Name:________________________________ Class:____________

JURONG PIONEER JUNIOR COLLEGE
JC1 End-of-Year Examination 2021

MATHEMATICS **9758/01**
Higher 2 **29 September 2021**
Paper 1 **3 hours**

Additional materials: Answer Paper
Cover Page
List of Formulae (MF 26)

READ THESE INSTRUCTIONS FIRST

Write your name and civics class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given by [ ] at the end of each question or part question.

This document consists of **5** printed pages and 1 blank page.

1 The parametric equations of a curve C are

$$x = t + 2e^{t},\ y = 2t + e^{-t},\ t > -2.$$

(i) Sketch C. [2]

(ii) Find the equation of the normal to the curve at the point P where $t = 0$. [4]

2 A curve C has equation $y = \dfrac{a}{x+3} + bx + c$, where a, b and c are constants. It is given that C passes through the point with coordinates (1, –3) and has a stationary point (–5, –21).

(i) Find the values of a, b and c. [4]
(ii) Hence find the equations of the asymptotes of C. [2]

3 It is given that $x^2 y - \tan^{-1} y = \dfrac{3}{4}\pi$. Find $\dfrac{dy}{dx}$ in terms of x and y. [4]

Hence, find the exact value of $\dfrac{dy}{dx}$ when $y = 1$, given that $x > 0$. [3]

4 **(i)** Sketch the curve with equation $y = \ln(x+1), x > -1$, stating the equation of asymptote and intercepts with the axes. On the same diagram, sketch the curve with equation $y = \sqrt{9 - x^2}, -3 \le x \le 3$, stating the intercepts with the axes. [4]

(ii) Use your answer in part **(i),** solve the inequality $\ln(x+1) < \sqrt{9 - x^2}$. [2]

(iii) Hence solve the inequality $\ln(x^2+1) < \sqrt{9 - x^4}$. [3]

5 The function f is defined by $f : x \to \dfrac{1}{3}\left(e^{x-2} - 1\right), x > 2$.

(i) Find f^{-1} and write down the its domain. [3]

(ii) Explain why the solution of $f(x) = f^{-1}(x)$ satisfies the equation $e^{x-2} = 3x + 1$ and find the value of this solution. [3]

The function g is defined by $g : x \to 1 + x^2,\ \ x \in \mathbb{R}$.

(iii) Show that gf exists . Hence find the composite function gf , stating its domain and the corresponding range. [4]

6 **(a)** Given that $\mathbf{u} \bullet \mathbf{v} = 0$, what can be deduced about the vectors **u** and **v**? [2]

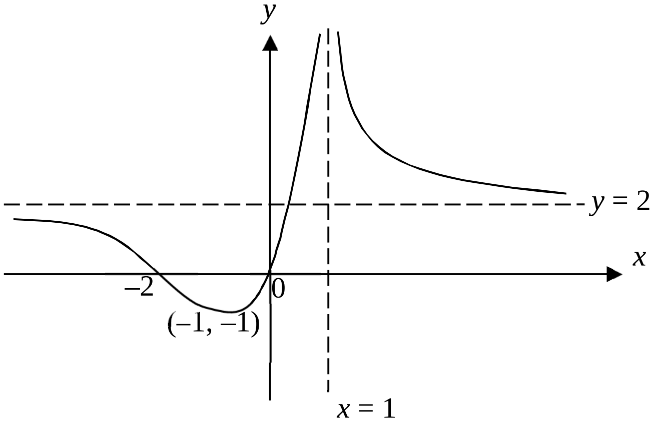
(b) Referred to an origin O, the position vectors of two points A and B are **a** and **b** respectively. A line l has vector equation given by $\mathbf{r} = \frac{1}{3}\mathbf{a} + \lambda\left(2\mathbf{b} - \mathbf{a}\right)$, where $\lambda \in \mathbb{R}$.

The point N is the foot of perpendicular from A to l. It is given that $|\mathbf{a}| = 2$, $|\mathbf{b}| = 1$ and **a** is perpendicular to **b**.

(i) Find the position vector of N in terms of **a** and **b**. [5]

(ii) Find the exact area of triangle OAN. [3]

7 **(a)** The graph of $y = \mathrm{f}(x)$ has asymptotes $x = 1$, $y = 2$ and a minimum point at $(-1, -1)$ as shown in the diagram. It cuts the x-axis at the origin and at $(-2, 0)$.



Sketch the following graphs on separate diagrams, labeling clearly the asymptotes, turning points and intercepts on the axes where applicable.

(i) $y = \mathrm{f}\left(2x\right) + 1$, [3]

(ii) $y = \dfrac{1}{\mathrm{f}(x)}$. [3]

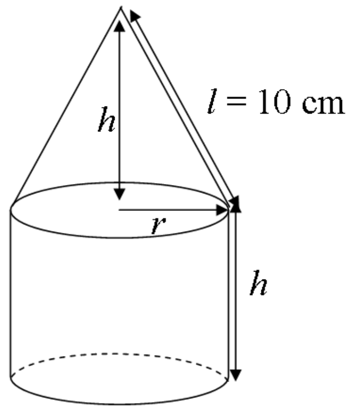
(b) The curve whose equation is $y = \dfrac{1}{x+1}$ undergoes, in succession, the following transformations:

A: A reflection in the y – axis.

B: A translation of 5 units in the negative x – direction.

Give the equation of the resulting curve. [3]

8 **(a)**



A closed container is constructed using a sheet of metal with area 100π cm^2. The container comprises 2 shapes, a cone and a cylinder. The slant height, l, of the cone is 10 cm. Given that the cone and the cylinder share the same height h, and radius r,

(i) show that $h = \dfrac{100 - r^2 - 10r}{2r}$, [1]

(ii) use differentiation to find, the exact maximum volume of the container, proving that it is a maximum. [6]

[Volume of Cone $= \dfrac{1}{3}\pi r^2 h$, Curved Surface Area of Cone = πrl]

(b) The height of an upright cone is twice the radius, r, of its circular base. It is known that the volume of the cone is increasing at the rate of 15 cm^3 min^{-1} when the radius is 3 cm. Find the rate of increase of the base area of the cone at this instant. [4]

9 A plane Π_1 has equation $x + y + z = 3$. A line passes through the points P and Q with position vectors $\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $4\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ respectively.

(i) Find the exact length of projection of $\overrightarrow{PQ}$ onto Π_1. [3]

(ii) Find the position vector of the point of intersection of line PQ and Π_1. [3]

A plane Π_2 is parallel to the y-z plane and contains the point $(-2, 1, 4)$.

(iii) Find the cartesian equation of Π_2. [2]

(iv) A point $S(a, 7, b)$ lies on both Π_1 and Π_2. Write down the values of a and b. [2]

(v) Hence or otherwise, find a vector equation of the line of intersection of Π_1 and Π_2. [2]

10 **(a)** Find $\displaystyle\int \frac{e^{\sin^{-1} x}}{\sqrt{1-x^2}}\,dx$. [2]

(b) Express $\mathrm{f}(x) = \dfrac{x^2 - 6x + 1}{(3x+1)(x^2+3)}$ in the form

$$\frac{A}{3x+1} + \frac{Bx + C}{x^2 + 3},$$

where the values of A, B and C to be determined. [3]

Hence find $\displaystyle\int \mathrm{f}(x)\,dx$. [2]

(c) Find the exact value of $\displaystyle\int_{\pi}^{2\pi} \sin^2\left(\frac{x}{8}\right) dx$. [3]

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