Anglo-Chinese School

(Independent)

THE BEST IS YET TO BE

PRELIMINARY EXAMINATION 2019

YEAR 6 IB DIPLOMA PROGRAMME

MATHEMATICS

HIGHER LEVEL

PAPER 1

Tuesday 17 September 2019

2 hours

Candidate Session Number

0	0	2	3	2	9				

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the writing paper provided. Fill in your session number on each answer sheet, and attach them to this examination paper using the string provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **Mathematics HL and Further Mathematics HL formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[100 marks]**

<table>
<tr><th colspan="2">Section A
(50 Marks)</th><th colspan="2">Section B
(50 Marks)</th></tr>
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This question paper consists of 11 printed pages including this cover page.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.

1. [Maximum mark: 5]

The discrete random variable X has probability density function $P(X = x) = k\left(\frac{2}{5}\right)^x$ for $x = 2, 3, \ldots$

(a) Show that $k = \frac{15}{4}$. [2]

(b) Find the cumulative distribution function $P(X \le x)$. [3]

2. [Maximum mark: 6]

Given that $f(x)=\sin x$, for $x\in[0,2\pi]$ and $g(x)=2x+\frac{\pi}{2}$ for $x\in[0,1]$,

(a) Determine if $fg(x)$ exists and find $fg(x)$, stating its domain clearly. [3]

(b) Sketch the graph of $y=fg(x)$, indicating clearly the positions of the axial intercepts and turning points. [3]

3. [Maximum mark: 6]

The points P , Q and R are $(2,5,1),(0,2,-1)$ and $(3,7,-2)$ respectively.

(a) Find the cartesian equation of the plane PQR. [4]

(b) Find the volume of the pyramid formed by the origin and the points $P,Q,$ and R. [2]

4. [Maximum mark: 6]

The curve C has equation $e^{x+2y} = 2$. Find the equation of the normal to C at the point where $x = 0$.

5. [Maximum mark: 5]

Consider the three planes below, where $k \in \mathbb{R}$ is a constant.

$$x + y + z = 0$$

$$2x + 3y + 3z = 2$$

$$3x - 10y + kz = -1$$

(a) Find the value of k for which the three planes do not meet at a unique point. [3]

(b) If $k = -9$, find the coordinates of the point of intersection of the three planes. [2]

6. [Maximum mark: 6]

By using a suitable substitution, find $\int \sqrt{x}\cos\sqrt{x}\ dx$.

7. [Maximum mark: 9]

(a) Use mathematical induction to prove that $\sum_{r=1}^{n}(-1)^r \frac{r^2+r+1}{r!}=(-1)^n \frac{n+1}{n!}-1$ for any positive integer n. [6]

(b) Hence, find the value of $\sum_{r=3}^{20}(-1)^r \frac{r^2+r+1}{r!}$ in the form $\frac{A}{B!}+C$. [3]

8. [Maximum mark: 7]

A continuous random variable X has probability density function defined by

$$f(x) = \begin{cases} e^{-x}, & 0 \le x \le 1 \\ \dfrac{1}{2e}, & 1 < x \le 3 \\ 0, & \text{otherwise} \end{cases} .$$

(a) Find $P(0 \le X \le 2)$, leaving your answer in terms of e. [3]

(b) Hence, explain why the median lies in $0 \le x \le 1$. [2]

(c) Find the median of X. [2]

Do **not** write solutions on this page.

Section B

Answer **all** questions in the answer sheets provided. Please start each question on a new page.

9. [Maximum mark: 17]

Let $y = \dfrac{x-1}{e^{x-3}}$ for $x > 0$.

(a) Prove that $\dfrac{dy}{dx} = e^{3-x} - y.$ [3]

(b) Find the maximum and minimum values of y in the interval $[1, 4]$. [4]

(c) Find the coordinates of the point of inflexion on the graph of $y = \dfrac{x-1}{e^{x-3}}$. [4]

[You do not need to justify that it is a point of inflexion]

(d) (i) State the range of values of x where the graph is concave down.

(ii) Give a reason why the graph of $y = \dfrac{x-1}{e^{x-3}}$ is decreasing for $x > 2$.

(iii) Find the equation of the horizontal asymptote of $y = \dfrac{x-1}{e^{x-3}}$. [6]

10. [Maximum mark: 15]

Let $f\left(x\right)=\dfrac{3x-8}{x-3}$, where $x\neq 3$.

(a) (i) Sketch the graph of $y=f\left(x\right)$, indicating clearly the positions of the axial intercepts and asymptotes.

(ii) Hence, sketch the graph of $y=\dfrac{3|x|-8}{|x|-3}$, indicating clearly the positions of the axial intercepts and asymptotes.

(iii) The graph of $y=f\left(x\right)$ undergoes the following transformations in succession

A : a translation by the vector $\begin{pmatrix}1\\2\end{pmatrix}$

B : a stretch by a factor of 2 units parallel to the y-axis.

Find the equation of the resulting function. [9]

(b) The equation $f\left(x\right)=x^2$ has roots α,β,γ.

(i) Write down the values of $\alpha+\beta+\gamma$ and of $\alpha\beta\gamma$.

(ii) Find the value of $\alpha^2+\beta^2+\gamma^2$.

(iii) Find, in the form $Ay^3+By^2+Cy+D=0$, a cubic equation whose roots are $\dfrac{1}{\alpha},\dfrac{1}{\beta}$ and $\dfrac{1}{\gamma}$. [6]

11. [Maximum mark: 18]

Let $\alpha=\left(\sqrt{3}+i\right)\left(1-i\right)$.

(a) (i) Find the modulus and argument of α.

(ii) Hence, find the exact value of $\cot\dfrac{\pi}{12}$. [8]

(b) Given that $n\in\mathbb{Z}^+$, find the smallest value of n that makes α^n a positive real number. [3]

(c) (i) Show that, for any complex number $z, r>0, -\pi<\theta\leq\pi$,

$$\left(z-re^{i\theta}\right)\left(z-re^{-i\theta}\right)=z^2-\left(2r\cos\theta\right)z+r^2$$

(ii) Solve the equation $z^4+|\alpha|^8=0$, listing the roots individually in the form $re^{i\theta}$, where $r>0$ and $-\pi<\theta\leq\pi$.

(iii) Hence, or otherwise, express $z^4+|\alpha|^8=0$ as the product of two quadratic factors with real coefficients, simplifying your answer as far as possible. [7]

END OF PAPER