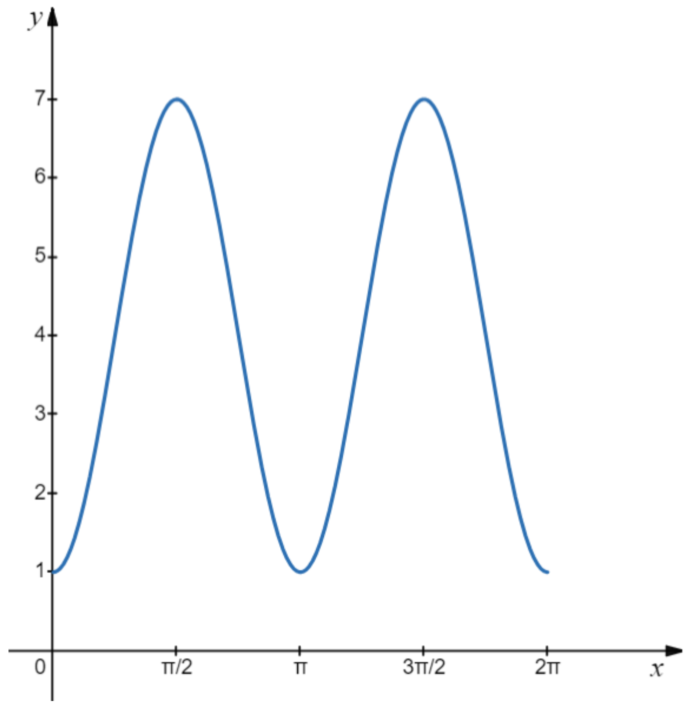
Marking Scheme AM P1 (4049/01)- Prelim 2024

Qn	Answer	Marks	Partial Marks	Guidance
1	$b^2-4ac<0$ $\left[2(p+2)\right]^2-4p(p+7)<0$ $4(p^2+4p+4)-4p^2-28p<0$ $4p^2+16p+16-4p^2-28p<0$ $-12p+16<0$ $12p-16>0$ $p>\frac{4}{3}$			
2	$y=3e^{2x}+2e^{-x}$ $\frac{dy}{dx}=6e^{2x}-2e^{-x}$ $\frac{d^2y}{dx^2}=12e^{2x}+2e^{-x}$ $\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=12e^{2x}+2e^{-x}+6e^{2x}-2e^{-x}+3e^{2x}+2e^{-x}$ $=21e^{2x}+2e^{-x}$ $A=21$ $B=2$			
3(a)	$\text{Period}=180^\circ \quad (or\ \ \pi)$ $\text{Amplitude}=3$			

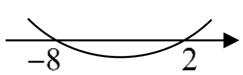
3(b)				
4	$\frac{d^2y}{dx^2}=3x-2$ $\frac{dy}{dx}=\int(3x-2)dx$ $=\frac{3x^2}{2}-2x+c$ $c=-13$ $\frac{dy}{dx}=\frac{3x^2}{2}-2x-13$ $y=\int\frac{3x^2}{2}-2x-13\ dx$ $=\frac{x^3}{2}-x^2-13x+d$ $-40=\frac{(4)^3}{2}-(4)^2-13(4)+d$ $d=-4$ $y=\frac{x^3}{2}-x^2-13x-4$			

5(a)	$\frac{1}{2}(3\sqrt{2}+\sqrt{5})BC=\frac{16+7\sqrt{10}}{2}$ $BC=\frac{16+7\sqrt{10}}{3\sqrt{2}+\sqrt{5}}$ $=\frac{\left(16+7\sqrt{10}\right)\left(3\sqrt{2}-\sqrt{5}\right)}{\left(3\sqrt{2}+\sqrt{5}\right)\left(3\sqrt{2}-\sqrt{5}\right)}$ $=\frac{48\sqrt{2}-16\sqrt{5}+21\sqrt{20}-7\sqrt{50}}{13}$ $=\frac{48\sqrt{2}-16\sqrt{5}+42\sqrt{5}-35\sqrt{2}}{13}$ $=\frac{13\sqrt{2}+26\sqrt{5}}{13}$ $=\sqrt{2}+2\sqrt{5}$			
5(b)	$AC^2=\left(3\sqrt{2}+\sqrt{5}\right)^2+\left(\sqrt{2}+2\sqrt{5}\right)^2$ $=18+6\sqrt{10}+5+2+4\sqrt{10}+20$ $=45+10\sqrt{10}$			
6(a)	$f'(x)=3\cos x+8\sin 2x$ $\cos x>0$ and $\sin 2x>0$ for $0<x<\frac{\pi}{2}$ $\therefore 3\cos x+8\sin 2x>0$ for $0<x<\frac{\pi}{2}$ $f'(x)>0$ for $0<x<\frac{\pi}{2}$ Hence f is an increasing function			
6(b)	$\frac{dy}{dx}=3\cos\frac{\pi}{6}+8\sin\left(2\times\frac{\pi}{6}\right)$ $=3\times\frac{\sqrt{3}}{2}+8\times\frac{\sqrt{3}}{2}$ $=11\frac{\sqrt{3}}{2}$ $\frac{dy}{dt}=\frac{11\sqrt{3}}{2}\times 2$ $=11\sqrt{3}$			

7(a)	General term $= \binom{10}{r}\left(\frac{2}{x^3}\right)^{10-r}\left(-x^2\right)^r$ $= \binom{10}{r}2^{10-r}\left(-1\right)^r x^{5r-30}$ when $5r-30=6$ $r=\frac{36}{5}$ Hence there is no term in x^6			
7(b)	$5r-30=0$ $r=6$ $\binom{10}{6}2^{10-6}\left(-1\right)^6=3360$ $\left(....+3360+...\right)\left(3-\frac{x^6}{8}\right)$ Coefficient of $x^6 = 3360\times\frac{-1}{8}$ $=-420$			

8(a)	$\frac{4}{y}=\frac{x}{x+9}$ (similar triangles) $xy=4x+36$ $y=4+\frac{36}{x}$			
8(b)	$A=\frac{1}{2}(x+9)y$ $=\frac{1}{2}(x+9)\left(4+\frac{36}{x}\right)$ $=2x+36+\frac{162}{x}$ $\frac{dA}{dx}=2-\frac{162}{x^2}$ when $\frac{dA}{dx}=0$, $2-\frac{162}{x^2}=0$ $2=\frac{162}{x^2}$ $x^2=81$ $x=9$ Minimum Area = $2\times 9+36+\frac{162}{9}$ $=72\text{ m}^2$			
9(a)	$\angle BCA=\angle ABC$ (given $AB=AC$) $\angle ABC=\angle CAF$ (Alternate segment theorem) $\therefore \angle BCA=\angle CAF$			
9(b)	$\angle OAE=\angle OAF=90^\circ$ (Radius perpendicular to tangent) $\angle BAE=\angle BCA$ (Alternate segment theorem) $\angle BAE=\angle CAF$ (Using part a) $\angle OAB=\angle OAC$ (Both 90° – Equal angles) Hence OA bisects angle BAC			
10	$\log_3(x-8)+\log_3 x=2$			

(a)	$\log_3 x(x-8)=2$ $x(x-8)=3^2$ $x^2-8x-9=0$ $(x-9)(x+1)=0$ $x=9 \quad or \quad x=-1$ (NA) $\therefore x=9$			
10 (b)	$\log_{\frac{1}{b}} a=\dfrac{1}{\log_a\left(\frac{1}{b}\right)}$ $=\dfrac{1}{\log_a 1-\log_a b}$ $=\dfrac{1}{-\log_a b}$ $=\dfrac{-1}{\left(\frac{1}{\log_b a}\right)}$ $=\dfrac{-1}{\left(\frac{1}{c}\right)}$ $=-c$			
11 (a)	$16-ax-x^2=16-\left(x^2+ax\right)$			

	$=16-\left[\left(x+\frac{a}{2}\right)^2-\frac{a^2}{4}\right]$ $=16+\frac{a^2}{4}-\left(\frac{a}{2}+x\right)^2$ $16+\frac{a^2}{4}=25$ $\frac{a^2}{4}=9$ $a^2=36$ $a=6$ $b=\frac{a}{2}$ $=3$ **Alternate solution** $16-ax-x^2=25-\left(b^2+2bx+x^2\right)$ $16-ax-x^2=25-b^2-2bx-x^2$ B2 $16=25-b^2$ $b^2=9$ $b=3$			
11 (b)	$y=25-(3+x)^2$ Max value of $y=25$ Corresponding value of $x=-3$			
11 (c)	$y>0$ $16-6x-x^2>0$ $x^2+6x-16<0$ $(x+8)(x-2)<0$  $-8<x<2$			
12 (a)	$\sin\left(\frac{\pi}{3}+x\right)-\sin\left(\frac{\pi}{3}-x\right)$			

<table>
<tr><td></td><td>$= \sin\frac{\pi}{3}\cos x + \cos\frac{\pi}{3}\sin x - \left(\sin\frac{\pi}{3}\cos x - \cos\frac{\pi}{3}\sin x\right)$
$= \sin\frac{\pi}{3}\cos x + \cos\frac{\pi}{3}\sin x - \sin\frac{\pi}{3}\cos x + \cos\frac{\pi}{3}\sin x$
$= 2\cos\frac{\pi}{3}\sin x$
$= 2\times\frac{1}{2}\times\sin x$
$= \sin x$</td><td></td><td></td><td></td></tr>
<tr><td>12
(b)</td><td>$\sin\left(\frac{\pi}{3}+2x\right) - \sin\left(\frac{\pi}{3}-2x\right) = \sin 2x$
$\sin 2x - 2 = 4\sin 2x$
$3\sin 2x = -2$
$\sin 2x = -\frac{2}{3}$
$\text{Basic angle} = 0.72973$
$2x = \pi + 0.72973, \quad 2\pi - 0.72973$
$x = 1.94, \quad 2.78$</td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>13
(a)</td><td>$\lg y = (\lg b)x + \lg a$</td><td></td><td></td><td></td></tr>
<tr><td>13
(b)</td><td colspan="3"><table><tr><td>$\lg y$</td><td>0.792</td><td>0.672</td><td>0.568</td><td>0.462</td><td>0.342</td><td>0.230</td></tr></table>
Straight line graph</td><td></td></tr>
<tr><td>13
(c)</td><td>$\lg a \approx 0.9$
$a \approx 7.94$

$\lg b \approx \frac{0.3-0.5}{2.7-1.8} \approx -0.222$
$b = 0.599$</td><td></td><td></td><td></td></tr>
<tr><td>13
(d)</td><td>when $x = 0.8$
$\lg y = 0.72$
$y = 5.23$</td><td></td><td></td><td></td></tr>
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