Tampines Secondary School

Sec 4&5 Express Additional Math Paper 1 2024 Marking Scheme

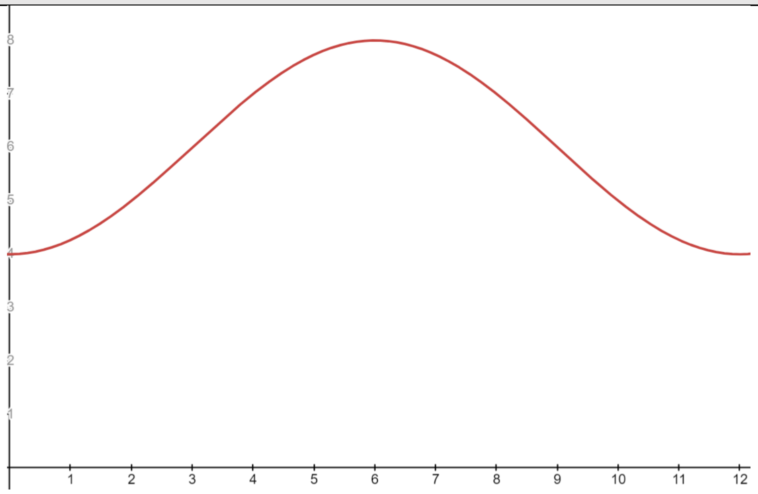
Total Marks: 90

√ = follow through

No.	Answers	Marks
1	$y = \frac{2x^2}{x+1}$ $\frac{dy}{dx} = \frac{(x+1)(4x) - (2x^2)(1)}{(x+1)^2}$ $= \frac{2x^2 + 4x}{(x+1)^2}$ Decreasing function → $\frac{dy}{dx} < 0$ Since $(x+1)^2 > 0$, $2x^2 + 4x < 0$ $2x(x+2) < 0$ $\therefore -2 < x < 0$	 M1 A1 M1 A1
2(a)	$-20x^2 + 120x + 3 = -20(x^2 - 6x) + 3$ $= -20[(x-3)^2 - 3^2] + 3$ $= -20(x-3)^2 + 183$	 M1 A1
(b)	$h_1(0) = 3$ $h_2(0) = 6.6$ TP-1 was fired from a height of 3 metres above ground while TP-2 was fired from a height of 6.6 metres above ground.	B1 B1
(c)	From TP-1's max pt (3, 183) and TP-2's max pt (6, 183), they both reach the <u>same height</u>. TP-1: $h = 0$ → $x = 6.02$ m TP-2: $h = 0$ → $x = 12.1$ m > 6.02 m Since TP-2 could reach a further distance from the launched position, compared to TP-1, TP-2 should be acquired.	B1 M1 B1
3	Height $= \frac{(2x+1)^2}{x^2(2x-1)}$ $= \frac{4x^2+4x+1}{x^2(2x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{2x-1}$ $4x^2 + 4x + 1 = Ax(2x-1) + B(2x-1) + Cx^2$ Let $x = 0$, $B = -1$ Let $x = \frac{1}{2}$, $4 = \frac{1}{4}C$ → $C = 16$ Compare coeff of x^2, $4 = 2A + 16$ → $A = -6$ $\therefore$ Height $= \frac{16}{2x-1} - \frac{6}{x} - \frac{1}{x^2}$	M1 A1 M1: either sub mtd or compare coeff A3

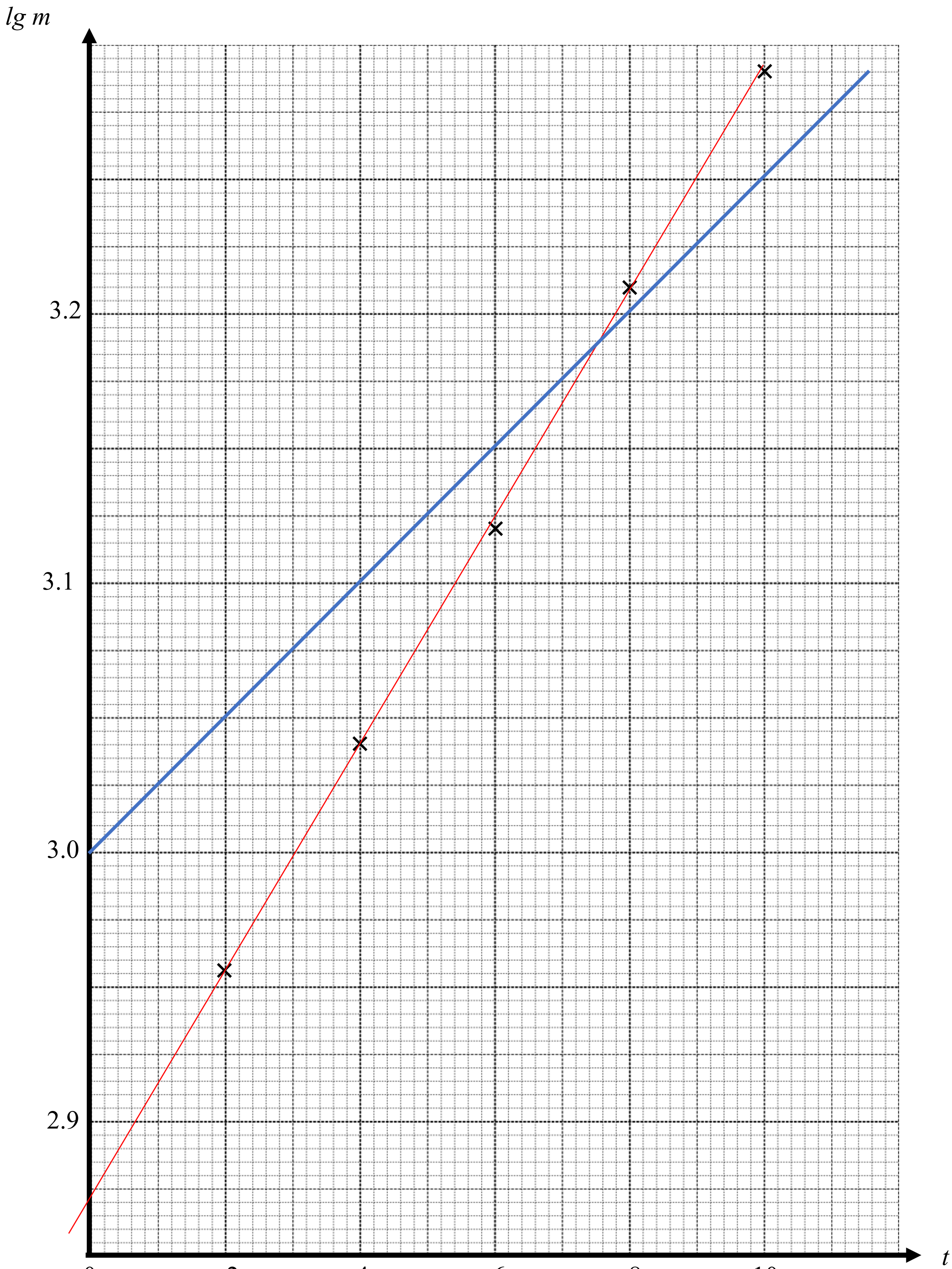
No.	Answers	Marks
4a(i)	$(2 + qx)^6 = 64 + 192qx + 240q^2x^2 + \ldots$	B3
(ii)	$(2 + px)(2 + qx)^6 = (2 + px)(64 + 192qx + 240q^2x^2 + \ldots)$ Term in x: $384q + 64p = 0$ $\rightarrow$ $p = -6q$ Term in x^2: $480q^2x^2 + 192pqx^2 = -168$ $480q^2 + 192(-6q)q = -168$ $-480q^2 = -168$ $q = \frac{1}{2}$ or $q = -\frac{1}{2}$ (rej since $q > 0$) $p = -3$	 M1 M1 A1 A1
(b)	$T_{r+1} = {}^{12}C_r\,(x^3)^{12-r}(-2)^r(x^{-1})^r$ $36 - 3r - r = 0$ $r = 9$ Term $= {}^{12}C_9\,(-2)^9$ $= -112\,640$	M1 M1 A1
5(a)	$f'(x) = \frac{18e^{-3x}}{-3} + c$ $= -6e^{-3x} + c$ When $x = 0$, $f'(x) = 2$, $-6e^{-3(0)} + c = 2$ $c = 8$ Stationary point, $f'(x) = -6e^{-3x} + 8 = 0$ $6e^{-3x} = 8$ $e^{-3x} = \frac{4}{3}$ $x = -\frac{1}{3}\ln\frac{4}{3}$ When $x = -\frac{1}{3}\ln\frac{4}{3}$, $f''(x) = 18e^{-3x}$ $= 24 > 0$ $\rightarrow$ point is minimum	M1 M1: subt $x = 0$ & $f'(x) = 2$ M1 A1 M1 A1
(b)	$f'(x) = -6e^{-3x} + 8$ $f(x) = \frac{-6e^{-3x}}{-3} + 8x + d$ $= 2e^{-3x} + 8x + d$ Subt $\left(1, \frac{2}{e^3}\right)$, $\frac{2}{e^3} = 2e^{-3} + 8 + d$ $\rightarrow$ $d = -8$ Hence eqn of curve is $f(x) = 2e^{-3x} + 8x - 8$	 M1 A1

No.	Answers	Marks
6(a)	$\frac{dy}{dx} = 2x + 6 - 2m$ At $x = 4$, $\frac{dy}{dx} = 0$ → $14 - 2m = 0$ $m = 7$ OR $y = (x + 3 - m)^2 - (3 - m)^2 + m + 5$ Min pt is at $x = 4$ → $(3 - m) = 4$ $m = 7$	M1 M1 A1 OR M1(complete the sq) M1 A1
(b)	When $m = 8$, $y = x^2 - 10x + 13 + p$ Lies above x-axis, discriminant < 0 → $(-10)^2 - 4(13 + p) < 0$ $48 - 4p < 0$ $p > 12$	B1 B1
8(a)	$\angle EDC = \angle DAC$ (alt seg thm) $\angle DAC = \angle DCA$ ($AD = CD$, isos triangle) $\therefore \angle EDC = \angle DCA$ Hence AC is parallel to DE (alt angles)	B1 B1 B1
(b)	$\angle BCD = \angle BAD$ (angles in semicircle) Let $\angle EDC = x$ $\angle EDC = \angle CBD = x$ (alt seg thm) $\angle DCA = \angle DAC = x$ (shown in part (a)) $\angle DAC = \angle ADM = x$ (alt angle, AD parallel DE) $\angle ADM = \angle ABD = x$ (alt seg thm) $\therefore \angle ABD = \angle CBD$ By AA, triangle ABD is similar to triangle CBD.	B1 A1 A1
9(a)	Period $= \frac{2\pi}{p}$ $12 = \frac{2\pi}{p}$ $p = \frac{\pi}{6}$ (shown)	A1
(b)	$a = 6$ $b = 2$	B2

No.	Answers	Marks
(c)	 	G1: Shape G1: period G1: max & min points
(d)	03 00 to 09 00 or 3 a.m. to 9 a.m. 15 00 to 21 00 or 3 p.m. to 9 p.m.	B1 B1
10(a)	$s = \int t^2 - 6t + 5 \; dt$ $= \frac{t^3}{3} - \frac{6t^2}{2} + 5t + c$ When $t = 0$, $s = 3$ $\therefore$ $s = \frac{t^3}{3} - 3t^2 + 5t + 3$	 M1 A1
(b)	When $v = 0$, $t^2 - 6t + 5 = 0$ $t = 5$ or $t = 1$ When $t = 0$, $s = 3$ When $t = 1$, $s = \frac{16}{3}$ When $t = 5$, $s = -\frac{16}{3}$ Total dist $= \left(\frac{16}{3} - 3\right) + \frac{16}{3} \times 2$ $= 13$ m	M1 A1 M1: for $t = 1$ or 5 M1 A1
(c)	$a = 2t - 6$ At $a = 0$, $t = 3$, $s = 0$ Hence it is <u>nearer to its initial starting position</u> which is 3 m away compared to point B which is $\frac{16}{3}$ m away.	M1 M1 A1
11(a)	Grad $= -3$ Eqn: $0 = -3(2) + c$ → $c = 6$ Equation of AD is $y = -3x + 6$	B1 B1

No.	Answers	Marks
(b)	$D(0, 6)$ Midpoint of AD is $(1, 3)$ Grad of bisector = $\frac{1}{3}$ Eqn: $3 = \frac{1}{3}(1) + d$ → $d = \frac{8}{3}$ Equation of perpendicular bisector is $y = \frac{1}{3}x + \frac{8}{3}$	M1 B1 A1
(c)	$11 - 3x = \frac{1}{3}x + \frac{8}{3}$ $11 - \frac{8}{3} = \frac{1}{3}x + 3x$ $x = \frac{5}{2}$ $y = \frac{7}{2}$ $\therefore C\left(\frac{5}{2}, \frac{7}{2}\right)$	M1 A1
(d)	$BC = \frac{1}{2}AD$ $B\left(\frac{5}{2}+1, \frac{7}{2}-3\right) = B\left(\frac{7}{2}, \frac{1}{2}\right)$ (shown)	A1
(e)	Area = $\frac{1}{2}\begin{vmatrix} 2 & \frac{7}{2} & \frac{5}{2} & 0 & 2 \\ 0 & \frac{1}{2} & \frac{7}{2} & 6 & 0 \end{vmatrix}$ = 7.5 sq units	M1 A1
12(a)	$\frac{dy}{dx} = \frac{12}{(4x-5)^2}$ For curve to have stationary point, $\frac{dy}{dx} = 0$. In this case, $\frac{dy}{dx} = \frac{12}{(4x-5)^2} \neq 0$ since $12 \neq 0$. Hence this curve <u>does not have</u> a stationary point.	M1 A1 B1
(b)	$\frac{dy}{dx} = \frac{12}{(4x-5)^2} = \frac{4}{3}$ $(4x-5)^2 = 9$ $x = 2$ *or* $\frac{1}{2}$ (rej since $x > \frac{5}{4}$) $y = 1$ $\therefore P(2, 1)$	M1 M1 A1

No.	Answers	Marks
(c)	Eq of normal: $1 = -\frac{3}{4}(2) + c$ $c = \frac{5}{2}$ → $y = -\frac{3}{4}x + \frac{5}{2}$ When $y = 0$, $x = \frac{10}{3}$ Area = $\frac{1}{2} \times \left(\frac{10}{3} - \frac{5}{4}\right) \times 1$ $= \frac{25}{24}$ or 1. 04 sq units	M1 M1 A1
7(a)	<table><tr><td>t</td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td></tr><tr><td>lg m</td><td>2.96</td><td>3.04</td><td>3.12</td><td>3.21</td><td>3.29</td></tr></table>	
(b)	$\lg m = \lg a + \frac{\lg b}{3} t$ lg a = 2.87 → a = 741 [accept 724, 733, 750] grad = $\frac{\lg b}{3} = \frac{3.04 - 2.96}{4 - 2}$ b = 1.32	M1 A1 A1
(c)	lg $m^{40} = t + 120$ lg $m = 0.025t + 3$ $\therefore t = 7.5$ weeks	G1 (correct line drawn on grid) A1


lg m
3.2
3.1
3.0
2.9
0
2
4
6
8
10
t