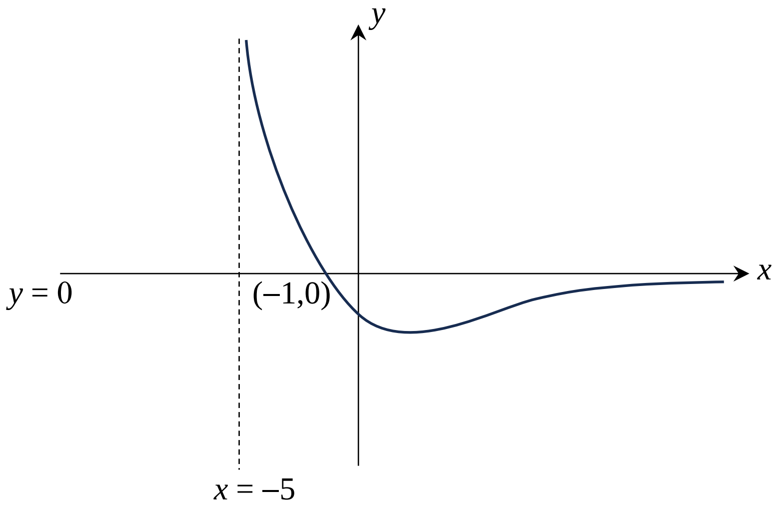
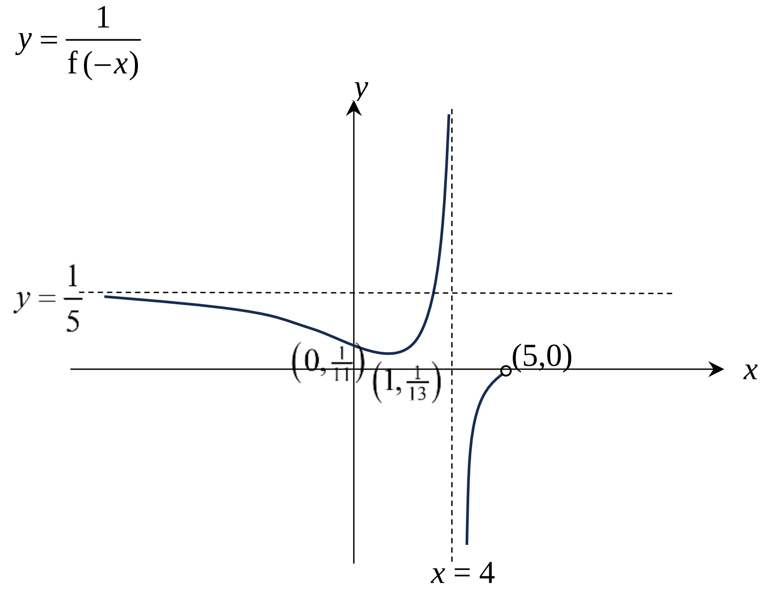
	Solution
1	Note that $\frac{x^2}{25}-(y-7)^2=4 \Leftrightarrow \frac{x^2}{10^2}-\frac{(y-7)^2}{2^2}=1$ $x^2-y^2=1$ $\downarrow$ replace x by $\frac{x}{10}$ $\frac{x^2}{10^2}-y^2=1$ $\downarrow$ replace y by $\frac{y}{2}$ $\frac{x^2}{10^2}-\frac{y^2}{2^2}=1$ $\downarrow$ replace y by $y-7$ $\frac{x^2}{10^2}-\frac{(y-7)^2}{2^2}=1$ The sequence of transformation are 1. Scaling parallel to x-axis by scale factor 10. 2. Scaling parallel to y-axis by scale factor 2. 3. Translation in the positive y-direction by 7 units **Alternatively,** $x^2-y^2=1$ $\downarrow$ replace x by $\frac{x}{10}$ $\frac{x^2}{10^2}-y^2=1$ $\downarrow$ replace y by $y-\frac{7}{2}$ $\frac{x^2}{10^2}-\left(y-\frac{7}{2}\right)^2=1$ $\downarrow$ replace y by $\frac{y}{2}$ $\frac{x^2}{10^2}-\left(\frac{y}{2}-\frac{7}{2}\right)^2=1 \Rightarrow \frac{x^2}{10^2}-\frac{(y-7)^2}{2^2}=1$ The sequence of transformation are 1. Scaling parallel to x-axis by scale factor 10. 2. Translation in the positive y-direction by $\frac{7}{2}$ units. 3. Scaling parallel to y-axis by scale factor 2.

2	Let x, y and z be the number of plates of chicken rice, duck rice and pork rice sold. Total profit = \$167.40 $\Rightarrow \quad 50x+80y+110z=16740$ $\Rightarrow \quad 5x+8y+11z=1674 \quad \cdots(1)$ Mr Chan earned \$28.80 more in profit from the sale of pork rice than the chicken rice $\Rightarrow \quad 110z-50x=2880$ $\Rightarrow \quad -5x+11z=288 \quad \cdots(2)$ Total number of plates of rice sold being 3 times the plates of duck rice sold $\Rightarrow \quad x+y+z=3y$ $\Rightarrow \quad x-2y+z=0 \quad \cdots(3)$ Solving (1), (2), (3), we have $x=81,\ y=72,\ z=63$ Mr Chan sold 81 plates of chicken rice, 72 plates of duck rice and 63 plates of pork rice that day.

3a

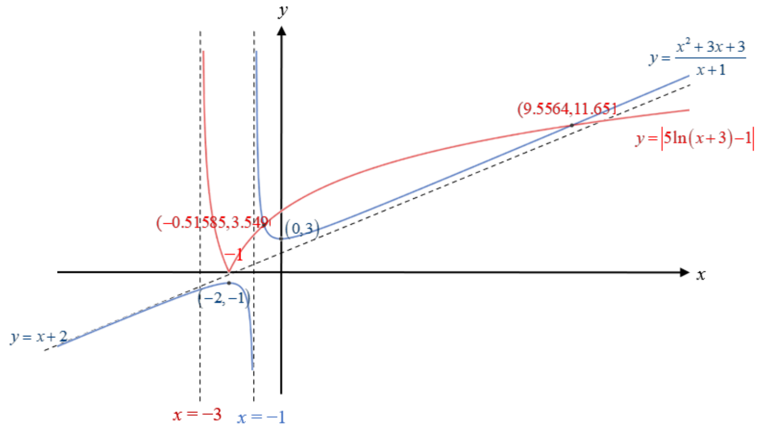

y
x
y = 0
(−1,0)
x = −5

3b


$y = \frac{1}{f(-x)}$
y
x
$y = \frac{1}{5}$
$\left(0, \frac{1}{11}\right)$
$\left(1, \frac{1}{13}\right)$
(5,0)
x = 4

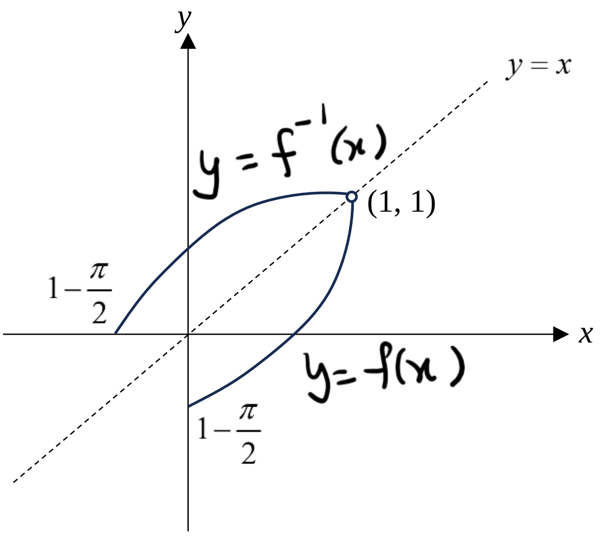
4a	$\dfrac{x^2+4}{x^2-2x+5}=1+\dfrac{2x-1}{x^2-2x+5}$ $=1+\dfrac{2x-2}{x^2-2x+5}+\dfrac{1}{x^2-2x+5}$ $\int\dfrac{x^2+4}{x^2-2x+5}\,\mathrm{d}x$ $=\int 1+\dfrac{2x-2}{x^2-2x+5}+\dfrac{1}{(x-1)^2+2^2}\,\mathrm{d}x$ $=x+\ln\left\|x^2-2x+5\right\|+\dfrac{1}{2}\tan^{-1}\left(\dfrac{x-1}{2}\right)+C$
4b	$\dfrac{\mathrm{d}}{\mathrm{d}x}\sec^2 3x=2(\sec 3x)(\sec 3x\tan 3x)(3)$ $=6\tan 3x\sec^2 3x$ $\int x\tan 3x\sec^2 3x\,\mathrm{d}x$ $=\dfrac{1}{6}\int x\left(6\tan 3x\sec^2 3x\right)\mathrm{d}x$ $=\dfrac{1}{6}\left[x\sec^2 3x-\int 1\left(\sec^2 3x\right)\mathrm{d}x\right]$ $=\dfrac{1}{6}\left(x\sec^2 3x-\dfrac{1}{3}\tan 3x\right)+C$

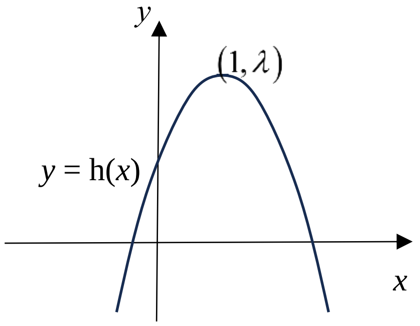
5(ai)	$\frac{d}{dx}\ln\left(\frac{1+\sin x}{1-\sin x}\right)$ $=\frac{d}{dx}\left(\ln(1+\sin x)-\ln(1-\sin x)\right)$ $=\frac{\cos x}{1+\sin x}-\frac{(-\cos x)}{1-\sin x}$ $=\frac{\cos x(1-\sin x)+\cos x(1+\sin x)}{(1+\sin x)(1-\sin x)}$ $=\frac{2\cos x}{1-\sin^2 x}$ $=\frac{2\cos x}{\cos^2 x}$ $=2\sec x$
5(aii)	$\frac{d}{dx}\tan^{-1}\left(\frac{1}{\sqrt{1-x}}\right)$ $=\frac{1}{1+\left(\frac{1}{\sqrt{1-x}}\right)^2}\frac{d}{dx}\left(\frac{1}{\sqrt{1-x}}\right)$ $=\frac{(1-x)}{(1-x)+1}\left(-\frac{1}{2}\frac{(-1)}{(1-x)^{\frac{3}{2}}}\right)$ $=\frac{1}{2(2-x)\sqrt{1-x}}$
5(b)	$y=x^{3x}$. Taking logarithm on both sides : $\ln y=\ln x^{3x}$, $\ln y=3x\ln x$. Differentiating implicitly w.r.t. x on both sides : $\frac{1}{y}\frac{dy}{dx}=(3)\ln x+3x\left(\frac{1}{x}\right)$ $\frac{1}{y}\frac{dy}{dx}=3\ln x+3$, $\frac{dy}{dx}=y\ (3\ln x+3)$, $=3x^{3x}(\ln x+1)$.

6(a)	
6(b)	We need to add the graph of $y=\|5\ln(x+3)-1\|$ The intersection points are $(-0.51585, 3.5496)$ and $(9.5564, 11.651)$. From graph, $-1<x\le -0.516$ (3 s.f.) or $x\ge 9.56$ (3 s.f.)
6(c)	Substituting x with $-\sin x$, we get $-1<-\sin x\le -0.516$ or $-\sin x\ge 9.56$ $0.516\le \sin x<1$ or $\sin x\le -9.56$ (N.A. since $-1\le \sin x\le 1$) $0.542\le x<\frac{\pi}{2}$

7(ai)	$\mathbf{a}+\mathbf{b}=\frac{1}{2}\mathbf{c} \quad \Rightarrow \quad \mathbf{a}=\frac{1}{2}\mathbf{c}-\mathbf{b}$ $\mathbf{a}\times\mathbf{c}=\left(\frac{1}{2}\mathbf{c}-\mathbf{b}\right)\times\mathbf{c}$ $=\frac{1}{2}\mathbf{c}\times\mathbf{c}-\mathbf{b}\times\mathbf{c}$ $=\mathbf{c}\times\mathbf{b}$ since $\mathbf{c}\times\mathbf{c}=\mathbf{0}$ Alternatively, Crossing both sides with **c**, we have $(\mathbf{a}+\mathbf{b})\times\mathbf{c}=\frac{1}{2}\mathbf{c}\times\mathbf{c}$ $\mathbf{a}\times\mathbf{c}+\mathbf{b}\times\mathbf{c}=\mathbf{0}$ $\mathbf{a}\times\mathbf{c}=-\mathbf{b}\times\mathbf{c}$ $\mathbf{a}\times\mathbf{c}=\mathbf{c}\times\mathbf{b}$
7(aii)	Dot both sides with **c**, we have $(\mathbf{a}+\mathbf{b})\cdot\mathbf{c}=\frac{1}{2}\mathbf{c}\cdot\mathbf{c}$ $\mathbf{a}\cdot\mathbf{c}+\mathbf{b}\cdot\mathbf{c}=\frac{1}{2}\lvert\mathbf{c}\rvert^2=\frac{1}{2}(1)^2=\frac{1}{2}$ $0+\lvert\mathbf{b}\rvert\lvert\mathbf{c}\rvert\cos\frac{\pi}{3}=\frac{1}{2}$ (since $\mathbf{a}\perp\mathbf{c}$) $\lvert\mathbf{b}\rvert(1)\left(\frac{1}{2}\right)=\frac{1}{2}$ $\lvert\mathbf{b}\rvert=1$ $\therefore \mathbf{b}$ is also a unit vector.

7(b)	Since *P* lies on line segment *DE*, by Ratio Theorem, let $\overrightarrow{OP}=\lambda\overrightarrow{OE}+(1-\lambda)\overrightarrow{OD}=\lambda\begin{pmatrix}5\\0\\1\end{pmatrix}+(1-\lambda)\begin{pmatrix}-3\\4\\-3\end{pmatrix}=\begin{pmatrix}8\lambda-3\\-4\lambda+4\\4\lambda-3\end{pmatrix}$ **Alternatively,** $\overrightarrow{DE}=\overrightarrow{OE}-\overrightarrow{OD}=\begin{pmatrix}5\\0\\1\end{pmatrix}-\begin{pmatrix}-3\\4\\-3\end{pmatrix}=\begin{pmatrix}8\\-4\\4\end{pmatrix}$ Equation of line *DE* is $\mathbf{r}=\begin{pmatrix}-3\\4\\-3\end{pmatrix}+\lambda\begin{pmatrix}8\\-4\\4\end{pmatrix}$, $\lambda\in\mathbb{R}$ Since *P* is a point on line *DE*, $\overrightarrow{OP}=\begin{pmatrix}-3\\4\\-3\end{pmatrix}+\lambda\begin{pmatrix}8\\-4\\4\end{pmatrix}$ for some λ $\overrightarrow{FP}=\overrightarrow{OP}-\overrightarrow{OF}=\begin{pmatrix}8\lambda-3\\-4\lambda+4\\4\lambda-3\end{pmatrix}-\begin{pmatrix}2\\4\\1\end{pmatrix}=\begin{pmatrix}8\lambda-5\\-4\lambda\\4\lambda-4\end{pmatrix}$ $\left\|\overrightarrow{FP}\right\|=3 \quad\Rightarrow\quad \sqrt{(8\lambda-5)^2+(-4\lambda)^2+(4\lambda-4)^2}=3$ $\Rightarrow\quad 64\lambda^2-80\lambda+25+16\lambda^2+16\lambda^2-32\lambda+16=9$ $\Rightarrow\quad 96\lambda^2-112\lambda+32=0$ $\Rightarrow\quad 6\lambda^2-7\lambda+2=0$ $\Rightarrow\quad (3\lambda-2)(2\lambda-1)=0$ $\Rightarrow\quad \lambda=\frac{1}{2}$ **or** $\lambda=\frac{2}{3}$ Therefore, $\overrightarrow{OP}=\begin{pmatrix}8\left(\frac{1}{2}\right)-3\\-4\left(\frac{1}{2}\right)+4\\4\left(\frac{1}{2}\right)-3\end{pmatrix}=\begin{pmatrix}1\\2\\-1\end{pmatrix}$ or $\begin{pmatrix}8\left(\frac{2}{3}\right)-3\\-4\left(\frac{2}{3}\right)+4\\4\left(\frac{2}{3}\right)-3\end{pmatrix}=\frac{1}{3}\begin{pmatrix}7\\4\\-1\end{pmatrix}$

8 (ai)	$\mathrm{f}(x)=1-\cos^{-1}x$ $\mathrm{f}'(x)=\dfrac{1}{\sqrt{1-x^2}}$ [Refer to MF27] For $0\le x<1,\ \mathrm{f}'(x)>0$ meaning **f is strictly increasing** in the interval $0\le x<1$and **hence is a 1-1 function**. Let $\mathrm{f}^{-1}\left(1-\dfrac{\pi}{2}\right)=k$ which is equivalent to solving $\mathrm{f}(k)=1-\dfrac{\pi}{2}$ where $0\le k<1$ $1-\cos^{-1}k=1-\dfrac{\pi}{2}$ $\Rightarrow\quad \cos^{-1}k=\dfrac{\pi}{2}$ $\Rightarrow\quad k=\cos\dfrac{\pi}{2}=0$
	Alternative method to find $\mathrm{f}^{-1}\left(1-\dfrac{\pi}{2}\right)$: Let $y=1-\cos^{-1}x$ $\cos^{-1}x=1-y$ $x=\cos(1-y)$ $\mathrm{f}^{-1}(x)=\cos(1-x)$ $\mathrm{f}^{-1}\left(1-\dfrac{\pi}{2}\right)=\cos\left(1-\left(1-\dfrac{\pi}{2}\right)\right)$ $=\cos\dfrac{\pi}{2}$ $=0$
8(aii)	

8(bi)	$h(x)=\lambda-(x-1)^2$ Note that the maximum value of h is λ when $x = 1$. $\therefore R_h=(-\infty,\lambda]$ 

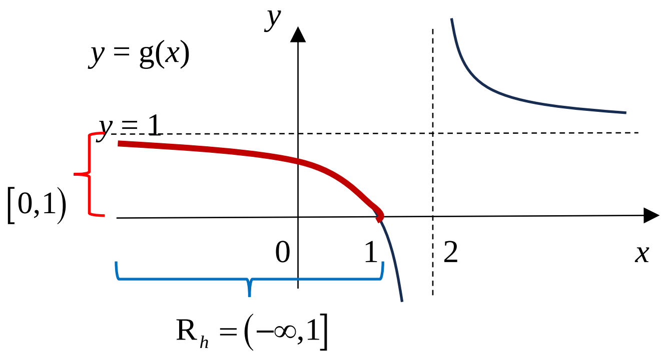
8(bii)

For the composite function gh to exist,
$(-\infty, \lambda] = R_h \subseteq D_g = (-\infty, 2) \cup (2, \infty)$.
So $\lambda < 2$.

But since λ is an integer, largest value of $\lambda = 1$.

To find the range of gh:
□ $\to (-\infty, 1] \to [0, 1)$
D_h R_h R_{gh}

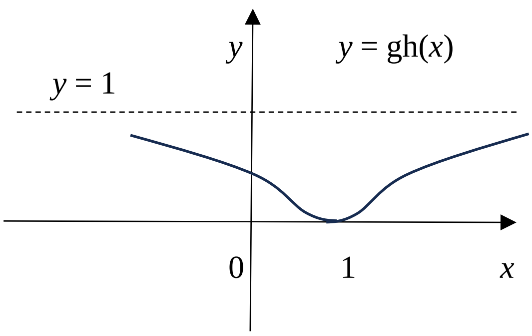
Explanation:
Using graph of g with $(-\infty, 1]$ as the "domain" to find R_{gh} :



Alternative method using graph of gh to find R_{gh} :

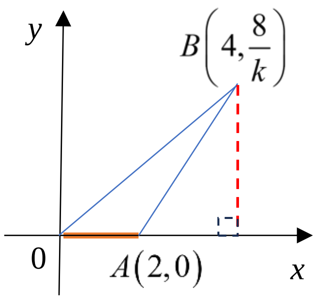
$$gh(x) = 1 + \frac{1}{1 - (x-1)^2 - 2} = 1 - \frac{1}{1 + (x-1)^2}$$

$D_{gh} = D_h =$ □



From graph of gh, Rgh = $[0, 1)$

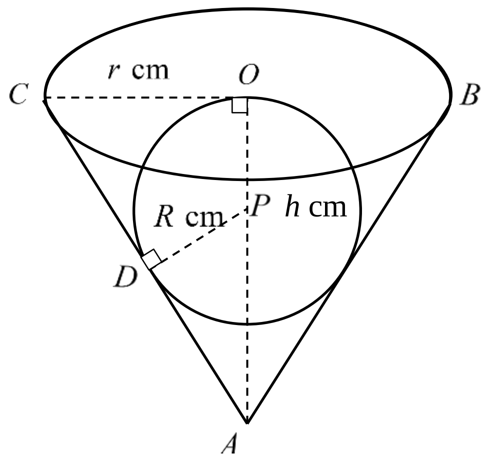
9(a)	Since A is on x-axis, therefore $y=\frac{t^2-1}{k}=0$. $\therefore t^2-1=0 \quad \Rightarrow \quad t=-1 \text{ or } 1$ Reject t = 1 as $x=\frac{3t-1}{t-1}$ is undefined. Let $t=-1$, we have $x=\frac{3(-1)-1}{(-1)-1}=2$ $\therefore A$ has coordinates (2, 0).
9(b)	$\frac{\mathrm{d}x}{\mathrm{d}t}=\frac{(t-1)(3)-(1)(3t-1)}{(t-1)^2}=-\frac{2}{(t-1)^2}$ $\frac{\mathrm{dy}}{\mathrm{d}t}=\frac{2t}{k}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}y}{\mathrm{d}t}/\frac{\mathrm{d}x}{\mathrm{d}t}=-\frac{t(t-1)^2}{k}$ At A, $t=-1$, $\therefore \frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{(-1)(-t-1)^2}{k}=\frac{4}{k}$. $\therefore$ equation of T is $y=\frac{4}{k}(x-2)$ [

9(c)	At B, $\frac{t^2-1}{k}=\frac{4}{k}\left(\frac{3t-1}{t-1}-2\right)$ $t^2-1=4\left(\frac{t+1}{t-1}\right)$ $(t-1)^2(t+1)=4(t+1)$ $\left[(t-1)^2-4\right](t+1)=0$ [or expand and solve using GC: $t^3-t^2-5t-3=0$] $\left[(t-1)^2-4\right]=0$ or $(t+1)=0$ $(t-1)=\pm 2$ or $t=-1$ $\therefore t=3$ or $t=-1$ (rejected as this is for point A) Therefore, when $t=3$, $x=\frac{3(3)-1}{3-1}=4,\quad y=\frac{3^2-1}{k}=\frac{8}{k}$ $x=\frac{3(3)-1}{3-1}=4,\quad y=\frac{3^2-1}{k}=\frac{8}{k}$ Therefore, the coordinates of B is $\left(4,\frac{8}{k}\right)$.
9(d)	Area of triangle OAB $=\frac{1}{2}(\text{base})(\text{height})$ $=\frac{1}{2}(OA)(y\text{-coord of }B)$ $=\frac{1}{2}(2)\left(\frac{8}{k}\right)=\frac{8}{k}$  .

10 (a)	Let $\dfrac{2x}{4+x^2}=\dfrac{1}{4}x$ $8x=x\left(4+x^2\right)$ $x\left(x^2-4\right)=0$ $x=0,\ 2\ \text{ or }\ -2\text{(rejected)}$ Area $R=\displaystyle\int_0^2 \frac{2x}{4+x^2}\,\mathrm{d}x-\int_0^2 \frac{x}{4}\,\mathrm{d}x$ $=\left[\ln\left(4+x^2\right)\right]_0^2-\left[\dfrac{1}{8}x^2\right]_0^2$ $=\ln 8-\ln 4-\dfrac{1}{2}$ $=\ln 2-\dfrac{1}{2}$

10(b)	$x = 2\tan\theta, \quad \frac{dx}{d\theta} = 2\sec^2\theta$ $\int \frac{x^2}{\left(4+x^2\right)^2}\, dx$ $= \int \frac{4\tan^2\theta}{\left(4+4\tan^2\theta\right)^2}\, \left(2\sec^2\theta\right) d\theta$ $= \int \frac{4\tan^2\theta}{16\sec^4\theta}\, \left(2\sec^2\theta\right) d\theta$ $= \int \frac{4\tan^2\theta}{8\sec^2\theta}\, d\theta$ $= \int \frac{1}{2}\sin^2\theta\, d\theta$ $= \frac{1}{4}\int 1-\cos 2\theta\, d\theta$ $= \frac{1}{4}\left(\theta - \frac{1}{2}\sin 2\theta\right) + C$ Method 1: The Volume $= \pi\int_0^2 \frac{4x^2}{\left(4+x^2\right)^2}\, dx - \pi\int_0^2 \frac{x^2}{16}\, dx$ $= \pi\left[\theta - \frac{1}{2}\sin 2\theta\right]_0^2 - \pi\left[\frac{1}{48}x^2\right]_0^2$ $= \pi\left[\frac{\pi}{4} - \frac{1}{2}\right] - \frac{\pi}{6}$ $= \frac{\pi^2}{4} - \frac{2\pi}{3}$ Method 2: The Volume $= \pi\int_0^2 \frac{4x^2}{\left(4+x^2\right)^2}\, dx - \frac{1}{3}\pi\left(\frac{1}{2}\right)^2(2)$ $= \pi\left[\theta - \frac{1}{2}\sin 2\theta\right]_0^2 - \frac{\pi}{6}$ $= \pi\left[\frac{\pi}{4} - \frac{1}{2}\right] - \frac{\pi}{6}$ $= \frac{\pi^2}{4} - \frac{2\pi}{3}$

11 (a)	$\overrightarrow{AB}=\begin{pmatrix}0\\k\\0\end{pmatrix}-\begin{pmatrix}-2\\0\\-1\end{pmatrix}=\begin{pmatrix}2\\k\\1\end{pmatrix}$ Since l_1 makes an angle of $\frac{\pi}{3}$ with l_2, we have AB making an acute angle of $\frac{\pi}{3}$ with line parallel to $\begin{pmatrix}3\\0\\4\end{pmatrix}$. Therefore, $\cos\frac{\pi}{3}=\dfrac{\left\lvert\begin{pmatrix}2\\k\\1\end{pmatrix}\cdot\begin{pmatrix}3\\0\\4\end{pmatrix}\right\rvert}{\sqrt{2^2+k^2+1^2}\sqrt{3^2+4^2}}$ $\frac{1}{2}=\dfrac{10}{\left(\sqrt{5+k^2}\right)(5)}$ $\sqrt{5+k^2}=4$ $5+k^2=16$ $k^2=11$ $k=\sqrt{11}$ or $-\sqrt{11}$ (rejected since k is +ve)
11(b)	**Method 1:** Since N is a point on l_1, we have $\overrightarrow{ON}=\begin{pmatrix}-2\\0\\-1\end{pmatrix}+\lambda\begin{pmatrix}3\\0\\4\end{pmatrix}=\begin{pmatrix}-2+3\lambda\\0\\-1+4\lambda\end{pmatrix}$ $\therefore\overrightarrow{BN}=\begin{pmatrix}-2+3\lambda\\0\\-1+4\lambda\end{pmatrix}-\begin{pmatrix}0\\\sqrt{11}\\0\end{pmatrix}=\begin{pmatrix}-2+3\lambda\\-\sqrt{11}\\-1+4\lambda\end{pmatrix}$. Since BN is perpendicular to l_1, we have $\overrightarrow{BN}\cdot\begin{pmatrix}3\\0\\4\end{pmatrix}=0$ $\Rightarrow\begin{pmatrix}-2+3\lambda\\-\sqrt{11}\\-1+4\lambda\end{pmatrix}\cdot\begin{pmatrix}3\\0\\4\end{pmatrix}=0$ $\Rightarrow -6+9\lambda-4+16\lambda=0$ $\Rightarrow \lambda=\frac{2}{5}$

	$\Rightarrow \quad \overrightarrow{ON} = \begin{pmatrix} -2+3\left(\frac{2}{5}\right) \\ 0 \\ -1+4\left(\frac{2}{5}\right) \end{pmatrix} = \begin{pmatrix} -\frac{4}{5} \\ 0 \\ \frac{3}{5} \end{pmatrix}$
11(c)	**Method 1:** $\overrightarrow{BN} = \begin{pmatrix} -\frac{4}{5} \\ -\sqrt{11} \\ \frac{3}{5} \end{pmatrix} \Rightarrow \left\lvert\overrightarrow{BN}\right\rvert = \sqrt{\left(-\frac{4}{5}\right)^2 + \left(-\sqrt{11}\right)^2 + \left(\frac{3}{5}\right)^2} = \sqrt{12}$ $\overrightarrow{AN} = \begin{pmatrix} -\frac{4}{5} \\ 0 \\ \frac{3}{5} \end{pmatrix} - \begin{pmatrix} -2 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} \frac{6}{5} \\ 0 \\ \frac{8}{5} \end{pmatrix} \Rightarrow \left\lvert\overrightarrow{AN}\right\rvert = \sqrt{\left(\frac{6}{5}\right)^2 + \left(\frac{8}{5}\right)^2} = 2$ Area of triangle $ABN = \frac{1}{2}\left\lvert\overrightarrow{AN}\right\rvert\left\lvert\overrightarrow{BN}\right\rvert = \frac{1}{2}(2)\left(\sqrt{12}\right) = 2\sqrt{3}$ **Method 2:** Area of triangle ABN $= \frac{1}{2}\left\lvert\overrightarrow{AN} \times \overrightarrow{BN}\right\rvert = \left\lvert \begin{pmatrix} \frac{6}{5} \\ 0 \\ \frac{8}{5} \end{pmatrix} \times \begin{pmatrix} -\frac{4}{5} \\ -\sqrt{11} \\ \frac{3}{5} \end{pmatrix} \right\rvert$ $= \frac{1}{2}\left\lvert \begin{pmatrix} \frac{8\sqrt{11}}{5} \\ 2 \\ -\frac{6\sqrt{11}}{5} \end{pmatrix} \right\rvert = \frac{1}{2}\sqrt{\left(\frac{8\sqrt{11}}{5}\right)^2 + 2^2 + \left(\frac{-6\sqrt{11}}{5}\right)^2} = 2\sqrt{3}$
12(a)	 Let centre of the sphere be P. Using similar triangles APD and ACO,

	$\frac{h-R}{AC}=\frac{R}{r}$ $AC=\frac{rh-rR}{R}$ By Pythagoras' Theorem, $AC^2=h^2+r^2$ $\left(\frac{rh-rR}{R}\right)^2=h^2+r^2$ $r^2h^2-2r^2hR+r^2R^2=h^2R^2+r^2R^2$ $r^2\left(h^2-2hR\right)=h^2R^2$ $\therefore r^2=\frac{h^2R^2}{h^2-2hR}$ **OR** Let centre of the sphere be P. $AD=\sqrt{\left(AP^2-PD^2\right)}$ $=\sqrt{\left((h-R)^2-R^2\right)}$ $=\sqrt{\left(h^2-2hR\right)}$ Using congruent triangles PCO and PCD, $CO=CD=r$ cm. $AC=AD+DC$ $=\sqrt{\left(h^2-2hR\right)}+r$ Since by Pythagoras' theorem, $AC^2=h^2+r^2$ $h^2+r^2=h^2-2hR+2r\sqrt{\left(h^2-2hR\right)}+r^2$ $\therefore r^2=\frac{h^2R^2}{h^2-2hR}$
12(b)	Volume of cone, V $=\frac{1}{3}\pi r^2h$ $=\frac{1}{3}\pi\frac{h^2R^2}{h^2-2hR}h$ $=\frac{1}{3}\pi R^2\frac{h^3}{h^2-2hR}$ $=\frac{1}{3}\pi R^2\frac{h^2}{h-2R}$ $\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{1}{3}\pi R^2\left[\frac{2h(h-2R)-h^2}{(h-2R)^2}\right]$ $=\frac{1}{3}\pi R^2\left[\frac{h^2-4hR}{(h-2R)^2}\right]$ $=\frac{1}{3}\pi R^2\frac{h(h-4R)}{(h-2R)^2}$

	$\frac{dV}{dh}=0$ $\frac{1}{3}\pi R^2\frac{h(h-4R)}{(h-2R)^2}=0$ $h=4R$ or $h=0$ (reject $\because h>0$) $\therefore$ minimum $V=\frac{1}{3}\pi R^2\frac{(4R)^2}{4R-2R}=\frac{8}{3}\pi R^3$ cm^3
12(c)	**Method 1** $\frac{1}{x^2}-\frac{1}{y^2}=\frac{1}{a^2}$ $x^{-2}-y^{-2}=\frac{1}{a^2}$ Differentiate w.r.t. t: $-2x^{-3}\frac{dx}{dt}+2y^{-3}\frac{dy}{dt}=0$ When $y=2a$, $\frac{1}{x^2}-\frac{1}{4a^2}=\frac{1}{a^2}\Rightarrow\frac{1}{x^2}=\frac{5}{4a^2}\therefore x=\frac{2a}{\sqrt{5}}$ $-2\left(\frac{\sqrt{5}}{2a}\right)^3 2+2\left(\frac{1}{2a}\right)^3\frac{dy}{dt}=0$ $\frac{dy}{dt}=2\sqrt{5}^3$ Thus chemical Y is added at the rate of increase of $2\sqrt{5}^3$ cm^{3}s^{-1}. **Method 2** When $y=2a$, $\frac{1}{x^2}-\frac{1}{4a^2}=\frac{1}{a^2}\Rightarrow\frac{1}{x^2}=\frac{5}{4a^2}\therefore x=\frac{2a}{\sqrt{5}}$ $x^{-2}-y^{-2}=\frac{1}{a^2}$ Differentiate w.r.t. x: $-2x^{-3}+2y^{-3}\frac{dy}{dx}=0$ $\frac{dy}{dx}=\frac{y^3}{x^3}$ $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}$ When $y=2a$, $\frac{dy}{dt}=\frac{(2a)^3}{\left(\frac{2a}{\sqrt{5}}\right)^3}\times 2=2\left(\sqrt{5}\right)^3=10\sqrt{5}$