JURONG PIONEER JUNIOR COLLEGE
9749 H2 PHYSICS

D.C. CIRCUITS

Content

- **Circuit symbols and diagrams**
- **Series and parallel arrangements**
- **Potential divider**
- **Balanced potentials**

Learning Outcomes

Candidates should be able to:

a) recall and use appropriate circuit symbols as set out in the ASE publication *Signs, Symbols and Systematics (The ASE Companion to 16-19 Science, 2000).*

b) draw and interpret circuit diagrams containing sources, switches, resistors, ammeters, voltmeters, and/or any other type of component referred to in the syllabus.

c) solve problems using the formula for the combined resistance of two or more resistors in series.

d) solve problems using the formula for the combined resistance of two or more resistors in parallel.

e) solve problems involving series and parallel circuits for one source of e.m.f.

f) show an understanding of the use of a potential divider circuit as a source of variable p.d.

g) explain the use of thermistors and light-dependent resistors in potential divider circuits to provide a potential difference which is dependent on temperature and illumination respectively.

h) recall and solve problems by using the principle of the potentiometer as a means of comparing potential differences.

Introduction

- In the previous topic on Current of Electricity, physical quantities such as electromotive force (e.m.f.), potential difference (p.d), current (I), resistance (R) and internal resistance (r) in electric circuits were discussed.

- This topic on Direct Current (D.C.) Circuits involves electric circuits with current that flows in one direction.

1 Circuit symbols and diagrams

(a) *Candidates should be able to* recall and use appropriate circuit symbols as set out in the ASE publication *Signs, Symbols and Systematics (The ASE Companion to 16-19 Science, 2000).*

(b) *Candidates should be able to draw and interpret circuit diagrams containing sources, switches, resistors, ammeters, voltmeters, and/or any other type of component referred to in the syllabus.*

1.1 Circuit symbols

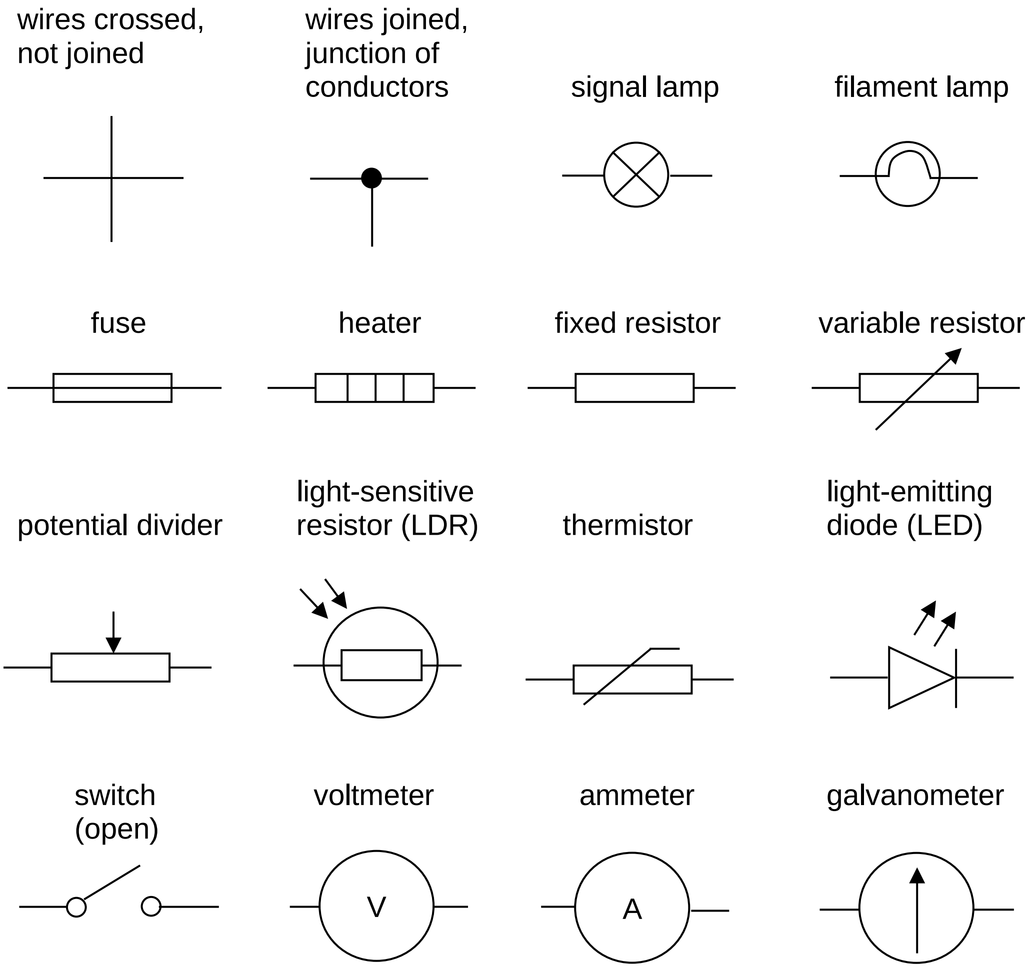


Fig. 1 Circuit Symbols

2 Series and parallel arrangements

(c) Candidates should be able to solve problems using the formula for the combined resistance of two or more resistors in series.

When two or more resistors are connected using connecting wires, the combined effect of these resistors can be represented by drawing a single resistor with a resistance called the combined resistance (equivalent or effective resistance). The connecting wires are assumed to have negligible resistance.

2.1 Resistors in series

Fig. 2 shows two resistors of resistance R_1 and R_2 connected in series between points A and B. The current passing through both resistors is the same. The corresponding potential difference (p.d.) across each of the resistors are V_1 and V_2. The p.d. across points A and B is V.

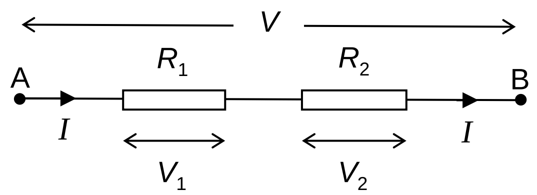


Fig. 2 Resistors in series

Applying $V = IR$ to each of the resistors, we get:

$V_1 = IR_1$ and $V_2 = IR_2$

For a series connection,
$$\begin{aligned} V &= V_1 + V_2 \\ &= IR_1 + IR_2 \\ &= I(R_1 + R_2) \end{aligned}$$

Rearranging: $(R_1 + R_2) = \dfrac{V}{I}$ ---------- (1)

To find the combined resistance of R_1 and R_2, re-draw Fig. 2 using a single combined resistance R between points A and B. With the same p.d. V and current I through points A and B, we can apply $V = IR$ between points A and B to get:

$$R = \frac{V}{I} \text{ ---------- (2)}$$

Comparing equations (1) and (2), we can easily calculate the combined resistance between A and B using:

$$\boxed{R = R_1 + R_2}$$

This equation can be extended to as many resistors as there are in series:

$R = R_1 + R_2 + R_3 + \ldots + R_N$ (where N is number of resistors connected in series)

Note: The effective resistance is always greater than the largest resistance in the series network.

(d) Candidates should be able to solve problems using the formula for the combined resistance of two or more resistors in parallel.

2.2 Resistors in parallel

Consider 2 resistors, R_1 and R_2, connected in parallel, as shown in Fig. 3,

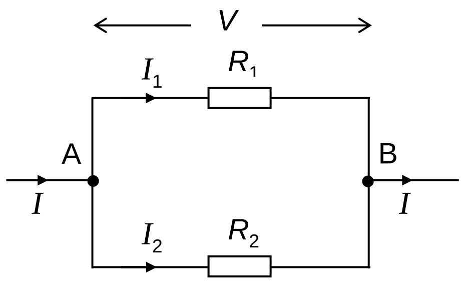


Fig. 3 Resistors in parallel

The p.d. V across both resistors is the same because they are across the same 2 points A and B. Hence,

$$I_1 = \frac{V}{R_1} \text{ ---------- (3)}$$

$$I_2 = \frac{V}{R_2} \text{ ---------- (4)}$$

To find the combined resistance of R_1 and R_2, re-draw Fig. 3 using a single combined resistance R between the points A and B. If the total current I is flowing between the points A and B, then the effective resistance R is given by

$$I = \frac{V}{R} \text{ ---------- (5)}$$

In the parallel circuit, the total current is the sum of the individual currents

$$I = I_1 + I_2 \text{ ---------- (6)}$$

Combining equations (3), (4), (5) into (6),

$$\frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2}$$

Cancelling V, we can easily calculate the combined resistance using:

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}} \text{ ------------ (7)}$$

This equation can be extended to as many resistors as there are in parallel.

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ \ldots \ + \frac{1}{R_N} \text{ ------------ (8)}$$

(where N is number of resistors connected in parallel)

Note: The effective resistance is always less than the smallest resistance in the parallel network.

Special cases:

(i) Only 2 resistors in parallel:

Equation (7) can be re-written as $\boxed{R = \dfrac{R_1 R_2}{R_1 + R_2}}$

(ii) *N* number of identical resistors (*R*) in parallel:

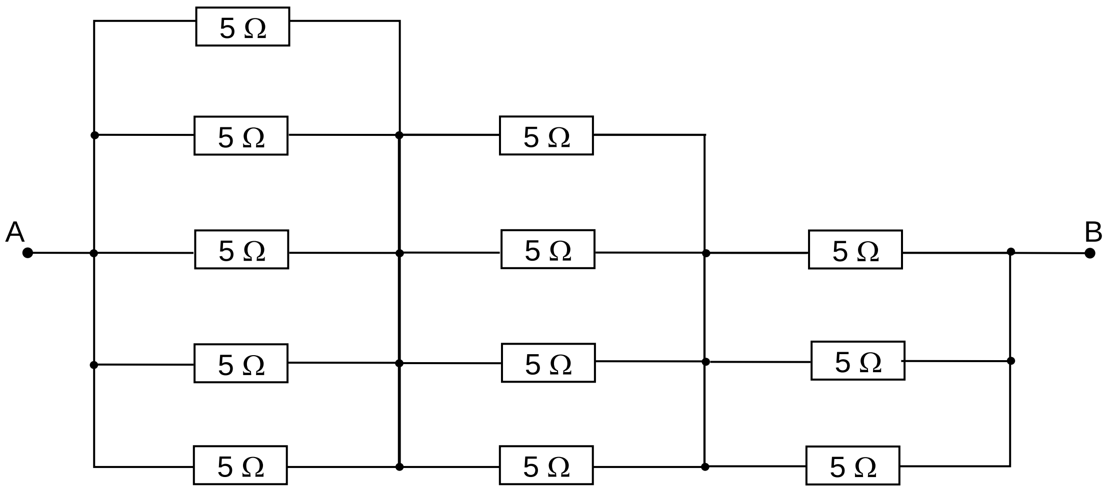
Equation (8) can be re-written as

$$\frac{1}{R_{\text{effective}}} = \frac{1}{R} + \frac{1}{R} + \ \dots \ + \frac{1}{R} = \frac{N}{R}$$

$$\boxed{R_{\text{effective}} = \frac{R}{N}}$$

Example 1

Calculate the effective resistance across A and B in the network below.



Effective resistance = $\dfrac{5}{5} + \dfrac{5}{4} + \dfrac{5}{3}$

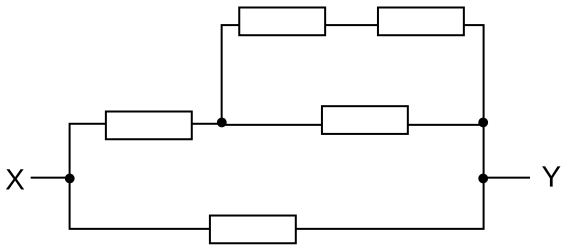
$= 3.9 \ \Omega$

For circuits involving many resistors connected in a combination of series and parallel network, more steps are required to determine the effective resistance of the overall circuit.

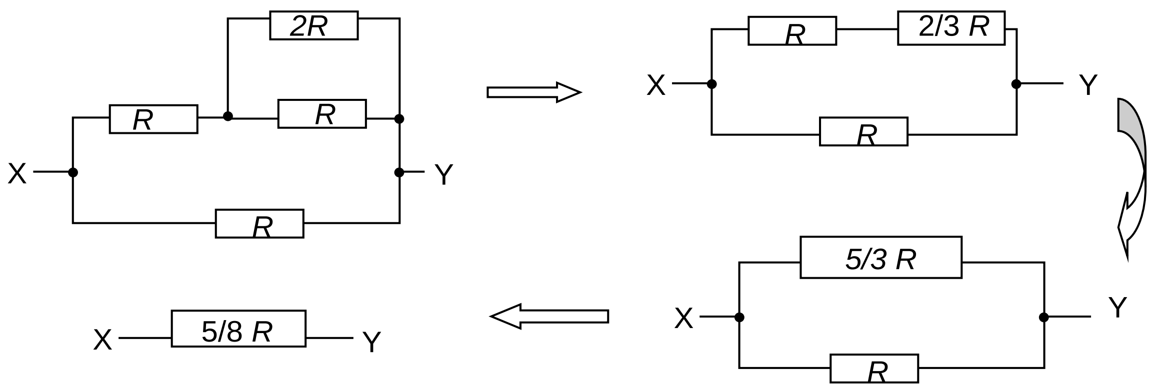
Example 2

The circuit diagram shows a network of five identical resistors, each of resistance *R*.

Calculate the effective resistance between points X and Y in terms of *R*.



Solution:



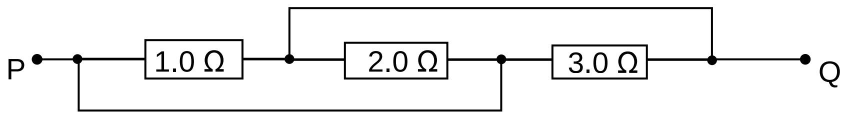
Effective resistance = $\frac{5R}{8}$

$= 0.625\,R$

Some circuits are not easy to understand, and must be redrawn to see the series or parallel relationship between the resistors. Example 3 is an illustration:

Example 3

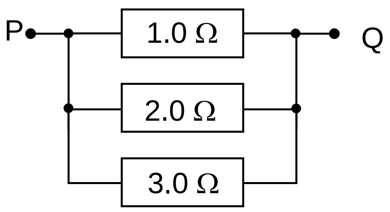
Three resistors are connected as shown in the diagram using connecting wires of negligible resistance. Calculate the resistance between points P and Q.



Solution:

Hints: 1) Consider the three paths which a current from P can flow to Q.
2) Current flows from higher potential to lower potential.

The above circuit can be arranged as three resistors in parallel.

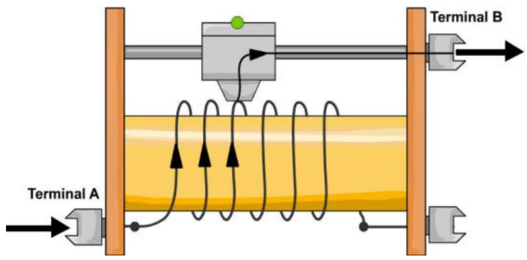


$$\frac{1}{R} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3}$$
$$R = 0.545\Omega$$

2.3 Variable resistors

1) Rheostat

A rheostat consists of a coil of resistance wire wound around an insulator cylinder. There are 3 terminals. A sliding contact varies the length of resistance wire which the current passes through.

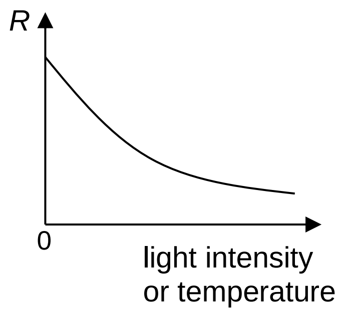


Recall that the resistance of a wire R can be written as:

$\boxed{R = \frac{\rho l}{A}}$ l: length, A: cross-sectional area, ρ : resistivity of the material

2) Light dependent resistor (LDR) or photoconductive cell

A LDR is a semiconductor whose resistance decreases as light intensity (illumination) falling on it increases. This is because electromagnetic radiation energy of light is absorbed to break the bonds between atoms and bound electrons. With more mobile charge carriers (negatively-charged electrons and positively-charged holes), the resistance of the semiconductor decreases.



3) Thermistor

A thermistor is made of a semiconductor material which has resistance that changes according to the surrounding temperature. Commonly used thermistors have a negative temperature coefficient, i.e. the resistance decreases as temperature increases.

(e) *Candidates should be able to solve problems involving series and parallel circuits for one source of e.m.f.*

Examples 1 to 3 are partial circuit diagrams which show only the resistors and not the e.m.f. sources. Partial diagrams are used mainly for calculating the combined or effective resistance between 2 points in the circuit. If we need to calculate more details such as the current or potential difference across parts of the circuit, the circuit diagram needs to be drawn with the e.m.f. source.

Example 4

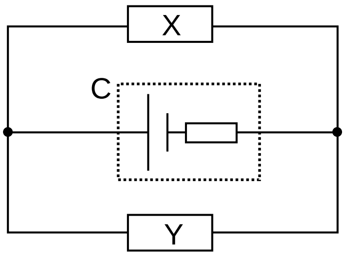
In the circuit shown, each of the resistors X and Y has resistance 6.0 Ω. The cell C has e.m.f. 12 V and internal resistance 1.0 Ω.

Calculate

(a) the total resistance of the circuit,

(b) the current through cell C and the current though resistor Y.

Solution:



(a) Since X and Y are parallel to each other.

Effective resistance of X and Y = $\frac{6}{2}$

= 3.0 Ω

Total resistance of circuit = 3.0 + 1.0 = 4.0 Ω

(b) Current through cell C = $\frac{12}{4}$ = 3.0 A

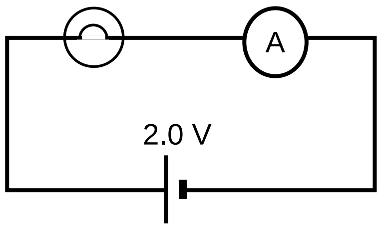
Since current splits equally at the junction before flowing through X and Y, current through Y is 1.5 A.

Example 5

The diagram shows a bulb and ammeter connected to a cell with an e.m.f. of 2.0 V and negligible internal resistance.

If the bulb and ammeter has a resistance of 5.0 Ω each, determine the ammeter reading.

Solution:



Combined resistance = 5.0 + 5.0 = 10.0 Ω

Ammeter reading = $\frac{2.0}{10.0}$

= 0.20 A

3 Potential divider

(f) Candidates should be able to show an understanding of the use of a potential divider circuit as a source of variable p.d.

An e.m.f. source usually provides an e.m.f. supply of a fixed value. A potential divider can be used to supply a variable p.d. across 2 points in a circuit. It is frequently used to control the temperature in a freezer, monitor changes in light in a room or as audio volume controls,

3.1 The Potential divider principle

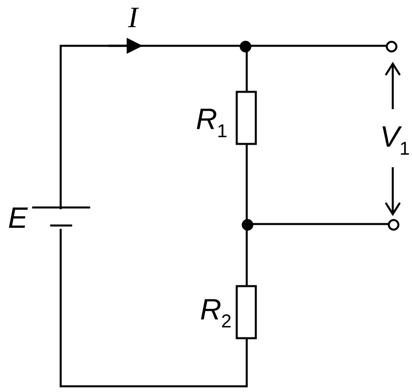


Fig. 4 Potential divider

Fig. 4 shows 2 resistors of resistances R_1 and R_2 connected in series, and a cell which has an e.m.f. E and negligible internal resistance.

By connecting 2 wires with 2 terminals across one of the 2 resistors, say R_1, the p.d. across R_1 can be obtained. This p.d. V_1 can be easily adjusted by varying the ratio of R_1 and R_2. V_1 therefore becomes a variable source of p.d.

The circuit above is call the potential divider as it divides the e.m.f. source E across resistors R_1 and R_2.

Apply $V = IR$ to obtain the following 2 equations:

$E = I(R_1 + R_2)$ --------- (1)
$V_1 = IR_1$ ---------(2)

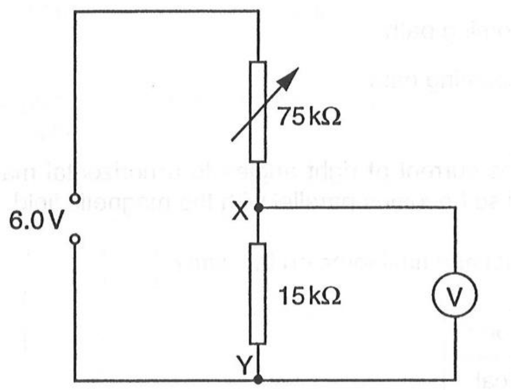
$\frac{(2)}{(1)}$ => $\frac{V_1}{E} = \frac{R_1}{R_1 + R_2}$

Hence we get $\boxed{V_1 = \frac{R_1}{R_1 + R_2} \times E}$

Note that V_1 depends on the ratio $\frac{R_1}{R_1 + R_2}$.

Example 6

The diagram shows two resistors connected in a circuit with a power supply and voltmeter.



What range of voltages can be obtained between points X and Y?

A zero to 1.0 V

B zero to 6.0 V

C 1.0 V to 5.0 V

D 1.0 V to 6.0 V

Solution:

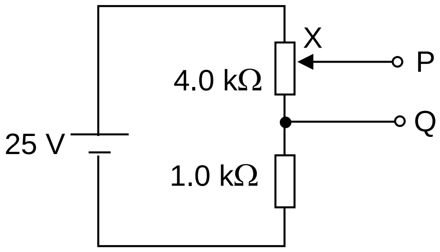
When variable resistor is set to zero, p.d. across 15 kΩ is $\left(\dfrac{15}{15+0}\right)6.0 = 6.0\ \text{V}$.

When variable resistor is set to 75 kΩ, p.d. across 15 kΩ is $\left(\dfrac{15}{15+75}\right)6.0 = 1.0\ \text{V}$.

Answer: **D**

Example 7

The diagram below shows a potential divider circuit involving a fixed 1.0 kΩ resistor and a rheostat with a maximum resistance of 4.0 kΩ. By adjusting the position of contact X, the circuit can be used to provide a variable potential difference between the terminals P and Q. Calculate the limits (minimum and maximum) of this variable potential difference between P and Q.



Solution:

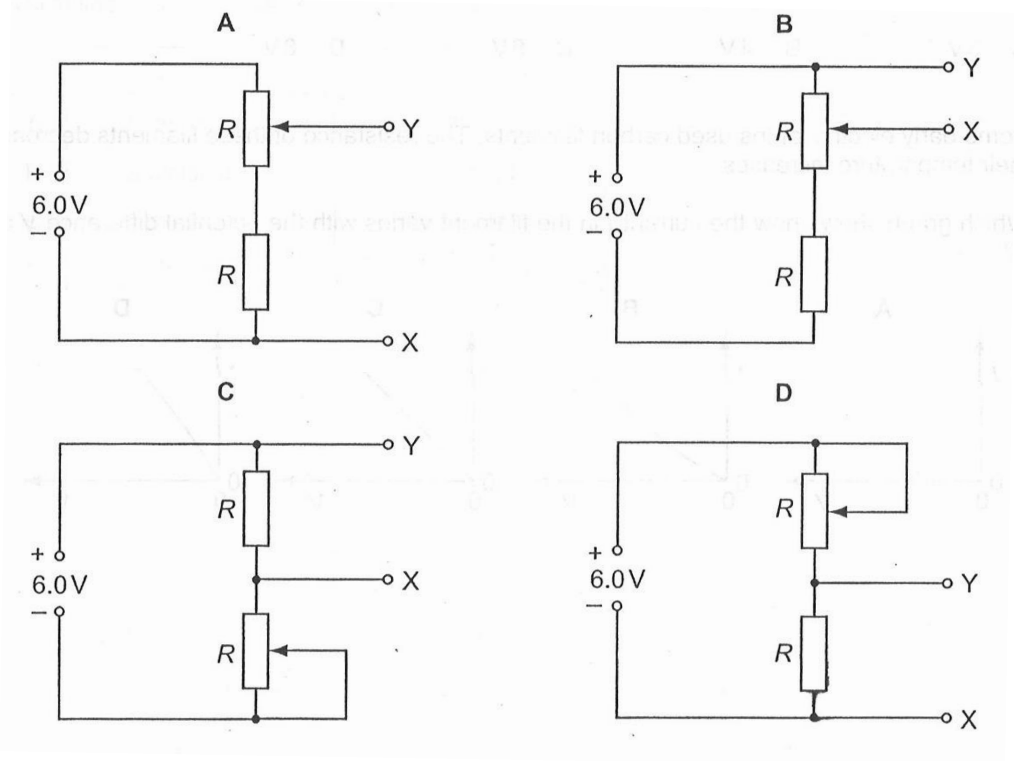
When contact X is moved to the top end of the rheostat, the resistance between PQ is max, at 4.0 kΩ, therefore p.d. between PQ, $V_{PQ} = \dfrac{4}{1+4}(25) = 20\ \text{V}$.

When contact X is moved to the bottom end of the rheostat, the resistance between PQ is min, at 0.0 kΩ, therefore p.d. between PQ, $V_{PQ} = \dfrac{0}{1+4}(25) = 0\ \text{V}$.

The limits of potential difference between P and Q are 0 V and 20 V.

Example 8

A potential divider has a constant supply of 6.0 V as shown in the diagrams.
Which circuit will provide a potential difference between X and Y that can be varied between zero and 3.0 V?



Solution:

In option B, when the movable contact X is at the highest position, the p.d. between X and Y is zero. When the movable contact X is at the lowest position, the p.d. between X and Y is $\left(\dfrac{R}{R+R}\right)6.0 = 3.0\ \text{V}$.

In options A, C and D, the range is between 3.0 V and 6.0 V.

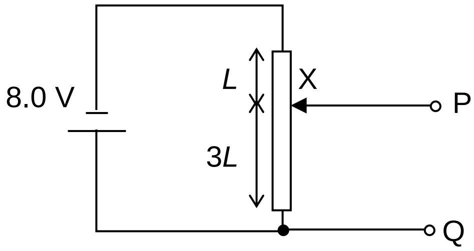
Answer: **B**

Example 9

A rheostat with a length of 4L and maximum resistance of 4.0 kΩ is connected in series to a 8.0 V supply. With contact X at the position shown below, calculate

(a) the potential difference across points P and Q,

(b) the potential at point P.



Solution:

(a) $V_{PQ} = \left(\dfrac{3}{1+3}\right)(8.0) = 6.0\ \text{V}$

(b) Let the negative terminal of the 8.0 V supply be a reference of potential of 0 V.
Therefore $V_Q = +8.0$ V

$V_P = +8.0 - 6.0$
$= +2.0$ V

3.2 Variable p.d. using light dependent resistors and thermistors

(g) *Candidates should be able to explain the use of thermistors and light dependent resistors in potential divider circuits to provide a potential difference which is dependent on temperature and illumination respectively.*

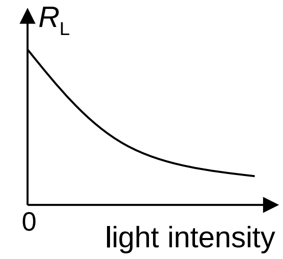
In Example 7, a rheostat is used to provide a variable output p.d.

Devices like LDRs and thermistors can also provide variable output p.d. This is because the resistance of the LDR changes with light intensity, and the resistance of the thermistor changes with temperature.

3.2.1 Light dependent resistor (LDR) or photoconductive cell

Recall that the resistance of a LDR decreases as light intensity (illumination) falling on it increases.

Fig. 5 and Fig. 6 show 2 ways of using a LDR to obtain a variable p.d. which varies according to light intensity. Each circuit serves a different purpose.



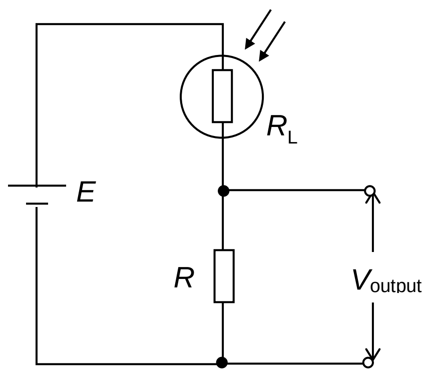


Fig. 5 V_{output} is across the resistor

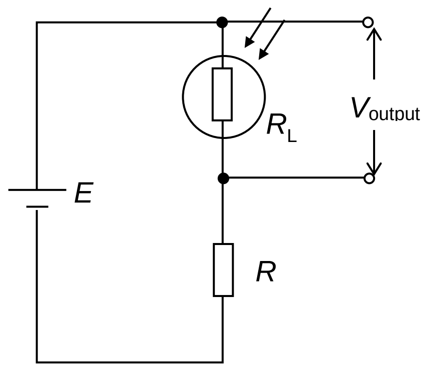


Fig. 6 V_{output} is across the LDR

<u>Burglar Alarm</u>
In Fig. 5, two wires connected across the fixed resistor give a V_{output} of

$$V_{output} = \frac{R}{R_L + R} \times E$$

During a break in at night, if the burglar's torch shines light on the LDR, R_L decreases. Hence V_{output} increases. If this V_{output} is applied across a siren, the siren will be activated when V_{output} reaches a certain value.

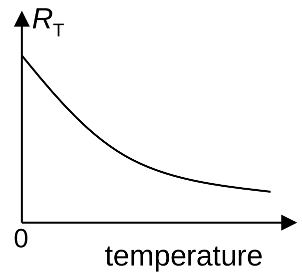
<u>Auto-ON night light</u>
Fig. 6, two wires connected across the LDR give a V_{output} of

$$V_{output} = \frac{R_L}{R_L + R} \times E$$

As the sun sets, light intensity decreases and R_L increases. Hence V_{output} across the LDR increases. If this V_{output} is applied across a lamp, it will light up when V_{output} reaches a certain value.

3.2.2 Thermistors

Recall that a thermistor (or thermally sensitive resistor) is made of a semiconductor material which has resistance that changes according to the surrounding temperature.



For a thermistor with a negative temperature coefficient, its resistance R_T decreases as temperature increases.

Fig. 7 and Fig. 8 show 2 ways of using a thermistor to obtain a variable p.d. which varies according to temperature. Each circuit serves a different purpose.

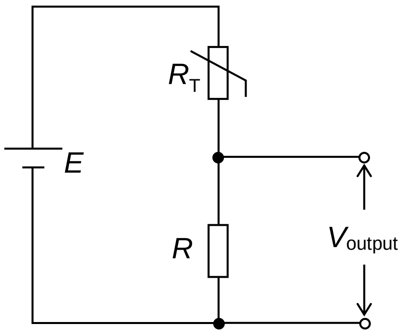


Fig. 7 V_{output} is across the resistor

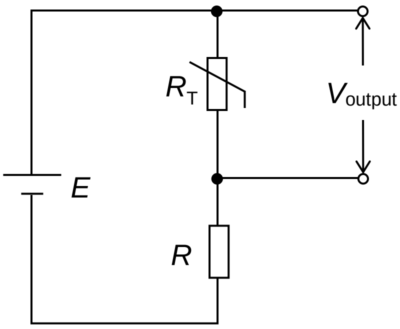


Fig. 8 V_{output} is across the thermistor

<u>Temperature detection fire alarm</u>
Fig. 7, two wires connected across the fixed resistor give a V_{output} of

$$V_{output} = \frac{R}{R_T + R} \times E$$

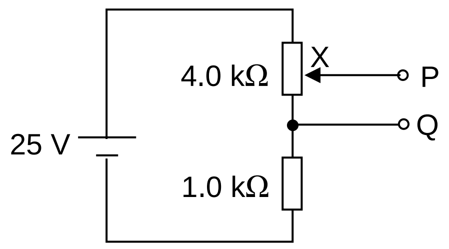
During a fire, the temperature increases and R_T decreases. Hence V_{output} across the fixed resistor increases. If this V_{output} is applied across a siren, the siren will be activated when V_{output} reaches a certain value.

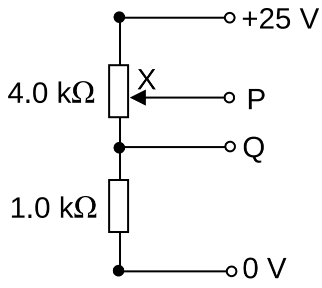
<u>Ice and snow melting device for frost protection</u>
Fig. 8, two wires connected across the thermistor give a V_{output} of

$$V_{output} = \frac{R_T}{R_T + R} \times E$$

During winter, the temperature decreases and R_T increases. Hence V_{output} across the thermistor increases. If this V_{output} is applied across a heater, the heater will turn on when V_{output} reaches a certain value.

Note : There are different ways to draw circuit diagrams. The following two diagrams represent the same electrical setup :

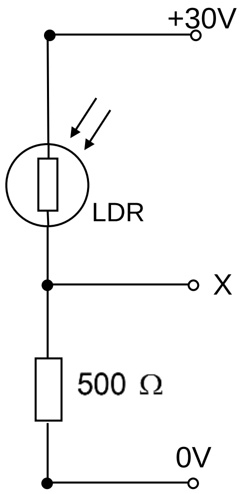




Example 10

(a) A light dependent resistor (LDR) and a 500 Ω resistor form a potential divider between voltage lines held at +30 V and 0 V as shown in the diagram. The resistance of the LDR is 1000 Ω in the dark but then decreases to 100 Ω in bright light.

Calculate the corresponding change in the potential at X.



Solution:

In the dark, resistance of LDR is 1000 Ω,

p.d. across 500 Ω resistor $= \dfrac{500}{500+1000} \times 30 = 10$ V

Hence potential at X, V_X = +10 V

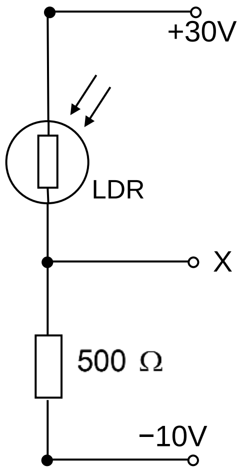
In bright light, resistance of LDR is 100 Ω,

p.d. across 500 Ω resistor $= \dfrac{500}{500+100} \times 30 = 25$ V

Hence the new value of V_X = +25 V

Change in V_X = 25 – 10 = +15 V

(b) If the potential at the lower end of the supply is changed to −10 V, determine the potential at X in the dark.



Solution:

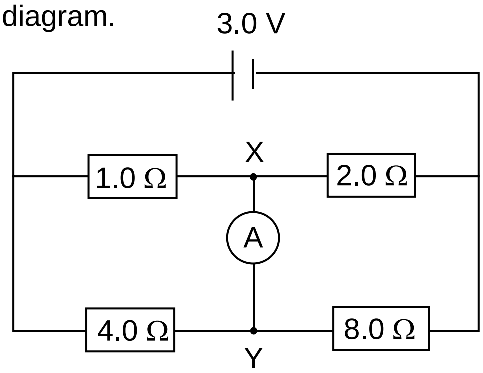
In the dark, resistance of LDR is 1000 Ω,
p.d. across 500 Ω resistor

$$= \frac{500}{500+1000} \times [30 - (-10)] = 13.33 \text{ V}$$

$$V_{500} = V_X - (-10)$$
$$13.33 = V_X - (-10)$$
$$V_X = 3.3 \text{ V}$$

Example 11

(a) Determine the ammeter reading shown in the diagram.



Solution:

Let the negative terminal of the 3.0 V cell be a reference of potential of 0 V.

p.d.across 1.0 Ω resistor

$$=\left(\frac{1.0}{1.0+2.0}\right)(3.0)=1.0\text{ V}$$

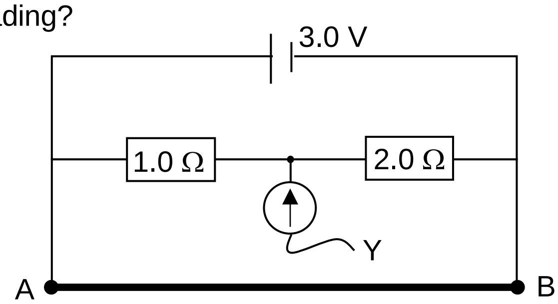
p.d.across 4.0 Ω resistor

$$=\left(\frac{4.0}{4.0+8.0}\right)(3.0)=1.0\text{ V}$$

Since the potential at X and Y are both +2.0 V, no current flows through the ammeter. Therefore the ammeter reading is zero.

(b) AB is a uniform resistance wire with a length of 100 cm. It is attached on a metre rule with one end at the 0 cm mark and the other at the 100 cm mark.

In the following diagram, which point of wire AB should the galvanometer be connected to so as to give a zero reading?



Solution:

p.d.across the 1.0 Ω resistor = $\left(\frac{1.0}{1.0+2.0}\right)(3.0)=1.0\text{ V}$

If the galvanometer reading is zero,

p.d. across AY = p.d across the 1.0 Ω resistor

$$\left(\frac{R_{AY}}{R_{AB}}\right)(3.0)=1.0\text{ V}$$

$$\left(\frac{\frac{\rho L_{AY}}{A}}{\frac{\rho L_{AB}}{A}}\right)(3.0)=1.0\text{ V}$$

$$\left(\frac{L_{AY}}{L_{AB}}\right)(3.0)=1.0\text{ V}$$

$$\left(\frac{L_{AY}}{100}\right)(3.0)=1.0\text{ V}$$

$$L_{AY}=33\text{ cm}$$

Hence point Y should be connected to the 33 cm mark.

4 Balanced potentials

(h) Candidates should be able to recall and solve problems by using the principle of the potentiometer as a means of comparing potential differences.

4.1 Potentiometer

Potentiometer is a device for comparing potential difference. The working principle for the potentiometer is the potential divider principle.

Fig. 9 shows a potentiometer. It consists of a length of resistance wire AB of uniform cross-sectional area, and a steady current passes through it. The resistance wire is stretched over a metre scale and connected in series to a driver cell of e.m.f. E_{driver}. The resistor (with resistance R) is added to change the sensitivity and range of the potentiometer.

Consider the potentiometer wire AB of length L_{AB}, uniform cross-sectional area A_0, resistivity ρ and resistance R_{AB}.

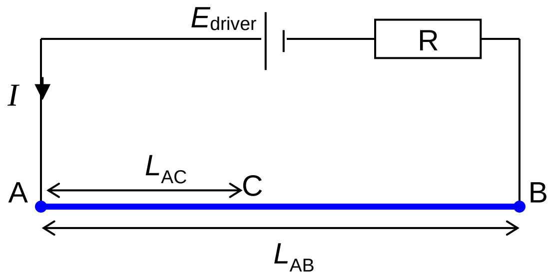


Fig. 9 Potentiometer

The p.d. across wire AB, $$V_{AB} = IR_{AB} = I\left(\frac{\rho L_{AB}}{A_0}\right) \text{ ---------- (1)}$$

The p.d. across two points A and C is $$V_{AC} = IR_{AC} = I\left(\frac{\rho L_{AC}}{A_0}\right) \text{ ---------- (2)}$$

$\frac{(2)}{(1)}$ gives $$\frac{V_{AC}}{V_{AB}} = \frac{L_{AC}}{L_{AB}}$$

Rearranging, we get : $$V_{AC} = \left(\frac{L_{AC}}{L_{AB}}\right)V_{AB} \text{ ---------- (3)}$$

Use the potential divider principle to get V_{AB} :

$$V_{AB} = \left(\frac{R_{AB}}{R_{AB} + R}\right) \times E_{\text{driver}} \text{ ---------- (4)}$$

combining equation (3) and (4), we get the **potentiometer equation:**

$$\boxed{V_{AC} = \frac{L_{AC}}{L_{AB}} \times \left(\frac{R_{AB}}{R_{AB} + R}\right) \times E_{\text{driver}}} \text{ ---------- (5)}$$

Potentiometer output = (fraction of length) × (fraction of resistance) × (e.m.f of driver cell)

External circuit to be investigated

Fig. 10 shows an external circuit to be investigated using the potentiometer. The circuit consists of a cell with e.m.f. E and internal resistance r, connected in series to a fixed resistance (of resistance R_1) and a switch S.

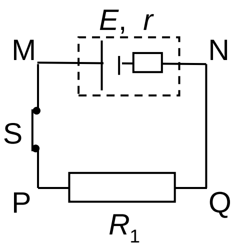


Fig. 10 External Circuit to be measured

How to use the potentiometer

Fig. 11 shows how the potential difference across points M and N is determined using the potentiometer.

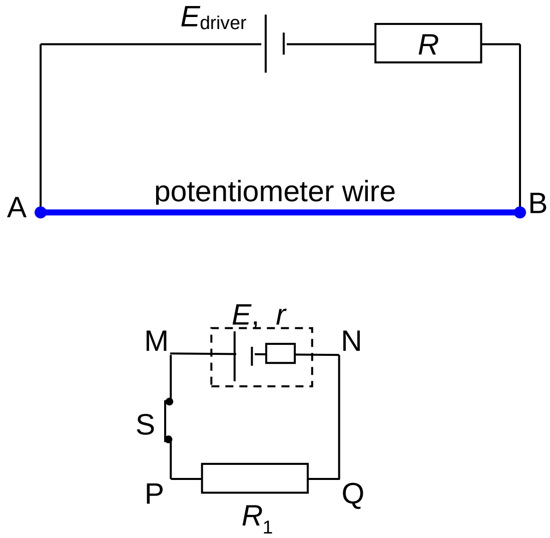


Fig. 11 Potentiometer connected to external circuit

Step 1: Connect point A on the potentiometer to the point M of the external circuit.

$\Rightarrow$ Potential at A = Potential at M

Step 2: Connect a galvanometer and a jockey to point N.

Step 3: Lightly tap the jockey on the potentiometer wire to find a point C such that no current flows through the galvanometer.

$\Rightarrow$ Potential at point C = Potential at point N

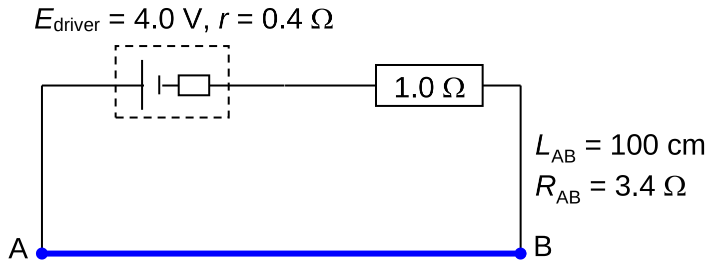
Step 4: Potential difference V_{MN} is determined through calculation :

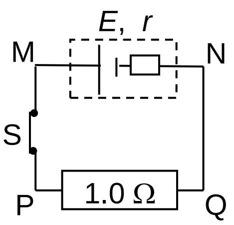
$$\text{Potential difference } V_{MN} = \text{Potential difference } V_{AC}$$
$$= \frac{L_{AC}}{L_{AB}} \times \left(\frac{R_{AB}}{R_{AB} + R} \right) \times E_{\text{driver}}$$

Example 12

A potentiometer consists of a dry cell (e.m.f. of 4.0 V and internal resistance of 0.4 Ω) connected to a 1.0 Ω resistor, and a uniform wire AB of length 100 cm and resistance 3.4 Ω.

The circuit to be investigated consists of a dry cell of e.m.f *E,* connected in series to a 1.0 Ω resistor and a switch S.





(a) Complete the diagram by drawing 2 wires, a jockey and a galvanometer to show how the potentiometer can be used to balance the dry cell.

(b) The balanced length (measured from point A) is 53 cm, when switch S is opened, and 50 cm when S is closed.

Determine the e.m.f. *E* and the terminal potential V_T, when S is closed.

Solution:

(b) Applying the potentiometer equation,

$$V_{AC} = \left(\frac{L_{AC}}{L_{AB}}\right)V_{AB} \qquad \text{where} \quad V_{AB} = \left(\frac{3.4}{0.4+1.0+3.4}\right)\times 4.0$$

$$= \frac{L_{AC}}{100}\times\left(\frac{3.4}{0.4+1.0+3.4}\right)\times 4.0$$

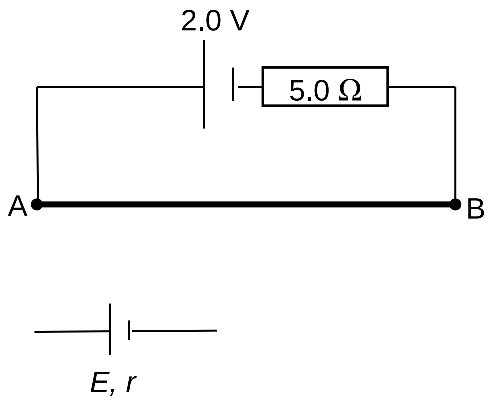
When S is opened, $V_{AC} = \frac{53}{100}\times\left(\frac{3.4}{0.4+1.0+3.4}\right)\times 4.0 = 1.5\ \text{V}$

When S is closed, $V'_{AC} = \frac{50}{100}\times\left(\frac{3.4}{0.4+1.0+3.4}\right)\times 4.0 = 1.4\ \text{V}$

Therefore E = 1.5 V and V_T =1.4 V

Example 13

A potentiometer consists of a 2.0 V dry cell with negligible internal resistance, a 5.0 Ω resistor, and a uniform wire AB of length 1.00 m and resistance 10.0 Ω. The potentiometer is used to measure the e.m.f *E,* and internal resistance *r,* of a dry cell.

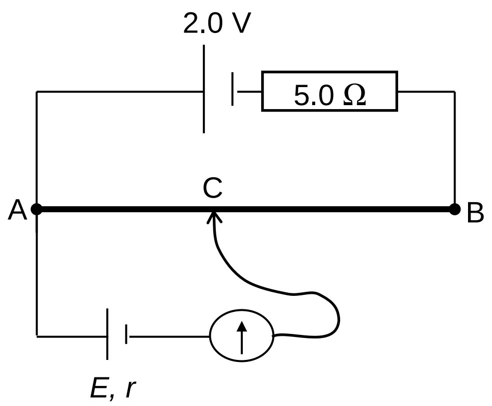


(a) Complete the diagram by drawing 2 wires, a jockey and a galvanometer to show how the potentiometer can be used to balance the dry cell.

(b) If the balanced length (measured from point A) is 23.0 cm, calculate the value of *E*.

Solution:

(a)



(b) Applying the potentiometer equation,

$$V_{AC} = \frac{L_{AC}}{L_{AB}} \times \left(\frac{R_{AB}}{R_{AB} + R} \right) \times E_{driver}$$

$$= \frac{23}{100} \times \left(\frac{10}{10 + 5} \right) \times 2.0 = 0.31 \text{ V}$$

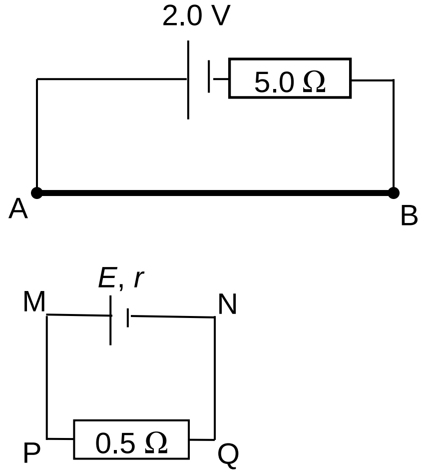
In this setup, no current flows through the external circuit when it is balanced.

Hence, $V_{AC} = E$

Therefore *E* = 0.31 V

(c) The external circuit is modified to include a 0.5 Ω resistor connected to the dry cell in **(a)**.

If the new balance length (measured from point A) is 12.6 cm, calculate *r*, the internal resistance of the dry cell.



Solution :

Applying the potentiometer equation,

$$V_{AC} = \frac{L_{AC}}{L_{AB}} \times \left(\frac{R_{AB}}{R_{AB} + R} \right) \times E$$

$$V_{AC} = \frac{12.6}{100} \times \left(\frac{10}{10+5} \right) \times 2.0 = 0.168 \text{ V}$$

Consider the external circuit,

$$V_{MN} = \left(\frac{0.5}{0.5+r} \right) \times E = \left(\frac{0.5}{0.5+r} \right) \times 0.31$$

When the setup is balanced, $V_{AC} = V_{MN}$,

$$0.168 = \left(\frac{0.5}{0.5+r} \right) \times 0.31$$

$$r = 0.423 \ \Omega$$

Important points to note when using the potentiometer:
When connecting the potentiometer to the external circuit,

(i) the higher potential ends of the e.m.f. sources of both the potentiometer and the external circuit must be jointed to the same end of the potentiometer wire.

(ii) the driver cell e.m.f. E_{driver} must be larger than the e.m.f. source of the external circuit.

(iii) to find the balanced point, the jockey is lightly tapped (and not slide or pressed) along the potentiometer wire. This is to ensure that the wire's circular cross sectional area is kept uniform.

(iv) If the driver e.m.f. is much larger than the potential difference to be measured, for example:

E_{driver} = 5 V but V_{AC} = 5 mV,

Then the **balanced length** L_{AC} will be very small :

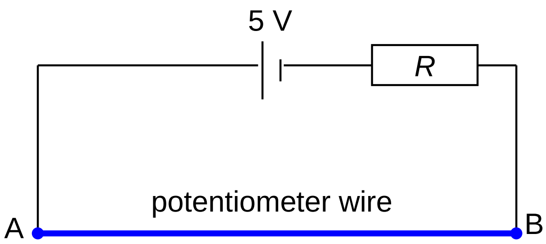
$$\frac{V_{AC}}{V_{AB}} = \frac{L_{AC}}{L_{AB}}$$

$$\frac{5\times10^{-3}}{5} = \frac{L_{AC}}{100\ \text{cm}}$$

$$L_{AC} = 0.1\ \text{cm}$$

The percentage error in the measurement of L_{AC} $= \dfrac{0.2\ \text{cm}}{0.1\ \text{cm}}\times100\% = 200\%$

To reduce this percentage error, a fixed resistor of high resistance can be connected in series so that the potential difference across wire AB is greatly reduced.



For example, if R = 100 Ω, R_{AB} = 1 Ω, then $V_{AB} = \left(\dfrac{1}{1+100}\right)\times5.0 = 0.05\ \text{V}$

The balanced length will be shorter: $\dfrac{V_{AC}}{V_{AB}} = \dfrac{L'_{AC}}{L_{AB}}$

$$\frac{5\times10^{-3}}{0.05} = \frac{L'_{AC}}{100\ \text{cm}}$$

$$L'_{AC} = 10\ \text{cm}$$

The percentage error in the balanced length L_{AC} is reduced to, $\dfrac{0.2\ \text{cm}}{10\ \text{cm}}\times100\%$ = 2%

4.2 Applications of potentiometer: To measure very small e.m.f.

A thermocouple is a device for measuring temperature. Fig. 12 shows the 2 ends of the thermocouple placed in two beakers at different temperatures. A very small e.m.f. of a few millivolts is produced in the thermocouple.

To measure the small e.m.f. using a potentiometer, a large resistance *R* is connected in series with the potentiometer wire.

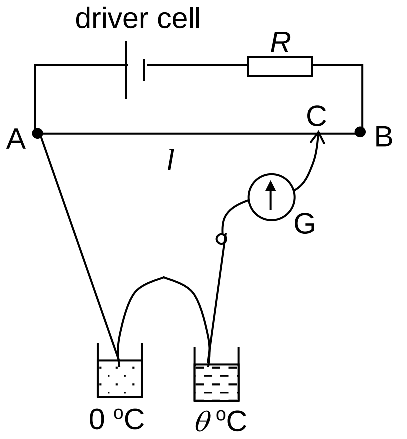
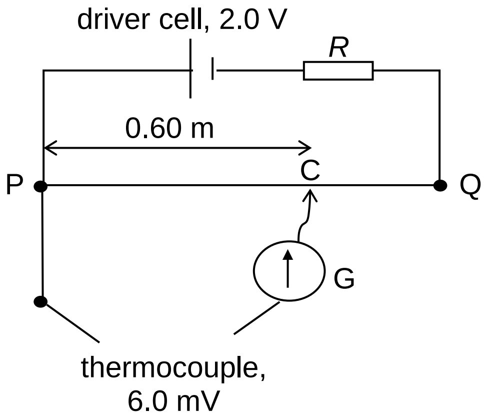


Fig. 12 Potentiometer connected to a thermocouple

Example 14

The diagram shows a simple potentiometer circuit to measure a small e.m.f. produced by a thermocouple. The meter wire PQ has a resistance of 5.0 Ω and length of 1.00 m. The driver cell has an e.m.f. of 2.0 V. If the balance point is obtained 0.60 m along PQ when measuring an e.m.f. of 6.0 mV, calculate the value of resistance *R*.



Solution:

Applying the potentiometer equation,

$$V_{\text{thermocouple}} = \frac{L_{PC}}{L_{PQ}} \times \left(\frac{R_{PQ}}{R_{PQ} + R} \right) \times E_{\text{driver}}$$

$$6.0 \times 10^{-3} = \frac{60}{100} \times \left(\frac{5.0}{5.0 + R} \right) \times 2.0$$

$$R = 995\ \Omega$$

4.3 Applications of potentiometer: To compare and measure resistance

Potentiometer can be used to find the values of unknown resistors. This is done by comparing the unknown resistor to a resistor of known value.

In Fig. 13 and 14, two resistors of resistance R_1 and R_2 are connected in series with a cell of e.m.f. E.

In Fig. 13, the potentiometer is connected across resistor R_1. The balanced length is recorded as l_1.

Applying the potentiometer equation, $V_{\text{AC1}} = \frac{l_1}{L} \times \left(\frac{R_{\text{AB}}}{R_{\text{AB}} + R} \right) \times E_{\text{driver}}$ ------ (1)

In Fig. 14, the potentiometer is connected across resistor R_2. The balanced length is recorded as l_2.

Applying the potentiometer equation, $V_{\text{AC2}} = \frac{l_2}{L} \times \left(\frac{R_{\text{AB}}}{R_{\text{AB}} + R} \right) \times E_{\text{driver}}$ ------ (2)

$\frac{(1)}{(2)}$: we get a ratio of $\frac{V_{\text{AC1}}}{V_{\text{AC2}}} = \frac{l_1}{l_2}$

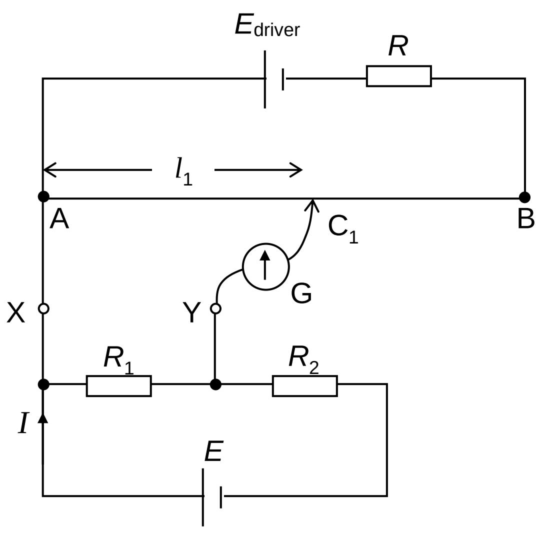


Fig. 13 (XY connected across R_1)

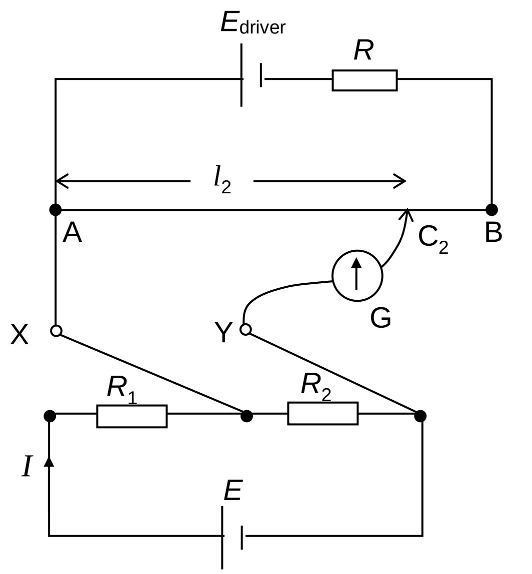


Fig. 14 (XY connected across R_2)

The same current I flows through resistance R_1 and R_2 since they are connected in series. Hence applying $V = IR$ to R_1 and R_2, we get:

For R_1 : $V_{\text{AC1}} = IR_1$ ------ (3)
For R_2 : $V_{\text{AC2}} = IR_2$ ------ (4)

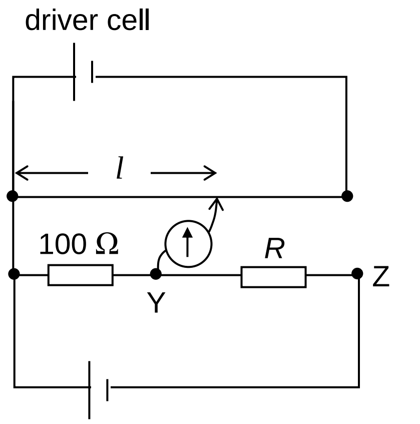
$\frac{(3)}{(4)}$: we get a ratio of $\frac{V_{\text{AC1}}}{V_{\text{AC2}}} = \frac{R_1}{R_2}$

Combining the two ratio $\frac{V_{\text{AC1}}}{V_{\text{AC2}}} = \frac{l_1}{l_2}$ and $\frac{V_{\text{AC1}}}{V_{\text{AC2}}} = \frac{R_1}{R_2}$, we get $\boxed{\frac{l_1}{l_2} = \frac{R_1}{R_2}}$

Example 15

The figure below shows a circuit used to compare the resistance R of an unknown resistor with a 100 Ω standard resistor. A galvanometer is used to obtain the balance length, l. The balance lengths l are 400 mm and 588 mm when the galvanometer is connected to Y and to Z respectively. The length of the wire is 1.00 m.

Calculate the value of resistance R.



Solution:

Using the concept $\frac{l_Y}{l_Z} = \frac{R_Y}{R_Z}$,

When galvanometer is connected to Y, l_Y = 400 mm and R_Y = 100 Ω,

When galvanometer is connected to Z, l_Z = 588 mm and R_Z = 100 Ω + R,

Taking ratio, we get $\frac{400}{588} = \frac{100}{100+R}$

Hence, $R = 47\ \Omega$

4.4 Applications of potentiometer: To measure internal resistance of a cell

Fig. 15 and Fig. 16 show an external circuit with a cell of e.m.f. E and internal resistance r connected in series with a resistor R. To measure the internal resistance, two balanced lengths are obtained, one with the switch open and one with the switch closed.

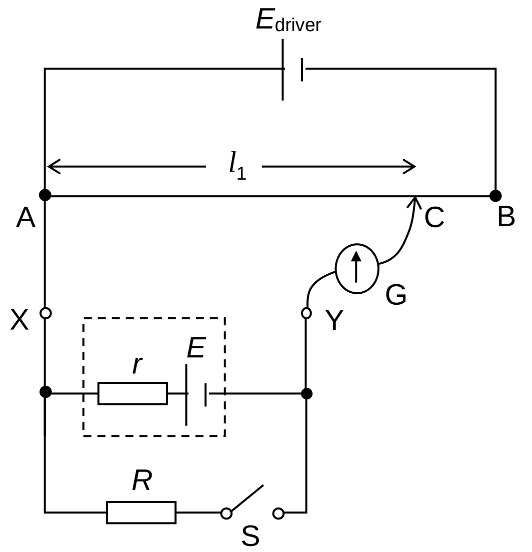


Fig. 15 Open circuit

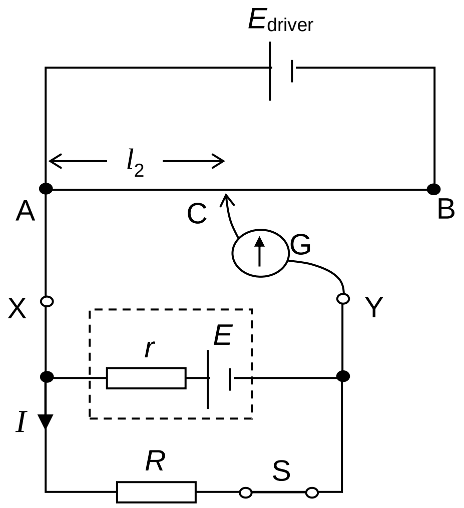


Fig. 16 Closed circuit

For the open circuit in Fig. 15, $E = \frac{l_1}{l_{AB}} \times E_{\text{driver}}$ -------- (1)

For the closed circuit in Fig. 16, $\frac{R}{R+r} E = \frac{l_2}{l_{AB}} \times E_{\text{driver}}$ -------- (2)

By taking $\frac{(1)}{(2)}$: $\frac{l_1}{l_2} = \frac{R+r}{R}$

Re-write the equation $\frac{l_1}{l_2} = \frac{R+r}{R}$ into a linear equation: $\frac{l_1}{l_2} = 1 + \frac{r}{R}$

$$\frac{l_1}{l_2} = r\left(\frac{1}{R}\right) + 1$$

Vary R and repeat the experiment to obtain six sets of readings for l_1, l_2 and $\frac{l_1}{l_2}$.

Plot a graph of $\frac{l_1}{l_2}$ against $\frac{1}{R}$. The straight line graph gives a gradient equal to r.