Chapter 1 Quadratic Functions

Name: ______________________________ (   ) Class: 3_______ Date: ________________________

1.1 Maximum and Minimum Values

We use the notation $y = \mathrm{f}(x)$ to mean that y is a _____________ of $\mathrm{f}(x)$.

To find all possible real values of $\mathrm{f}(x) = ax^2 + bx + c$, we first use the method of completing the square to express the function in the form ______________, where ____ and ____ are constants.

Activity One:

Sketch the graph of $\boldsymbol{y = mx + c}$ and the graph of $\boldsymbol{y = ax^2 + bx + c}$.

$y = mx + c$

I observe that the graph $y = mx + c$ is a

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$y = ax^2 + bx + c$

I observe that the graph $y = ax^2 + bx + c$ is a

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Example 1

1. Given $\mathrm{f}(x)^2 = 2x^2 - 4$, find
(i) its value when $x = 0$,
(ii) all possible real values of $\mathrm{f}(x)$.

Practice 1

1. Given $f(x)^2 = 2x^2 + 4$, find
(i) $f(-1)$,
(ii) all possible real values of $f(x)$.

Example 2

1. Express $f(x) = x^2 + 6x - 4$ in the form $(x - h)^2 + k$, where h and k are constants. Hence find
(i) all possible real values of $f(x)$,
(ii) the minimum value of $f(x)$,
(iii) the value of x at which the minimum value of $f(x)$ occurs.

Practice 2

1. Given $f(x) = x^2 + 3x - 6$, find
(i) all possible real values of $f(x)$,
(ii) the minimum value of $f(x)$,
(iii) the value of x at which the minimum value of $f(x)$ occurs.

Example 3

1. Express each of the following functions in the form $a(x - h)^2 + k$, where a, h and k are constants, and $a \neq 0$. Hence find the minimum or maximum value of $f(x)$. Justify your answers.

(a) $f(x) = 3x^2 - 6x + 2$

(b) $f(x) = -2x^2 - 3x + 3$

Practice 3

1. Express each of the following functions in completed square form. Hence find the maximum or minimum value of $f(x)$. Justify your answers.

(a) $f(x) = 2x^2 - 3x - 1$

(b) $f(x) = -3x^2 + 4x - 2$

Example 4

1. Show that $x^2 - 2x + 6 \geq 5$ for all real values of x.

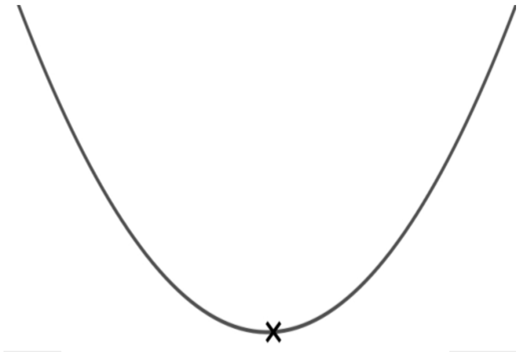
Practice 4

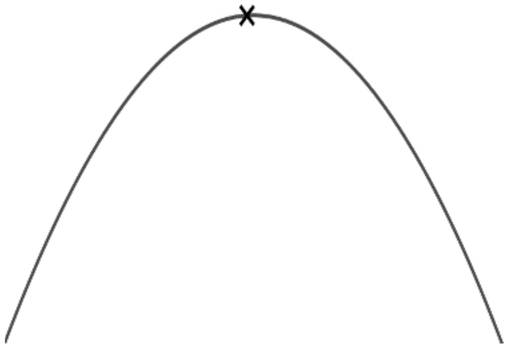
1. Show that $-x^2 - 4x - 10$ is negative for all real values of x.

1.2 Graphical Representation of Quadratic Functions

The graph of a quadratic function changes direction at a point called the turning point or the vertex of the graph.

Turning Point





Turning Point

The general form $ax^2 + bx + c$ is not very useful in sketching quadratic graphs, as it does not show features like intercepts and turning points. As such, we would convert quadratic functions from the general form to either the factorised form or the completed square form.

- The factorised form $a(x - p)(x - q)$, where p and q are the x-intercepts
- The completed square form $a(x - h) + k$, where (h, k) are the coordinates of the turning point.

Discriminant ($b^2 - 4ac$)	**Nature of Roots of $ax^2 + bx + c = 0$**	**Numbers of x-intercept**	**Graph of $y = ax^2 + bx + c$**
> 0	2 Real and Distinct Roots	2	
$= 0$	2 Real and Equal (Repeated) Roots	1	
< 0	No Real Roots	0	

Example 5

1. (i) Find the coordinates of the turning point and the y-intercept on the graph of $y = -2x^2 + 4x + 4$.
(ii) Find the exact values of x for which $y = 0$.

Practice 5

1. (i) Find the coordinates of the turning point and the y-intercept on the graph of $y = 4x^2 - 12x + 9$.
(ii) Solve the equation $y = 0$.

Example 6

1. (i) Find the x-intercepts and the coordinates of the turning point on the graph of $y = 2x^2 - 6x - 8$.
(ii) Hence explain why $2x^2 - 6x - 8 = 0$ has two distinct real roots.

Practice 6

1. (i) Find the x-intercepts and the coordinates of the turning point on the graph of $y = –2x^2 + 2x + 4$.
(ii) Hence explain why $–2x^2 + 2x + 4= 0$ has two distinct real roots.

Example 7

1. Use the discriminant to determine the number of x-intercepts for the graph of each given quadratic function.
(a) $y = x^2 + 6x + 9$
(b) $y = (2 – x)^2 + 1$

Practice 7

1. Use the discriminant to determine the number of x-intercepts for the graph of each given quadratic function.
(a) $y = x^2 + 2x – 6$
(b) $y = (1 – x)^2 + 2$

Example 8

1. Find the non-zero values of p for which the graph $y = px^2 - 2x + p$ touches the x-axis.

Practice 8

1. Find the non-zero values of p for which the graph $y = 3x^2 + 12 - px$ meets only the x-axis at one point only.

Example 9

1. Explain why the graph of $y = x^2 + (1 - p)x - p$ intersects the x-axis.

Practice 9

1. Explain why the graph of $y = x^2 - 2x + 2 - p$, where $p > 1$, intersects the x-axis at two points.

Example 10

1. (a) Show that the graph of $y = x^2 - 2kx + (k^2 + 1)$ is entirely above the x-axis.
(b) Show that $-2x^2 + 4kx - 2k^2 - 1$ is negative for all real values of x.

Practice 10

1. (a) Show that the graph of $y = -x^2 + 2kx - (k^2 + 1)$ is entirely above the x-axis.
(b) Show that $\frac{1}{4}x^2 + kx + k^2 + 1$ is always positive.

1.3 Applications of Quadratic Functions

Example 11

1. **Projectile Motion.** A ball was launched from a slingshot. Its height, h m, above the ground is given by $h = -2x^2 + 8x + 1$, where x m is the horizontal distance from the slingshot.
   (i) Find the height of the ball above the ground when it just left the slingshot.
   (ii) Find the greatest height of the ball after it was launched from the slingshot.
   (iii) If a toy is 3 m horizontally from the slingshot and 7 m above the ground, justify if the ball will hit the toy directly.

Practice 11

1. **Projectile Motion.** A projectile was launched from a catapult to smash a defence structure on a fort. Its height, h m, above the ground is given by $h = -\frac{1}{2500}x^2 + \frac{2}{25}x + 3$, where x m is the horizontal distance from the catapult.
   (i) Find the height of the projectile when it just left the catapult.
   (ii) Find the greatest height of the projectile after it was launched from the catapult.
   (iii) If the defence structure is 150 m horizontally from the catapult and 5 m above the ground, justify if the projectile will smash the structure.

End of Chapter 1