance level Under H_0, since $n = 50$ is large, by Central Limit Theorem, $\bar{X} \sim N\left(12.6, \frac{53}{50 \times 350}\right)$ approximately, where $s^2 = \frac{53}{350}$ is a good estimate of σ^2   Using GC, the test statistics $\bar{x} = 11.44$ gives $z_{calc} = -2.9074$ and p-value = 0.0018224 $\approx$ 0.00182 (3 sf) $\leq$ 0.05. Since $p-\text{value} = 0.0018224 \leq 0.05$, we reject H_0 and conclude there is sufficient evidence, at 5% level of significance, that the mean mass of tyres is less than 12.6 kg. Hence, the claim of the group of customers is not valid at 5% level of significance.
(c)	Let Y be the mass of a randomly chosen tyre (in kilograms) from the new shipment. $H_0 : \mu = 12.6$ $H_1 : \mu \neq 12.6$ Test at 5% significance level Under H_0, since $n = 75$ is large, by Central Limit Theorem, $\bar{Y} \sim N\left(12.6, \frac{0.5}{75}\right)$, approximately. Since there is sufficient evidence that the population mean mass of tyres from this shipment differs from 12.6 kg, we reject H_0.

	$\dfrac{k-12.6}{\sqrt{\dfrac{0.5}{75}}} \le -1.95996$ $\quad$ $\dfrac{k-12.6}{\sqrt{\dfrac{0.5}{75}}} \ge 1.95996$ $k-12.6 \le -0.160030$ or $k-12.6 \ge 0.160030$ $k \le 12.44$ (2 d.p) $\quad$ $k \ge 12.76$ (2 d.p)

Qn	Solutions
12(a)	$L \sim \mathrm{N}\left(16, 3^2\right)$ $\mathrm{P}\left(L > 17\right) = 0.36944$ (to 5 s.f.) Required probability $= \left[\mathrm{P}\left(L > 17\right)\right]^3$ $= 0.0504$ (to 3 s.f.)
(b)	p is more than the answer in part (i). The cases found in (i) are subsets of the cases found in (ii).
(c)	$M \sim \mathrm{N}\left(15, 1.5^2\right)$ Let $A = M_1 + M_2 + M_3 - 2L$ $\mathrm{E}\left(A\right) = 3\left(15\right) - 2\left(16\right) = 13$ $\mathrm{Var}\left(A\right) = 3\left(1.5^2\right) + 2^2\left(3^2\right) = 42.75$ $A \sim \mathrm{N}\left(13, 42.75\right)$ $\mathrm{P}\left(M_1 + M_2 + M_3 > 2L\right)$ $= \mathrm{P}\left(M_1 + M_2 + M_3 - 2L > 0\right)$ $= \mathrm{P}\left(A > 0\right)$ $= 0.977$ (to 3 s.f.)
(d)	Let $T = 1.05\left(L_1 + L_2 + L_3\right) + 1.1\left(M_1 + M_2\right)$ $\mathrm{E}\left(T\right) = 1.05\left(3\times16\right) + 1.1\left(2\times15\right) = 83.4$ $\mathrm{Var}\left(T\right) = 1.05^2\left(3\times3^2\right) + 1.1^2\left(2\times1.5^2\right) = 35.2125$ $T \sim \mathrm{N}\left(83.4, 35.2125\right)$ $\mathrm{P}\left(T < 83\right) = 0.473$ (to 3 s.f.)