NAME:		( )
CLASS:	TEACHING GROUP:	MARKS /100

PEI HWA SECONDARY SCHOOL

Diagnostic Assessment 2023

Secondary Four Express / Five Normal (Academic)

MATHEMATICS 4052

21 June 2023

2 hours 30 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your class, teaching group, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.
The number of marks is given in the brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total number of marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142, unless the question requires the answer in terms of π.

For Examiner's Use	
Category	**Question No.**
Correction tape	
Pencil written	
Arrows	
Units	
Others	

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer all the questions.

1 **(a)** Simplify $\left(\frac{8x^6}{y^2}\right)^{-\frac{2}{3}}$, leaving your answer in positive indices.

$\left(\frac{8x^6}{y^2}\right)^{-\frac{2}{3}}$

$= \left(\frac{y^2}{8x^6}\right)^{\frac{2}{3}}$ M1

$= \frac{y^{\frac{4}{3}}}{4x^4}$ A1

Answer ……..…… [2]

(b) Given that $2^x = 5$, find the value of $4^{-x} + 8^x$.

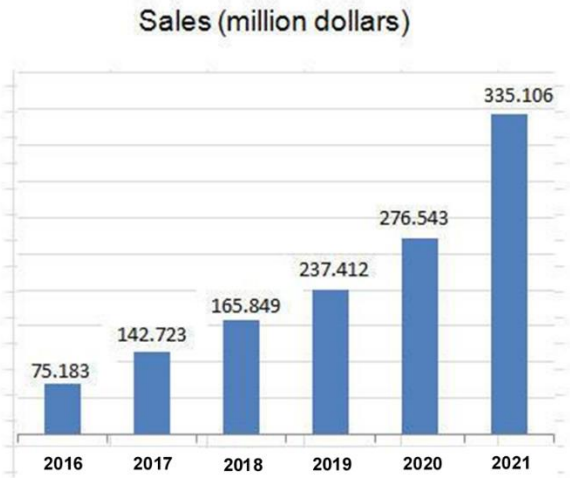
$4^{-x} + 8^x = \frac{1}{2^{2x}} + 2^{3x}$ M1

$= \frac{1}{5^2} + 5^3$

$= 125\frac{1}{25} / \frac{3126}{25}$ A1 (accept 125.04)

Answer ……..…… [2]

2 The following chart shows the sales of a company over the years.



State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph.

……. The difference in sales between 2016 and 2017 was 67.54 while the difference in sales between 2020 and 2021was 56.563. B1 .....

……. The height difference of the bars between 2020 and 2021 is greater than 2016 and 2017, misleads us to think that sales between 2020 and 2021 is greater than between 2016 and 2017. B1 ..... [2]

3 x is decreased by 50% and then increased by 40%. Find the percentage decrease in x from its original value.

x is decreased by 50% --- $0.5x$
then increased by 40% --- $1.4 \times 0.5x = 0.7x$ M1
$\%\ \text{change} = \dfrac{0.7x - x}{x} \times 100\%$
$= -30\%$ A1
Percentage change = 30%
decrease

Answer ……% [2]

4 Write as a single fraction in its simplest form $\dfrac{3x}{1-x} + \dfrac{5}{(x-1)^2}$.

$\dfrac{3x}{1-x} + \dfrac{5}{(x-1)^2}$
$= -\dfrac{3x}{x-1} + \dfrac{5}{(x-1)^2}$ M1
$= \dfrac{-3x(x-1)+5}{(x-1)^2}$ M1
$= \dfrac{-3x^2+3x+5}{(x-1)^2}$ or $\dfrac{-3x^2+3x+5}{(1-x)^2}$ A1

Answer …… [3]

5 There are 18 boys and x girls in the school's concert band. It is given that the probability of selecting a boy from the concert band is $\dfrac{6}{17}$.

(a) Find the value of x.

$\dfrac{18}{18+x} = \dfrac{6}{17}$
$6x + 108 = 306$
$x = 33$ B1

Answer x = …… [1]

(b) Two children are selected at random.

Find the probability that a boy and a girl are selected.

$\left(\dfrac{18}{51} \times \dfrac{33}{50}\right) \times 2$ M1
$= \dfrac{198}{425}$ A1

Answer = …… [2]

(c) Some boys joined the concert band so that the probability of selecting a boy at random now becomes $\frac{13}{24}$. Find the number of boys who joined the concert band.

$\frac{18+m}{51+m} = \frac{13}{24}$ M1

$11m = 231$

$m = 21$ A1

Answer ……..…… [2]

6 **(a)** Factorise the expression $10x^2 + 7x - 12$.

$10x^2 + 7x - 12$

$= (2x+3)(5x-4)$

Answer ……..…… [1]

(b) **Hence** solve the equation $10(y-1)^2 + 7y = 19$.

$10(y-1)^2 + 7y = 12 + 7$

$10(y-1)^2 + 7y - 7 - 12 = 0$

$10(y-1)^2 + 7(y-1) - 12 = 0$

Hence, $x = y - 1$

$[2(y-1)+3] = 0$ or $[5(y-1)-4] = 0$

$y = -\frac{1}{2}$ or $y = 1\frac{4}{5}$

Answer y = ……..... or ……. [3]

7 The value of a painting increased from \$74 000 in 2018 to \$$P$ in 2021.

The value increased by 8% every year.

Find the value of the painting in 2021.

$P = 74000\left(1 + \frac{8}{100}\right)^3$

$= \$93\ 218.69$ or \$93 000

Answer \$ ……..…… [2]

8 The braking distance, d, of a car is directly proportional to the square of its speed, s.

When the speed is p metres per second, the braking distance is 6 metres.

Find the percentage change in the breaking distance when the speed is increased to 300%.

$d = ks^2$

$d = 6$ and $s = p$, $k = \dfrac{6}{p^2}$ M1

$s = 3p$, $d = \dfrac{6}{p^2}(3p)^2$ M1

$= 54$

% increase in braking distance $= \dfrac{54-6}{6} \times 100\% = 800\%$ A1

Answer 800 % [3]

9 Factorise completely

(a) $ab - a - b + 1$,

$ab - a - b + 1 = a(b-1) - (b-1)$ M1

$= (b-1)(a-1)$ A1

Answer ..... [2]

(b) $x^2 - 2ax + a^2 - 4b^2$.

$x^2 - 2ax + a^2 - 4b^2 = (x-a)^2 - 4b^2$ M1

$= (x - a - 2b)(x - a + 2b)$ A2

Answer ..... [3]

10 Given that $\overrightarrow{AB} = \begin{pmatrix} 4 \\ -8 \end{pmatrix}$ and $\overrightarrow{AC} = \begin{pmatrix} m \\ 18 \end{pmatrix}$ find

(i) $\left|\overrightarrow{AB}\right|$,

$\left|\overrightarrow{AB}\right| = \sqrt{4^2 + (-8)^2}$

$= \sqrt{80}$

$= 8.94$ units B1

Answer .....units [1]

(ii) the value of m such that AB and AC are parallel,

$\begin{pmatrix} 4 \\ -8 \end{pmatrix} = k\begin{pmatrix} m \\ 18 \end{pmatrix}$

$4 = km$ and $-8 = 18k$ — M1

Thus, $k = -\frac{4}{9}$ and $m = \frac{4}{-\frac{4}{9}} = -9$ — A1

Answer ……..…… [2]

(iii) the coordinates of B if A is the point (–1, –3).

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$\begin{pmatrix} 4 \\ -8 \end{pmatrix} = \overrightarrow{OB} - \begin{pmatrix} -1 \\ -3 \end{pmatrix}$

$\overrightarrow{OB} = \begin{pmatrix} 3 \\ -11 \end{pmatrix}$

B is (3, −11) — B1

Answer B (……..., ……..…) [1]

11 The ratio of Abby's money to Betty's money is 4 : 7. The ratio of Betty's money to Claire's money is 4 : 9.

The three girls have $321 altogether.

Claire gives some money to Abby and Betty so that each of them has the same amount of money.

How much money will Abby receive from Claire?

Abby : Betty : Claire = 16 : 28 : 63 — M1

107 units --- $321

1 unit --- $3 — M1

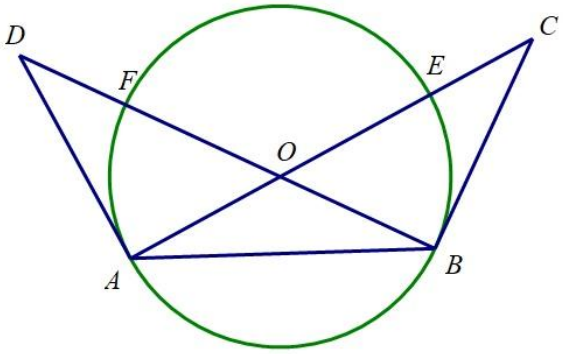
Abby's initial amount = $48

If each has the same amount of money, each should get $107 — A1

Therefore, amount Abby receive from Claire = $107 – $48 = $59

Answer $……..…… [3]

12 In the diagram, *AD* and *BC* are tangents to the circle, centre *O*, at the points *A* and *B* respectively. *AOEC* and *BOFD* are straight lines.



(a) Show that triangle *OAD* and triangle *OBC* are congruent.

Give a reason for each statement you make.

Answer

$\angle AOD = \angle BOC$ (vertically opposite angles)	M2 for any 3 statements M1 for 2 statements
$OA = OB$ (radii of circle)	
$\angle OAD = \angle OBC = 90°$ (radius perpendicular to tangent)	
Therefore, triangle *OAD* is congruent to triangle *OBC* (ASA congruency test)	A1

[3]

(b) Two similar solids have volumes V_1 and V_2 such that $8V_1 = 125V_2$.

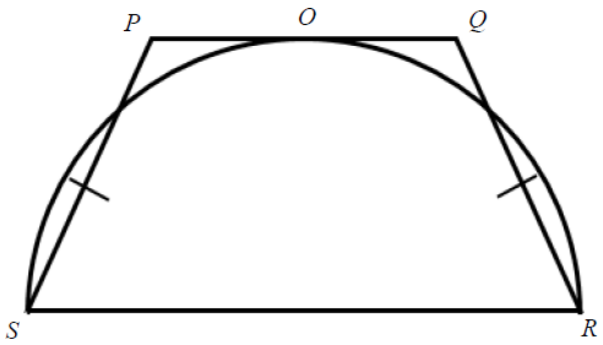
If their surface area, A_1 and A_2 respectively, are such that $A_1 = kA_2$, find the value of *k*.

$\frac{V_1}{V_2} = \frac{125}{8} = \left(\frac{l_1}{l_2}\right)^3$	
$\frac{l_1}{l_2} = \frac{5}{2}$	M1
$\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2 = \frac{25}{4}$	
$A_1 = \frac{25}{4}A_2$	
$k = \frac{25}{4}$	A1

Answer ……..…… [2]

13 The diagram shows a semicircle *SOR* and an isosceles trapezium *PQRS*, where $PS = (x + 3y)$ cm, $QR = (4y - 3x + 5)$ cm and $RS = (7y + 3x - 1)$ cm.

Given that the vertical height of the trapezium is $(2x + 3y)$ cm, find the perimeter of the semicircle *SOR*, leaving your answer in terms of π.



$x + 3y = 4y - 3x + 5$	M1
$4x - y = 5$ --- (1)	
$2(2x + 3y) = 7y + 3x - 1$	M1
$x - y = -1$ --- (2)	
(1) – (2): $x = 2$	
$y = 3$	A1
Radius = 2(2) + 3(3) = 13 cm	
Perimeter of semicircle = $26 + \pi(13)$	
$= 26 + 13\pi$ cm	A1

Answer ……..…… cm [4]

14 A line P is perpendicular to another line Q with equation $4y-3x-16=0$.

Line P also passes through the point (–15, 10).

The product (gradient of P) × (gradient of Q) = –1.

(a) Use this information to find the equation of line P.

Gradient of $P = -\frac{4}{3}$	M1
Equation of P: $y = -\frac{4}{3}x + c$, (–15, 10)	
$10 = -\frac{4}{3}(-15) + c$	
$c = -10$	
Hence $y = -\frac{4}{3}x - 10$	A1

Answer ……..…… [2]

(b) The equation of another line R is $4x+3y-15=0$.

Without solving for x and y, explain whether line R intersects the line P.

Answer

$4x+3y-15=0$	
$y = -\frac{4}{3}x + 5$	B1
Since the gradient of R is the same as the gradient of P, the lines are **parallel** and the lines have **different y-intercepts** so the lines are not overlapping. Hence, line R does not intersect line P.	B1

[2]

15 **(a)** $p = \dfrac{r^2 - 4}{5 - qr^2}$.

(i) Evaluate p when $q = 3\dfrac{1}{2}$ and $r = -1$.

$$p = \frac{(-1)^2 - 4}{5 - 3\frac{1}{2}(-1)^2} = -2 \quad \text{B1}$$

Answer ……..…… [1]

(ii) Express r in terms of p and q.

$$p = \frac{r^2 - 4}{5 - qr^2}$$

$$5p - pqr^2 = r^2 - 4 \quad \text{M1}$$

$$r^2(1 + pq) = 5p + 4$$

$$r^2 = \frac{5p + 4}{1 + pq} \quad \text{M1}$$

$$r = \pm\sqrt{\frac{5p + 4}{1 + pq}} \quad \text{A1}$$

Answer ……..…… [3]

(b) **(i)** Express $18 - 5x + x^2$ in the form $a + (b + x)^2$.

$$(x - \frac{5}{2})^2 - \left(\frac{5}{2}\right)^2 + 18 \quad \text{M1}$$

$$= (x - \frac{5}{2})^2 + \frac{47}{4}$$

$$= \frac{47}{4} + (-\frac{5}{2} + x)^2 \quad \text{A1/B2}$$

Answer ……..…… [2]

(ii) Write down the equation of the line of symmetry of $y = 18 - 5x + x^2$.

$$x = 2.5 \quad \text{B1}$$

Answer ……..…… [1]

(c) Solve $\frac{1}{x+3}+\frac{6}{3-x}=4$.

Give your solutions correct to 2 decimal places.

$3-x+6(x+3)=4(x+3)(3-x)$ | M1

$5x+21=-4x^2+36$

$4x^2+5x-15=0$

$x=\frac{-5\pm\sqrt{5^2-4(4)(-15)}}{2(4)}$ | M1

$x = 1.41$ or -2.66 (2dp) | A1

Answer ……..…… [3]

16 **(i)** Ashton wants to buy a pair of shoes from a shop in Singapore.
The price of the shoes is reduced by 10% in a sale.
The sale price of the shoes is $108.
Calculate the price of the pair of shoes before the sale.

$\frac{108}{90}\times 100$ | M1

$=\$120$ | A1

Answer $……..…… [2]

(ii) Ashton finds the same pair of shoes selling in an online shop from San Francisco with a mark price of 80 USD selling at a 15% discount.
The exchange rate between Singapore dollars ($) and American dollars (USD) is $1 = 0.73 USD.
Ashton will also have to pay a Currency Conversion Fee of 1.8% of the sale price and SGD $6 delivery fee.
Is it better for Ashton to buy the shoes from the shop or online?
Justify your decision you make and show your calculations clearly.

Answer

Cost in SGD$ $= 0.85\times 80\div 0.73$
$= \$\ 93.1506$ | M1

Include Currency Conversion Fee and delivery $= \frac{1.018}{100}\times 93.1506+6$
$= \$100.83$ < $108 | M1

Conclusion: **Ashton to buy the shoes online as it costs cheaper.** | A1

[3]

17 The variables x and y are connected by the equation $y = \frac{x^3}{5} - x + 2$.

Some corresponding values of x and y are given in the table below.

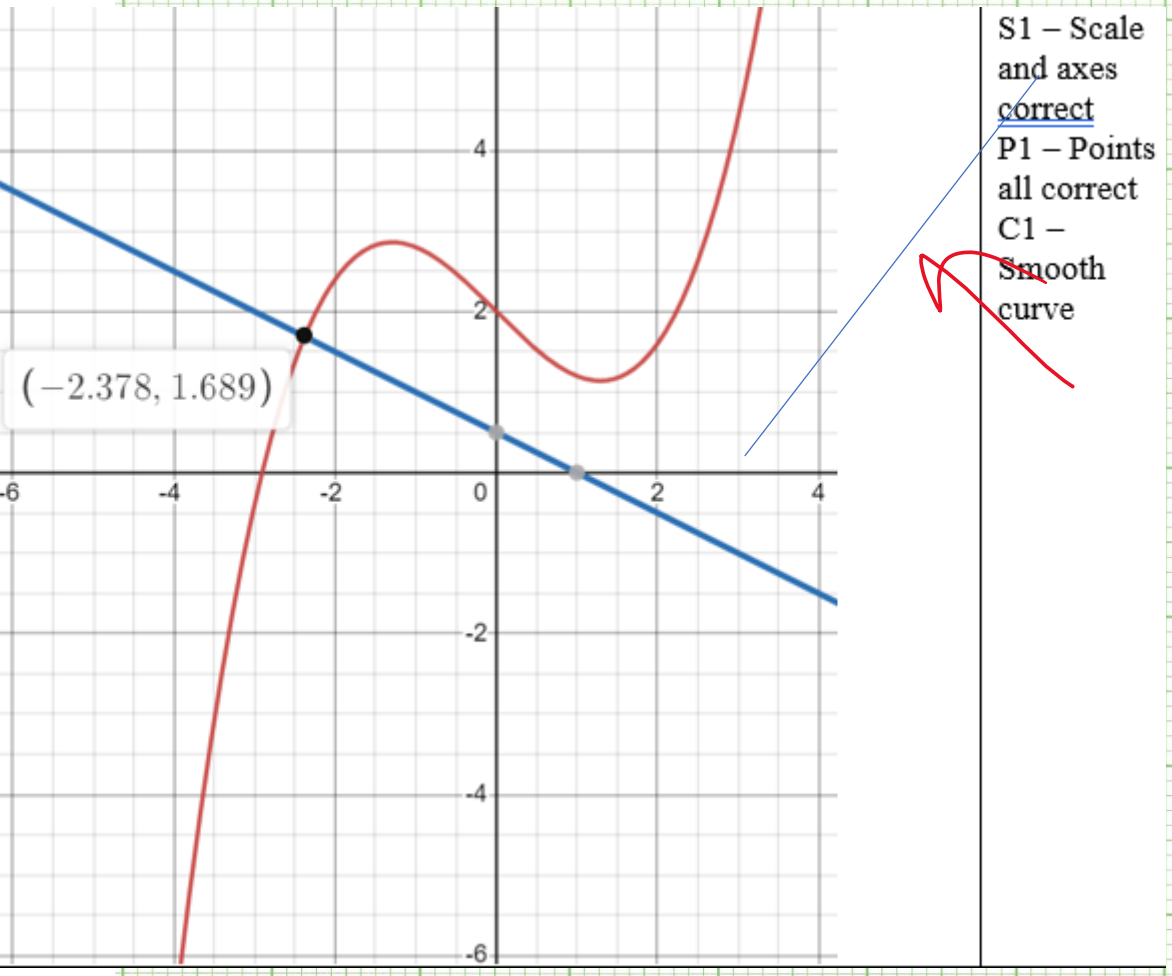
x	–4	–3	–2	–1	0	1	2	3
y	–6.8	p	2.4	2.8	2	1.2	1.6	4.4

(a) Calculate the value of p.

$p = -0.4$ | B1 *Answer* ..... [1]

(b) On the grid below, draw the graph of $y = \frac{x^3}{5} - x + 2$ for $-4 \leq x \leq 3$.

Use a scale of 2 cm to 1 unit for the horizontal x-axis and a scale of 1 cm to 1 unit for the vertical y-axis. [3]



S1 – Scale and axes correct
P1 – Points all correct
C1 – Smooth curve

(c) By drawing a tangent, find the gradient of the curve at (2, 1.6).

Exact gradient = 1.4 Give allowance of ~~0.7 to 2.1~~ 0.9 to 1.9	M1 – correct tangent that touches curve A1

Answer ……..…… [2]

(d) **(i)** On the same grid, draw the graph of $2y + x = 1$. [1]

(ii) Write down the x-coordinate of the point where the line intersects the curve.

$x = -2.3$ | B1

-2.5 to -2.1

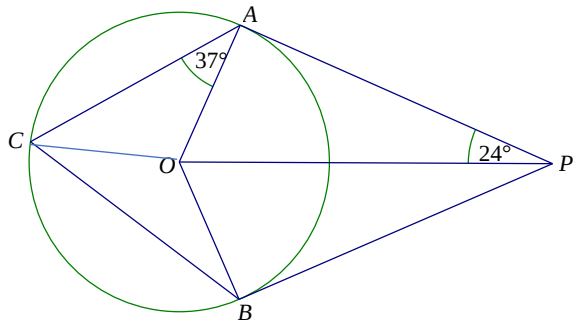
Answer x =……..…… [1]

(iii) Explain why the equation $2x^3 - 5x + 15 = 0$ has only 1 solution.

Evidence: $2x^3 - 5x + 15 = 0$ $2x^3 - 10x + 20 = -5x + 5$ $\frac{x^3}{5} - x + 2 = -\frac{x}{2} + \frac{1}{2}$	M1 for splitting into curve and line
Concept: At intersection, equation of curve = equation of line. Hence, since there is only 1 intersection, $2x^3 - 5x + 15 = 0$ only has 1 solution.	A1

[2]

18 *A*, *C* and *B* are points on a circle with centre *O*.
AP and *BP* are tangents to the circle.
Angle *CAO* = 37° and angle *APO* = 24°.



(a) Giving reasons for each step, find

(i) angle *AOB*,

$\angle PAO = 90°$ (tangent perpendicular to radius) $\angle AOP = 180° - 90° - 24°$ $= 66°$ (angle sum of triangle) $\angle AOB = 2 \times 66°$ $= 132°$ (tangents from external point)	M1 A1

Answer Angle *AOB* =…… [2]

(ii) angle *OBC*.

$\angle ACB = 132° \div 2 = 66°$ ($\angle$ at centre = 2 $\angle$ at circumference) Reflex $\angle AOB = 360° - 132° = 228°$ ($\angle$s at a point) $\angle OBC = 360° - 228° - 66° - 37° = 29°$ ($\angle$ sum of quadrilateral)	M1 M1 A1

or

$\angle ACB = 132° \div 2$ $= 66°$ ($\angle$ at centre = 2 $\angle$ at circumference) $\angle OPB = 24°$ (tangents from external point) $\angle OBC = 360° - 24° - 24° - 37° - 90° - 90° - 66°$ $= 29°$ ($\angle$ sum of quadrilateral)	M1 M1 A1

or

$\angle ACB = 132° \div 2$ $= 66°$ ($\angle$ at centre = 2 $\angle$ at circumference) $\angle OAB = \angle OBA$ $= (180° - 132°) \div 2$ $= 24°$ (base $\angle$s of isosceles Δ) $\angle OBC = 180° - 66° - 37° - 24° - 24°$ $= 29°$ ($\angle$ sum of Δ)	M1 M1 A1

Answer Angle *OBC* =…… [3]

The radius of the circle is 6 cm.

(b) Find the length of major arc *ACB*.

$\frac{360° - 132°}{360°} \times 2\pi(6)$	M1
$= 23.9$ cm (3 sf)	A1

Answer ……..……cm [2]

(c) **(i)** Explain why a second circle that passes through the points *A*, *O*, *B* and *P* can be drawn.

Answer ……

	Angles *PAO* and *PBO* are angles in semicircles *OAP* and *OBP* respectively.	B1
or &	$\angle OAP + \angle OBP = 90° + 90° = 180°$, so they are angles in opposite segments.	B1
or	$\angle AOB + \angle APB = 132° + 48° = 180°$, so they are angles in opposite segments	B1

…… [1]

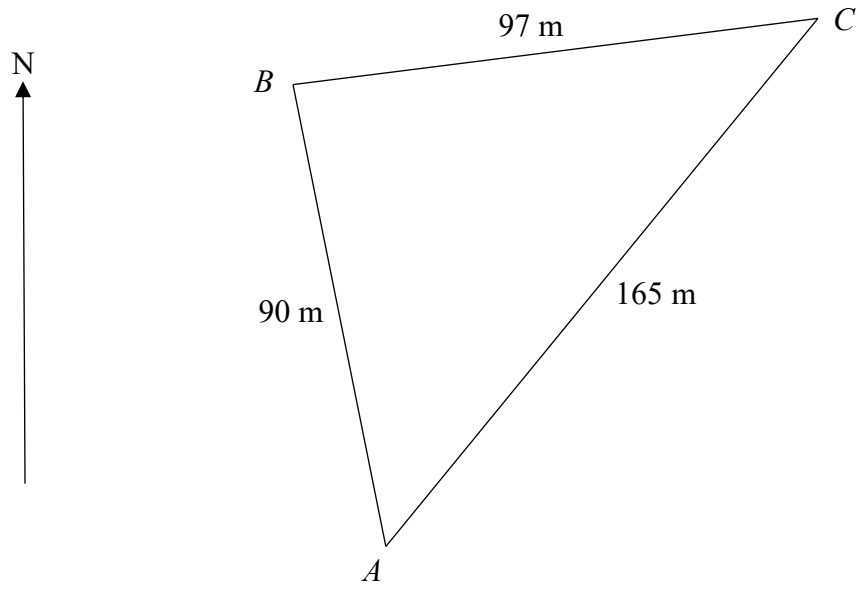
Therefore the 4 points forms a cyclic quadrilateral within the circle.

(ii) Calculate the area of the second circle.

$\sin 24° = \frac{AO}{OP}$	M1
$OP = \frac{6}{\sin 24°}$	M1
$= 14.751$ cm	
Area of second circle $= \pi(\frac{14.751}{2})^2 = 171$ cm^2 (3 sf)	A1

Answer ……..……cm^2 [3]

19 The diagram below shows a park *ABC* and a path is also located along *AC*.
$AB = 90\,\text{m}$, $BC = 97\,\text{m}$ and $AC = 165\,\text{m}$.



(a) Calculate

(i) angle *ABC*,

$$\text{Angle ABC} = \cos^{-1}\left(\frac{90^2 + 97^2 - 165^2}{2(90)(97)}\right) \quad \text{M1}$$
$$= 123.81$$
$$= 123.8° \quad \text{A1}$$

Answer ……..…….. [2]

(ii) angle *BAC*.

$$\text{Angle } BAC = \sin^{-1}\left(\frac{97 \times \sin 123.81}{165}\right) \quad \text{M1}$$
$$= 29.238$$
$$= 29.2° \quad \text{A1}$$

Answer ……..…….. [2]

(b) A tower is located at the point *A* and Mr Huang is standing at the point *B*.
Mr Huang estimates his angle of elevation to the top of the tower is 40°.
Calculate the estimated height of the tower.

Let height of tower be h.

$$\tan 40° = \frac{h}{90} \quad \text{M1}$$
$$h = 90 \tan 40$$
$$h = 75.5\,\text{m} \quad \text{A1}$$

Answer ……..……..m [2]

(c) From point *B*, Mr Huang decides to cut across the park to reach point *D* along the path *AC* in the shortest time.
Calculate the distance he would have to walk to reach *D* in the shortest time.

Let shortest distance be d.	
$\sin 29.238° = \frac{d}{90}$	M1
$d = 90 \sin 29.238$	
$d = 44.0$ m	A1
OR	
$\frac{1}{2} \times 90 \times 165 \times \sin 29.238° = \frac{1}{2} \times 165 \times d$	M1
$d = 44.0$ m	A1

Answer ……..……..m [2]

(d) Mr Huang claims that the angle of elevation to the top of the tower from *B* is larger than *D*.
Without any calculations and using mathematical concept, justify Mr Huang's claim.

Answer ……
……

Evidence: *AB* is the hypotenuse of the right angle triangle *ABD*.	M1
Concept: **Smaller ground distance will produce larger angle of elevation**.	
Conclusion: Since the height of tower is fixed, ground distance *AB* is larger than ground distance of *AD*, the **angle of elevation to the top of the tower from *B*** is **smaller** than *D*.	A1

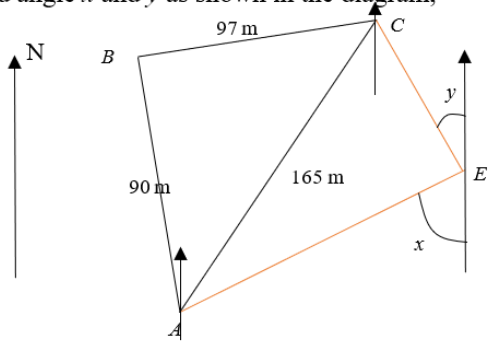
……*Therefore, his claim is incorrect.*
…… [2]

A toilet is located at point *E*.
The bearing of *E* from *A* is $070°$ and the bearing of *E* from *C* is $160°$.

(e) Show that triangle *CEA* is a right-angled triangle.

Answer

Marked angle x and y as shown in the diagram,



Angle $x = 70°$ (both alternate angles, // lines)	M1 for both angles
Angle $y = 20°$	
Angle $CEA = 180° - 70° - 20°$ (angle sum of triangle)	A1
$= 90°$	

Therefore triangle *CEA* is a right-angled triangle

[2]

(f) If $AE = 100$ m, find EC.

$EC = \sqrt{165^2 - 100^2}$ $= 131$ m	M1 A1

Answer ……..……..m [2]