A3: SURDS

- Four operations on surds, including rationalising the denominator
- Solving equations involving surds

1. ABCD is a trapezium where AB is parallel to DC. The area of the trapezium is $12 + 11\sqrt{3}\ cm^2$. $AB = 8 - \sqrt{48}$ cm and $DC = 12 - \sqrt{12}$ cm. Find the perpendicular distance between AB and DC, leaving your answer in the form $a + b\sqrt{3}$, where a and b are integers.	[5]
2. A rectangular block has a square base of side $\left(\sqrt{10} + \sqrt{2}\right)$ cm and a height of h cm. The volume of the rectangular block is $\left(52 + 28\sqrt{5}\right)\ cm^3$. **Without using a calculator**, find the height of the rectangular block, in cm. Give your answer in the form $\left(a + b\sqrt{5}\right)$ cm where a and b are integers.	[5]
3. It is given that $\sqrt{p - q\sqrt{5}} = \frac{12}{3+\sqrt{5}}$, where p and q are integers. Without using a calculator, find the value of p and q.	[6]
4. Simplify $7\sqrt{2} - 3\sqrt{8} + \sqrt{32}$.	[2]
5. A rectangle has an area of $11 - \sqrt{7}\ cm^2$. Given that the length is $3 + \sqrt{7}$ cm, find the breadth of the rectangle and express the breadth in the form $a - b\sqrt{7}$, where a and b are integers.	[4]
6. It is given that $2\sqrt{3}(x + 1) = 2 - x$. Find x in the form $a\sqrt{3} - b$, where a and b are real numbers.	[4]
7. Show $\frac{(2\sqrt{5}-3)(6+\sqrt{5})}{3+\sqrt{5}}$ can be written in the form $a + b\sqrt{5}$, where a and b are rational numbers.	[4]
8. Solve the equation $\frac{x}{3} - \frac{4}{x} = \sqrt{3}$, where $x \neq 0$, giving your answer in its simplest surd form.	[4]

9. (a) Without using a calculator, express $\left(2\sqrt{3}+\sqrt{2}\right)^2$ in the form $a+b\sqrt{6}$, where a and b are integers.	[2]
9. (b) Hence, or otherwise, express $\frac{\left(2\sqrt{3}+\sqrt{2}\right)^2}{3+\sqrt{6}}$ in the form $p+q\sqrt{6}$, where p and q are rational numbers.	[3]
10. It is given that $\sqrt{3}(x+3)=x+17$. Find x in the form $a+b\sqrt{3}$, where where a and b are integers.	[4]
11. Given that $\left(a-3\sqrt{5}\right)\left(2+\sqrt{5}\right)=b-2\sqrt{5}$, find the values of the integers a and b.	[3]
12. Solve $\sqrt{x+7}-3\sqrt{2}=1$, leaving your answer in the form $m+n\sqrt{2}$, where m and n are integers.	[3]
13. Simplify $\sqrt{108}-\frac{12}{\sqrt{3}}$, giving your answer in the form $k\sqrt{3}$, where k is an integer.	[2]
14. Without using a calculator, solve the equation $3x-\sqrt{2}=2\left(1+x\sqrt{2}\right)$.	[5]

A3: SURDS (MARKING SCHEME)

1. ABCD is a trapezium where AB is parallel to DC. The area of the trapezium is $12 + 11\sqrt{3}\ cm^2$. $AB = 8 - \sqrt{48}$ cm and $DC = 12 - \sqrt{12}$ cm. Find the perpendicular distance between AB and DC, leaving your answer in the form $a + b\sqrt{3}$, where a and b are integers. Area of trapezium $= \frac{1}{2}(a + b)h$, where a and b are the length of parallel sides $12 + 11\sqrt{3} = \frac{1}{2}\left(8 - \sqrt{48} + 12 - \sqrt{12}\right)h$ $24 + 22\sqrt{3} = \left(20 - \sqrt{4 \times 12} - \sqrt{12}\right)h$ ★ $\sqrt{48} = \sqrt{4 \times 12}$ $24 + 22\sqrt{3} = \left(20 - 2\sqrt{12} - \sqrt{12}\right)h$ ★ $\sqrt{2^2 \times 12} = 2\sqrt{12}$ $24 + 22\sqrt{3} = \left(20 - 3\sqrt{12}\right)h$ $24 + 22\sqrt{3} = \left(20 - 3\sqrt{4 \times 3}\right)h$ $24 + 22\sqrt{3} = \left(20 - 6\sqrt{3}\right)h$ ★ $3\sqrt{2^2 \times 3} = 3(2)\sqrt{3}$ $\frac{24+22\sqrt{3}}{20-6\sqrt{3}} = h$ $h = \frac{24+22\sqrt{3}}{20-6\sqrt{3}} \times \frac{20+6\sqrt{3}}{20+6\sqrt{3}}$ $h = \frac{12+11\sqrt{3}}{10-3\sqrt{3}} \times \frac{10+3\sqrt{3}}{10+3\sqrt{3}}$ ★ Rationalise the denominator $h = \frac{120+36\sqrt{3}+110\sqrt{3}+99}{100-27}$ $h = \frac{219+146\sqrt{3}}{73}$ $h = 3 + 2\sqrt{3}$ $h = 3 + 2\sqrt{3}$ cm	[5]

2. A rectangular block has a square base of side $\left(\sqrt{10}+\sqrt{2}\right)$ cm and a height of h cm. The volume of the rectangular block is $\left(52 + 28\sqrt{5}\right) cm^3$. **Without using a calculator**, find the height of the rectangular block, in cm. Give your answer in the form $\left(a + b\sqrt{5}\right)$ cm where a and b are integers. Volume of rectangular block $= lbh$ $\left(52 + 28\sqrt{5}\right) = \left(\sqrt{10}+\sqrt{2}\right)^2 h$ $h = \frac{52+28\sqrt{5}}{\left(\sqrt{10}+\sqrt{2}\right)^2}$ $h = \frac{52+28\sqrt{5}}{10+2\sqrt{20}+2}$ $h = \frac{52+28\sqrt{5}}{12+2\sqrt{4\times5}}$ ★ $2\sqrt{20} = 2\sqrt{4 \times 5}$ $h = \frac{52+28\sqrt{5}}{12+4\sqrt{5}}$ $h = \frac{52+28\sqrt{5}}{12+4\sqrt{5}} \times \frac{12-4\sqrt{5}}{12-4\sqrt{5}}$ ★ Rationalise the denominator $h = \frac{624-208\sqrt{5}+336\sqrt{5}-112(5)}{144-16(5)}$ $h = \frac{64+128\sqrt{5}}{64}$ $h = 1 + 2\sqrt{5}$ $h = 1 + 2\sqrt{5}$ cm	[5]

3. It is given that $\sqrt{p - q\sqrt{5}} = \frac{12}{3+\sqrt{5}}$, where p and q are integers. Without using a calculator, find the value of p and q. $\sqrt{p - q\sqrt{5}} = \frac{12}{3+\sqrt{5}}$ $p - q\sqrt{5} = \left(\frac{12}{3+\sqrt{5}}\right)^2$ $p - q\sqrt{5} = \frac{144}{9+2(3)(\sqrt{5})+(\sqrt{5})^2}$ $p - q\sqrt{5} = \frac{144}{9+6\sqrt{5}+5}$ $p - q\sqrt{5} = \frac{144}{14+6\sqrt{5}}$ $p - q\sqrt{5} = \frac{72}{7+3\sqrt{5}}$ $p - q\sqrt{5} = \frac{72}{7+3\sqrt{5}} \times \frac{7-3\sqrt{5}}{7-3\sqrt{5}}$ ★ Rationalise the denominator $p - q\sqrt{5} = \frac{504-216\sqrt{5}}{49-9(5)}$ $p - q\sqrt{5} = \frac{504-216\sqrt{5}}{4}$ $p - q\sqrt{5} = 126 - 54\sqrt{5}$ $p = 126,\ q = 54$	[6]
4. Simplify $7\sqrt{2} - 3\sqrt{8} + \sqrt{32}$. $7\sqrt{2} - 3\sqrt{8} + \sqrt{32}$ $= 7\sqrt{2} - 3\sqrt{4 \times 2} + \sqrt{2 \times 16}$ $= 7\sqrt{2} - 6\sqrt{2} + 4\sqrt{2}$ $= 5\sqrt{2}$	[2]

5. A rectangle has an area of $11 - \sqrt{7}\ cm^2$. Given that the length is $3 + \sqrt{7}$ cm, find the breadth of the rectangle and express the breadth in the form $a - b\sqrt{7}$, where a and b are integers. Area of rectangle $= lb$ $11 - \sqrt{7} = \left(3 + \sqrt{7}\right)b$ $\frac{11-\sqrt{7}}{3+\sqrt{7}} = b$ $b = \frac{11-\sqrt{7}}{3+\sqrt{7}} \times \frac{3-\sqrt{7}}{3-\sqrt{7}}$ ★ Rationalise the denominator $b = \frac{33-11\sqrt{7}-3\sqrt{7}+7}{9-7}$ $b = \frac{40-14\sqrt{7}}{2}$ $b = 20 - 7\sqrt{7}$ $b = 20 - 7\sqrt{7}$ cm	[4]
6. It is given that $2\sqrt{3}(x + 1) = 2 - x$. Find x in the form $a\sqrt{3} - b$, where a and b are real numbers. $2\sqrt{3}(x + 1) = 2 - x$ $2\sqrt{3}x + 2\sqrt{3} = 2 - x$ $2\sqrt{3}x + x = 2 - 2\sqrt{3}$ $x\left(2\sqrt{3} + 1\right) = 2 - 2\sqrt{3}$ $x = \frac{2-2\sqrt{3}}{2\sqrt{3}+1}$ $x = \frac{2-2\sqrt{3}}{2\sqrt{3}+1} \times \frac{2\sqrt{3}-1}{2\sqrt{3}-1}$ ★ Rationalise the denominator $x = \frac{4\sqrt{3}-2-4(3)+2\sqrt{3}}{4(3)-1}$ $x = \frac{6\sqrt{3}-14}{11}$ $x = \frac{6}{11}\sqrt{3} - \frac{14}{11}$	[4]

7. Show $\frac{(2\sqrt{5}-3)(6+\sqrt{5})}{3+\sqrt{5}}$ can be written in the form $a + b\sqrt{5}$, where a and b are rational numbers. $\frac{(2\sqrt{5}-3)(6+\sqrt{5})}{3+\sqrt{5}}$ $= \frac{12\sqrt{5}+2(5)-18-3\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$ ★ Rationalise the denominator $= \frac{9\sqrt{5}-8}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$ $= \frac{27\sqrt{5}-9(5)-24+8\sqrt{5}}{9-5}$ $= \frac{35\sqrt{5}-69}{4}$ $= \frac{35\sqrt{5}}{4} - \frac{69}{4}$ $=- \frac{69}{4} + \frac{35\sqrt{5}}{4}$	[4]
8. Solve the equation $\frac{x}{3} - \frac{4}{x} = \sqrt{3}$, where $x \neq 0$, giving your answer in its simplest surd form. $\frac{x^2-12}{3x} = \sqrt{3}$ $x^2 - 12 = 3\sqrt{3}x$ $x^2 - 3\sqrt{3}x - 12 = 0$ $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x = \frac{-(-3\sqrt{3})\pm\sqrt{(3\sqrt{3})^2-4(1)(-12)}}{2(1)}$ $x = \frac{3\sqrt{3}\pm\sqrt{27+48}}{2}$ $x = \frac{3\sqrt{3}\pm\sqrt{75}}{2}$ $x = \frac{3\sqrt{3}\pm\sqrt{25\times3}}{2}$ $x = \frac{3\sqrt{3}\pm5\sqrt{3}}{2}$ $x = 4\sqrt{3}$ ot $x =- \sqrt{3}$	[4]

9. (a) Without using a calculator, express $\left(2\sqrt{3}+\sqrt{2}\right)^2$ in the form $a+b\sqrt{6}$, where a and b are integers. $\left(2\sqrt{3}+\sqrt{2}\right)^2$ $=4(3)+2\left(2\sqrt{3}\right)\left(\sqrt{2}\right)+2$ $=14+4\sqrt{6}$	[2]
9. (b) Hence, or otherwise, express $\frac{\left(2\sqrt{3}+\sqrt{2}\right)^2}{3+\sqrt{6}}$ in the form $p+q\sqrt{6}$, where p and q are rational numbers. $\frac{14+4\sqrt{6}}{3+\sqrt{6}}\times\frac{3-\sqrt{6}}{3-\sqrt{6}}$ ★ Rationalise the denominator $=\frac{42-14\sqrt{6}+12\sqrt{6}-4(6)}{9-6}$ $=\frac{18-2\sqrt{6}}{3}$ $=6-\frac{2}{3}\sqrt{6}$	[3]
10. It is given that $\sqrt{3}(x+3)=x+17$. Find x in the form $a+b\sqrt{3}$, where where a and b are integers. $\sqrt{3}(x+3)=x+17$ $\sqrt{3}x+3\sqrt{3}=x+17$ $\sqrt{3}x-x=17-3\sqrt{3}$ $x\left(\sqrt{3}-1\right)=17-3\sqrt{3}$ $x=\frac{17-3\sqrt{3}}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$ ★ Rationalise the denominator $x=\frac{17\sqrt{3}+17-3(3)-3\sqrt{3}}{3-1}$ $x=\frac{14\sqrt{3}+8}{2}$ $x=7\sqrt{3}+4$ $x=4+7\sqrt{3}$	[4]

11. Given that $\left(a - 3\sqrt{5}\right)\left(2 + \sqrt{5}\right) = b - 2\sqrt{5}$, find the values of the integers a and b. $\left(a - 3\sqrt{5}\right)\left(2 + \sqrt{5}\right) = b - 2\sqrt{5}$ $2a + a\sqrt{5} - 6\sqrt{5} - 3(5) = 2a - 15 + a\sqrt{5} - 6\sqrt{5}$ By comparison, $a\sqrt{5} - 6\sqrt{5} =- 2\sqrt{5}$ $a\sqrt{5} = 4\sqrt{5}$ $a = 4$ $b = 2(4) - 15$ $b =- 7$	[3]
12. Solve $\sqrt{x + 7} - 3\sqrt{2} = 1$, leaving your answer in the form $m + n\sqrt{2}$, where m and n are integers. $\sqrt{x + 7} - 3\sqrt{2} = 1$ $\sqrt{x + 7} = 1 + 3\sqrt{2}$ $x + 7 = \left(1 + 3\sqrt{2}\right)^2$ $x + 7 = 1 + 2(1)\left(3\sqrt{2}\right) + 9(2)$ $x + 7 = 1 + 6\sqrt{2} + 18$ $x = 19 - 7 + 6\sqrt{2}$ $x = 12 + 6\sqrt{2}$	[3]

13. Simplify $\sqrt{108} - \frac{12}{\sqrt{3}}$, giving your answer in the form $k\sqrt{3}$, where k is an integer. $\sqrt{108} - \frac{12}{\sqrt{3}}$ $= \sqrt{3 \times 36} - \frac{12}{\sqrt{3}}$ $= 6\sqrt{3} - \frac{12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$ ★ Rationalise the denominator $=6\sqrt{3} - \frac{12\sqrt{3}}{3}$ $= \frac{18\sqrt{3}-12\sqrt{3}}{3}$ $= \frac{6\sqrt{3}}{3}$ $= 2\sqrt{3}$	[2]
14. Without using a calculator, solve the equation $3x - \sqrt{2} = 2(1 + x\sqrt{2})$. $3x - \sqrt{2} = 2(1 + x\sqrt{2})$ $3x - \sqrt{2} = 2 + 2\sqrt{2}x$ $3x - 2\sqrt{2}x = 2 + \sqrt{2}$ $x(3 - 2\sqrt{2}) = 2 + \sqrt{2}$ $x = \frac{2+\sqrt{2}}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}}$ ★ Rationalise the denominator $x = \frac{6+4\sqrt{2}+3\sqrt{2}+2(2)}{9-4(2)}$ $x = 10 + 7\sqrt{2}$	[5]