Name:______________________________________ Class:_____________

JURONG PIONEER JUNIOR COLLEGE

JC2 Preliminary Examination 2024

FURTHER MATHEMATICS 9649/02

Higher 2

16 September 2024

Paper 2

3 hours

Additional materials: 12-page Answer Booklet
(additional 4-page Answer Booklet(s), if applicable)
List of Formulae (MF 26)

READ THESE INSTRUCTIONS FIRST

Write your name and civics class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **8** printed pages.

Section A: Pure Mathematics [50 marks]

1 **(a)** By considering $\int_0^{\frac{\pi}{5}} \cos x \,\mathrm{d}x$, use Simpson's rule for two strips to show that $\cos\frac{\pi}{10}$ is approximately a root of

$$\left[\left(\frac{\pi}{30}\right)^2+1\right]x^2+\left(\frac{\pi}{15}\right)^2 x+\left[\left(\frac{\pi}{15}\right)^2-1\right]=0.$$ **[6]**

(b) Hence find an approximation to $\cos\frac{\pi}{10}$, giving your answer to 4 decimal places. **[1]**

2 A curve C has polar equation $r=\dfrac{4}{k+3\sin\theta}$, where $0 \le \theta < 2\pi$ and k is a constant, $k \ge 0$.

(i) What can be said about the value of k in each of the cases

- C is a parabola,
- C is an ellipse,
- C is a hyperbola. **[3]**

(ii) Given that $k=2$, find the equation of this curve in a standard cartesian form. **[4]**

(iii) Determine the eccentricity of the curve and the coordinates of the two foci. **[2]**

3 **(a)** **(i)** The function f is such that f(a)f(b) < 0 where $a < b$. A student concludes that the equation f(x) = 0 has at least one root in the interval (a, b). Draw a sketch to illustrate why the student could be wrong. **[1]**

(ii) The equation $\operatorname{cosec}^2 x - 3\ln x = 0$ has a root α in the interval [3, 4]. A student uses linear interpolation once on the interval to find an approximation to α. Find the approximation to α given by this method and comment on the suitability of the method in this case. **[3]**

(b) The equation $\operatorname{cosec}^2 x - 3\ln x = 0$ also has a root β in the interval [1.0, 2.4]. A student finds a sequence of approximations $\{x_1, x_2, x_3, ...\}$ to β using the recurrence relation

$$x_0 = 2 \text{ and } x_n = \mathrm{G}(x_{n-1}) \text{ for } n \geq 1.$$

(i) In his first attempt, the student uses $\mathrm{G}(x) = \sin^{-1}\sqrt{\frac{1}{3\ln x}}$. Calculate x_1 and explain why the student will fail when attempting to calculate x_2. **[2]**

(ii) In his second attempt, the student uses $\mathrm{G}(x) = \mathrm{e}^{\frac{1}{3}\operatorname{cosec}^2 x}$. Find the value of β to 3 decimal places, and demonstrate how the student could verify its correctness. **[3]**

4 **(a)** **(i)** The complex number z is given by $z = \cos\theta + \mathrm{i}\sin\theta$, where $0 \leq \theta \leq \frac{\pi}{2}$, show that $\cos\theta = \frac{1}{2}\left(z + z^{-1}\right)$. **[2]**

(ii) Prove that

$$\cos^n \theta = \frac{1}{2^n}\sum_{k=0}^{n}\binom{n}{k}\cos(n-2k)\theta,$$

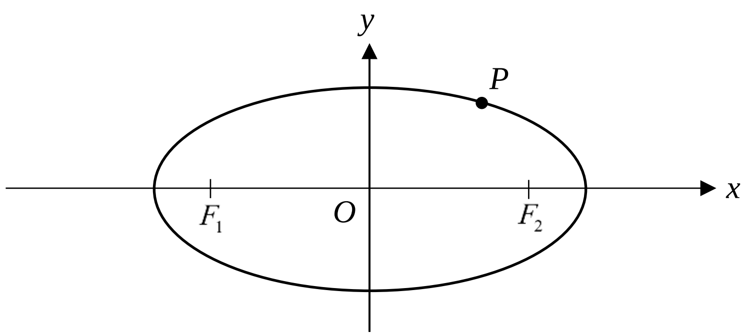
where n is an integer. **[5]**

(iii) Hence find the exact value of $\int_0^{\frac{\pi}{2}} \cos^3\theta \,\mathrm{d}\theta$. **[3]**

(b) Illustrate in an Argand diagram, the sets of points z for which $z = 2 + \lambda(2+\mathrm{i})$, where λ is a real parameter. **[2]**

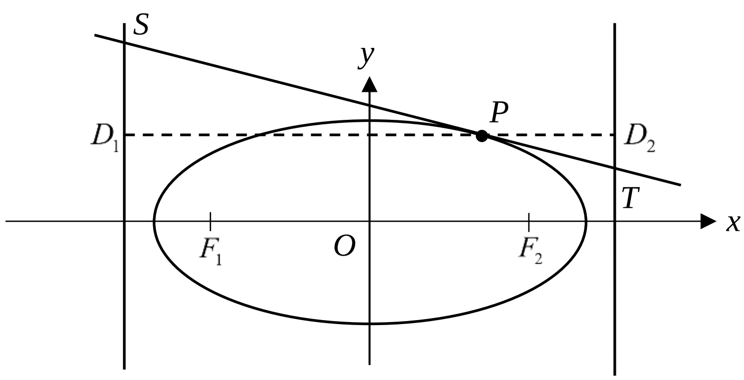
[Turn over

5 A diagram below shows an ellipse C with equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, where $0<b<a$, with foci F_1 and F_2 and eccentricity e. The point $P(x_0, y_0)$ lies on C.



(i) Show that the equation of the tangent at P is $\frac{x_0 x}{a^2}+\frac{y_0 y}{b^2}=1$. **[3]**

The tangent at P meets the directrices at S and T and points D_1 and D_2 are foot of perpendiculars from P to the directrices of C.



Given that $\angle SPD_1 = \angle TPD_2 = \phi$.

(ii) Show that $\angle PF_2T = 90^{\circ}$. **[5]**

(iii) Show that $\frac{PF_2}{PT} = e\cos\phi$ and $\frac{PF_1}{PS}=\frac{PF_2}{PT}$. **[3]**

(iv) Hence prove that $\angle SPF_1 = \angle TPF_2$. **[2]**

Section B: Statistics [50 marks]

6 A track and field coach is interested in determining if his new training programme will improve his athletes' 400 meter sprint time and wishes to conduct an analysis. A random sample of 10 athletes is selected. Each athlete is assessed and their sprint times both before and after his new training programme are recorded. The sprint times, in seconds, are given in the table.

Athlete	A	B	C	D	E	F	G	H	I	J
Before training	86	84	88	90	92	77	89	91	90	86
After training	80	80	78	79	92	82	88	84	92	83

(a) Explain why it might not be appropriate to use a test based on the *t*-distribution in this situation. **[1]**

(b) Carry out a suitable Wilcoxon test, using a 5% significance level, to investigate whether or not the new training programme appears to be effective. **[5]**

7 The number of delays on a railway line occurring in a randomly chosen month is denoted by D.

(a) State two assumptions needed for D to be well modelled by a Poisson distribution. **[2]**

Now assume that D can be well modelled by the distribution Po(λ).

(b) Find in terms of λ, the probability that D takes one of the values 0, 2, 4 or 6 and show that the probability that D is even is $\frac{1}{2}\left(1+e^{-2\lambda}\right)$. **[5]**

8 A mall manager investigates whether the time, T hours, spent by customers shopping in the mall is greater during the holiday season than the non-holiday season. Data from a random sample of 200 customers were collected during both seasons. The results are recorded in the following table.

	n	$\sum t$	$\sum t^2$
Holiday Season	100	341	1563
Non-holiday Season	100	257	1032

(i) Calculate a 95% confidence interval for the mean time spent by a customer during the holiday season. **[3]**

(ii) Carry out a suitable hypothesis test, defining any symbols you use. State the p-value for the test and explain what it indicates. **[5]**

(iii) On examining the individual time spent during the two seasons, it is clear that they are not normally distributed. Explain what implications, if any, this has for the test carried out in part **(ii)**. **[2]**

A statistician advises that it would have been better to measure the time spent in the mall of all 200 customers once during the holiday season and once during the non-holiday season.

(iv) Explain briefly what the test procedure would have been if the statistician's advice had been followed. Explain also why that procedure would have been better. **[2]**

9 Data from a random sample of adults were collected by a cinema company. For each person, we know the type of movie they saw and whether or not they bought snacks. The percentages of people in the various categories were as follows.

		Snacks	No Snacks
Type of movie	Action	10.70%	9.80%
	Comedy	27.25%	22.25%
	Family	10.30%	9.70%
	Horror	4.75%	5.25%

(i) Given that the sample size was 2000, carry out a chi-squared test at the 10% level of significance to investigate the independence of the two factors, type of movie they saw and whether or not they bought snacks. Show in your working the contribution to the test statistic in each cell of the table. **[7]**

(ii) Suppose now that the sample size had been 4000. State the new value of the test statistic and explain whether or not the conclusion of the test would be changed. **[2]**

(iii) A manager of the company believes that the figures shown in the table are broadly accurate for the proportion as a whole, and he wishes to have strong evidence that the two factors are not independent. Assuming he is correct, estimate the sample size required to show, at the 0.5% level of significance, that the two factors are not independent. **[3]**

10 The random variable X has probability density function (pdf)

$$f(x) = \begin{cases} \dfrac{k}{(x+1)^4} & \text{for} \quad x \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = 3$, and find the cumulative distribution function. Find the value of x such that $P(X < x) = \dfrac{7}{8}$. **[5]**

(ii) Sketch the pdf. **[2]**

(iii) Find $E(X+1)$, and deduce that $E(X) = \dfrac{1}{2}$. **[3]**

(iv) Find $\text{Var}(X)$. **[3]**

~End of Paper~