PRELIMINARY EXAMINATION 2024 SECONDARY 4 MATHEMATICS 4052/02 Paper 2 Tuesday 20 August 2024 2 hours 15 minutes Candidates answer on the Question Paper MARKS SCHEME	Chung Cheng High School Chung	CHOOL (MAIN) chool Chung Cheng High School Chung Cheng High School chool Chung Cheng High School Chung Cheng High School
MATHEMATICS       4052/02         Paper 2       Tuesday 20 August 2024         Candidates answer on the Question Paper       2 hours 15 minutes	PRELIMINARY EXAM SECONDAF	INATION 2024
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Class:

**Class Register Number:** 

Name:

# Mathematical Formulae

Compound interest

Total amount = 
$$P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = 
$$\pi r l$$
  
Surface area of a sphere =  $4 \pi r^2$   
Volume of a cone =  $\frac{1}{3} \pi r^2 h$   
Volume of a sphere =  $\frac{4}{3} \pi r^3$   
Area of triangle  $ABC = \frac{1}{2} a b \sin C$   
Arc length =  $r \theta$ , where  $\theta$  is in radian  
Sector area =  $\frac{1}{2} r^2 \theta$ , where  $\theta$  is in radian

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$

**Statistics** 

Trigonometry

$$Mean = \frac{\Sigma f x}{\Sigma f}$$

Standard deviation = 
$$\sqrt{\frac{\Sigma f x^2}{\Sigma f} - \left(\frac{\Sigma f x}{\Sigma f}\right)^2}$$

# **TURN OVER FOR QUESTION 1**

1 (a) The table shows the total electricity consumption in Singapore over three years.

Year	2020	2021	2022
Total Electricity Consumption in Gigawatt Hours (GWh)	50 779	53 483	54 884

(i) In 2021, the electricity consumed by households took up 15.5% of the total electricity consumption. Calculate the amount of electricity consumed by households in 2021, correct to two significant figures.

Electricity consumed by household  $=15.5\% \times 53483$ 

= 8289.865 = 8300 GWh (2 s.f.)

Answer ..... GWh [1]

(ii) Calculate the percentage increase in the total electricity consumption from 2020 to 2022.

Percentage increase =  $\frac{54884 - 50779}{50779} \times 100\%$ = 8.08%

*Answer* .....% [2]

(iii) Express the 2020 electricity consumption in kilowatt hours (kWh), leaving your answer in standard form, correct to two significant figures.

 $50779 \times 10^9 \div 10^3 = 5.0779 \times 10^{10}$ = 5.1×10<sup>10</sup> kWh (2 s.f.)

Answer ..... kWh [1]

- (b) A microprocessor is in the shape of a cube where the sides are 5 mm in length.
  - (i) Find the maximum number of microprocessors that can be placed into a container with dimensions 10 cm by 2 cm by 8 cm.

$$\frac{10}{0.5} = 20$$
$$\frac{2}{0.5} = 4$$
$$\frac{8}{0.5} = 16$$
$$\therefore 20 \times 4 \times 16$$
$$= 1280$$

(ii) A model of the microprocessor was made to a scale of 10 : 1. Given that the surface area of microprocessor is 150 mm<sup>2</sup>, find the surface area of the model microprocessor in square centimetres.

Model : Actual (Linear Scale) 10:1 (Area Scale) 100:1

The surface area of the model is 100 times of the actual object.

Surface Area of Model =  $100 \times 150$ 

$$= 15000 \text{ mm}^{2}$$
$$= \frac{15000}{100} \text{ cm}^{2}$$
$$= 150 \text{ cm}^{2}$$

#### [Turn over

2 (a) Solve 
$$2x-7=3(1-3x)$$
.

$$2x-7 = 3(1-3x)$$
$$2x-7 = 3-9x$$
$$11x = 10$$
$$x = \frac{10}{11}$$

$$Answer x = \dots [2]$$

(b) Solve the inequalities 7x-1<13 and  $\frac{x+1}{2} \ge -2(x-2)$ .

$$\frac{x+1}{2} \ge -2(x-2)$$

$$7x-1 < 13 \qquad (x+1) \ge -4(x-2)$$

$$7x < 14 \qquad \text{and} \qquad x+1 \ge -4x+8$$

$$x < 2 \qquad 5x \ge 7$$

$$x \ge 1.4$$

$$\therefore 1.4 \le x < 2$$

(c) Rearrange the formula  $y = \frac{x^2 + 5}{7x^2}$  to make x the subject.

$$y = \frac{x^{2} + 5}{7x^{2}}$$

$$7x^{2}y = x^{2} + 5$$

$$7x^{2}y - x^{2} = 5$$

$$x^{2}(7y - 1) = 5$$

$$x^{2} = \frac{5}{7y - 1}$$

$$x = \pm \sqrt{\frac{5}{7y - 1}}$$

(d) Solve the equation 
$$\frac{2x}{1-x} - 3 = \frac{1}{2x-3}$$
.

Give your solutions correct to two decimal places.

$$\frac{2x}{1-x} - 3 = \frac{1}{2x-3}$$

$$2x(2x-3) - 3(1-x)(2x-3) = 1-x$$

$$4x^2 - 6x - 3(2x-3-2x^2+3x) = 1-x$$

$$4x^2 - 6x - 6x + 9 + 6x^2 - 9x = 1-x$$

$$10x^2 - 20x + 8 = 0$$

$$5x^2 - 10x + 4 = 0$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(5)(4)}}{2(5)}$$

$$x = 1.4472... \text{ or } 0.552786...$$

$$x = 1.45 \text{ or } 0.55 (2 \text{ s.f.})$$

*Answer x* = ..... or ...... [5]

3 The diagram shows two open containers, a right cylinder and an inverted frustum. The right cylinder has a base radius of 10 cm and a height of 20 cm. The frustum is formed by cutting a smaller cone off the bottom of a larger cone. The smaller cone that was cut off has a height of 4 cm. The larger cone has a base radius of 15 cm and a height of 12 cm.

Three identical spherical marbles, each of radius 3 cm, are placed into the cylindrical container and water is poured in to a depth of h cm.



(a) Find the volume of each marble, leaving your answer in terms of  $\pi$ .

Volume of each marble  $=\frac{4}{3}\pi(3)^3$ =  $36\pi$  cm<sup>3</sup>

(b) Find the volume of the frustum, leaving your answer in terms of  $\pi$ .

Volume of big cone 
$$=\frac{1}{3}\pi (15)^2 (12)$$
  
 $= 900\pi \text{ cm}^3$   
 $\frac{V_1}{V_2} = \left(\frac{l_1}{l_2}\right)^3$   
 $\frac{V_1}{900\pi} = \left(\frac{4}{12}\right)^3$   
 $V_1 = \frac{100}{3}\pi \text{ cm}^3$   
Volume of frustum  $= 900\pi - \frac{25}{6}\pi$   
 $= \frac{2600}{3}\pi \text{ cm}^3$ 

(c) All the water from the cylinder, without the marbles, is then poured into the empty frustum, filling it completely without any overflow.

Find the value of h, correct to two decimal places.

Volume of water and marbles 
$$=\frac{2600}{3}\pi + 3(36\pi)$$
  
 $=\frac{2924}{3}\pi \text{ cm}^3$   
 $\pi (10)^2 h = \frac{2924}{3}\pi$   
 $h = \frac{2924}{3}\pi$   
 $h = \frac{2924}{3}\pi$   
 $h = 9.75$ 

 $Answer h = \dots \qquad [3]$ 

(d) The exterior surface area of each container is painted. Find the total area painted.

Using Pythagoras' Theorem,  $l^2 = 12^2 + 15^2$  $l = \sqrt{369}$  cm Surface area of cone =  $\pi(15)(\sqrt{369})$ =905.22036.. cm<sup>2</sup>  $\frac{A_1}{A_2} = \left(\frac{l_1}{l_2}\right)^2$  $\frac{A_{\rm l}}{\pi(15)(\sqrt{369})} = \left(\frac{4}{12}\right)^2$  $A_1 = 100.58004..\,\mathrm{cm}^2$  $\frac{l_1}{15} = \frac{4}{12}$  $l_1 = 5 \text{ cm}$ Surface Area of frustum =  $(905.22036..-100.58004..) + \pi (5)^2$ = 804.6403..+78.53981= 883.18013... Total Surface area =  $883.18013... + (2 \times 10 \times \pi \times 20) + \pi (10)^2$ = 883.18013...+1256.63706..+314.1592... = 2453.9764..  $= 2450 \text{ cm}^2 (3s.f)$ 

Answer .....  $cm^2$  [6]

4 (a) 12 students from class A took a Mathematics test.

The table below shows the test marks of the students. However, **two** of the students' marks are covered with ink.



(i) Given that the box-and-whisker plot below shows the distribution of the results, explain why the information may not be sufficient to find the missing marks of the two students.



## Answer

The box-and-whisker plot is not able to provide individual data point. It can only provide the minimum, maximum mark, 25<sup>th</sup>, 50<sup>th</sup> and 75<sup>th</sup> percentile and these data are insufficient to calculate the missing marks.

(ii) Given further that the modal mark is 63, find the two missing marks.

Since mode = 63, one of the missing marks = 63 40 47 52 52 55 61 63 63 63 \_ 84 \_ 95

Since upper quartile = 72,

$$\frac{63+x}{2} = 72$$
$$63+x = 144$$
$$x = 81$$

11



Make a comment comparing the averages and a comment comparing the distribution of the Mathematics Test marks between the two classes. Use figures to support your answers.

Generally, Class A did better. The median of Class A is 62 marks which is higher than the median of Class B which is 56 marks.

Interquartile range of Class A = 72 - 52 = 20

Interquartile range of Class B = 60 - 52 = 8

Since the interquartile range of Class A = 20 > the interquartile range of Class B = 8, the distribution of Class A's marks is more spread out than Class B's.

[3]

**(b)** Alice rolled a six-sided die, X, 100 times. Bala rolled another six-sided die, Y, 80 times. The number of times they each obtained a '6' is recorded in the table.

Die	Number of rolls	Number of times '6' is obtained
X	100	16
Y	80	18

(i)

Find the probability of rolling a '6' by Alice and Bala respectively.  $\frac{4}{25}$  or 0.16 Answer P(obtaining a '6' by Alice) =  $\frac{9}{40}$  or 0.225 P(obtaining a '6' by Bala) =  $\frac{9}{40}$  or 0.225 [1]

One of the dice is biased. Using your answers in (b)(i), determine which die, X or (ii) *Y*, is likely to be the unbiased 6-sided die. Explain your answer. Answer

Alice's die is more likely to be an unbiased die as the probability of getting a '6'

is closer to that of an unbiased die.

[1]

5 (a) Complete the table of values for  $y = \frac{2}{x^2} - 2x$ .

Values are given to one decimal place where appropriate.

)	ĸ	-3	-2	-1	-0.5	-0.4	0.5	1	2	3
J	V	6.2	4.5	4	9	13.3	7	0	-3.5	-5.8

[1]

[1]

- (**b**) On the grid opposite, draw the graph of  $y = \frac{2}{x^2} 2x$  for  $-3 \le x \le 3$ . [3]
- (c) (i) On the same grid, draw the graph of y + x = 6 for  $-3 \le x \le 3$ .

x	-3	3
у	9	3

(ii) Write down the *x*-coordinates of the points where the line intersects the curve.

(iii) These values of x are solutions of the equation  $x^3 + Ax^2 + B = 0$ .

Find the value of *A* and of *B*.

$$6-x = \frac{2}{x^2} - 2x$$
  
Multiply  $x^2$  throughout,  
$$6x^2 - x^3 = 2 - 2x^3$$
$$6x^2 - 2 = -x^3$$
$$x^3 + 6x^2 - 2 = 0$$
$$A = 6, B = -2$$

*Answer A* = .....

 $B = \dots$ 





*A*, *B*, *C* and *D* are points on the circle with centre *O* and radius 5 cm. *AE* is a tangent to the circle at *A* and *OE* is a straight line that passes through *C*. Angle  $OEA = 28^{\circ}$  and CD = 5 cm.

(a) Find, giving a reason for each step of your working,

(i) angle OAD,

6

 $\angle OAE = 90^{\circ} \text{ (tangent perpendicular radius)}$   $\angle COA = 180^{\circ} - 90^{\circ} - 28^{\circ} \text{ (angle sum of triangle)}$   $= 62^{\circ}$   $\angle COD = 60^{\circ} \text{ (angle in an equilateral triangle)}$   $\angle AOD = 60^{\circ} + 62^{\circ}$   $= 122^{\circ}$   $\angle OAD = \frac{180^{\circ} - 122^{\circ}}{2} \text{ (base angles of an isosceles triangle)}$   $= 29^{\circ}$ 

Answer Angle  $OAD = \dots \circ [3]$ 

reflex 
$$\angle COA = 360^\circ - 62^\circ$$
 (angles at a point)  
= 298°  
 $\angle ABC = \frac{298^\circ}{2}$  (angle at center = twice angle at circumference)  
= 149°

Answer Angle  $ABC = \dots \circ [2]$ 

angle ABC.

**(ii)** 

(b) Find the area of the shaded region.

Area of sector 
$$OCD = \frac{60^{\circ}}{360^{\circ}} (\pi) (5)^{2}$$
  
 $= \frac{25}{6} \pi = 13.089969.. \text{ cm}^{2}$   
 $\tan 28^{\circ} = \frac{5}{AE}$   
 $AE = \frac{5}{\tan 28^{\circ}}$   
Area of triangle  $= \frac{1}{2} \times \frac{5}{\tan 28^{\circ}} \times 5$   
 $= \frac{25}{2 \tan 28^{\circ}}$   
 $= 23.50908.. \text{ cm}^{2}$   
Area of shaded region  $= 13.089969..+23.50908..$   
 $= 36.6 \text{ cm}^{2}$  (3s.f)

*Answer* ..... cm<sup>2</sup> [4]

7 (a) A, B, C and D are four points such that the coordinates of A and C are (4,2) and

(10,-34) respectively. 
$$\overrightarrow{AB} = \begin{pmatrix} -9 \\ -12 \end{pmatrix}$$
 and  $\overrightarrow{AD} = \begin{pmatrix} 24 \\ -12 \end{pmatrix}$ .

(i) Find the coordinates of *B*.

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$$
$$\begin{pmatrix} -9\\ -12 \end{pmatrix} = \overrightarrow{OB} - \begin{pmatrix} 4\\ 2 \end{pmatrix}$$
$$\overrightarrow{OB} = \begin{pmatrix} -5\\ -10 \end{pmatrix}$$
$$\therefore B(-5, -10)$$

Answer (.....) [1]

(ii) Find 
$$|\overrightarrow{AB}|$$
.  
 $|\overrightarrow{AB}| = \sqrt{9^2 + 12^2}$   
= 15

Answer  $\left| \overrightarrow{AB} \right| = \dots$  units [1]

It is given that *E* is a point on *BD* such that  $\overrightarrow{BE} = \frac{1}{3}\overrightarrow{BD}$ .

(iii) Show that *A*, *C* and *E* are collinear. *Answer* 

$$\overrightarrow{BE} = \frac{1}{3} \begin{pmatrix} 33 \\ 0 \end{pmatrix} = \begin{pmatrix} 11 \\ 0 \end{pmatrix}$$
$$\overrightarrow{BE} = \overrightarrow{BA} + \overrightarrow{AE}$$
$$\begin{pmatrix} 11 \\ 0 \end{pmatrix} = \begin{pmatrix} 9 \\ 12 \end{pmatrix} + \overrightarrow{AE}$$
$$\overrightarrow{AE} = \begin{pmatrix} 2 \\ -12 \end{pmatrix}$$
$$\overrightarrow{AC} = \begin{pmatrix} 10 \\ -34 \end{pmatrix} - \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$
$$= \begin{pmatrix} 6 \\ -36 \end{pmatrix}$$
$$\therefore \overrightarrow{AC} = 3 \begin{pmatrix} 2 \\ -12 \end{pmatrix} = 3\overrightarrow{AE}$$

Since  $\overrightarrow{AE} = \frac{1}{3}\overrightarrow{AC}$  and with a common point *A*, therefore *A*, *C* and *E* are collinear.

- (b) A is the point (2, 6), B is the point (-4, -2) and C is the point (6, -2). A line L passes through point C and is parallel to AB.
  - (i) Find the equation of line *L*.

$$m_{AB} = \frac{6 - (-2)}{2 - (-4)}$$
$$= \frac{8}{6} = \frac{4}{3}$$

equation of line L:

$$y - (-2) = \frac{4}{3}(x - 6)$$
$$y + 2 = \frac{4}{3}x - 8$$
$$y = \frac{4}{3}x - 10$$

Point D lies on line L such that AD //BC.

(ii) Find the coordinates of point *D*.

$$AD = BC = 10 \text{ units}$$
  
Since  $AB / / CD$  and  $AD = AC$ ,  $ABCD$  is a parallelogram.  
thus  $AD / / BC$ .  
 $x_D = 2 + 10 = 12$   
 $y_D = \frac{4}{3}(12) - 10$   
 $= 6$   
 $\therefore D(12, 6)$ 

Answer D ( ..... ) [2]

(iii) Show that *ABCD* is a rhombus.

Answer

$$AB = \sqrt{(2 - (-4))^2 + (6 - (-2))^2}$$
  
=  $\sqrt{100} = 10$  units  
$$CD = \sqrt{(6 - (12))^2 + (-2 - 6)^2}$$
  
=  $\sqrt{100} = 10$  units  
$$BC = \sqrt{(6 - (-4))^2 + (-2 - (-2))^2}$$
  
=  $\sqrt{100} = 10$  units  
Since  $AD = BC = CD = AB = 10$  units,  $\therefore ABCD$  is a rhombus.

[2]

## [Turn over

8 (a) The figure shows the first three figures of a sequence formed by sticks of the same size.



The number of small squares formed in each of the figures is recorded in the table.

Figure Number	Number of Squares
1	$(1+1)^2 - 1$
2	$(2+1)^2 - 1$
3	$(3+1)^2 - 1$

(i) Find the number of small squares formed in Figure 5.number of small squares formed in Figure 5 = 35

(ii) Find, in terms of *n*, an expression for the number of squares in Figure *n*.

expression for the number of squares =  $(n+1)^2 - 1$ 

(iii) Explain why the sum of the number of squares in two consecutive figures is always odd.

Answer

$$\left( \left( n+1 \right)^2 -1 \right) + \left( \left( n+1+1 \right)^2 -1 \right)$$
  
=  $n^2 + 2n + 1 - 1 + n^2 + 4n + 4 - 1$   
=  $2n^2 + 6n + 3$   
=  $2(n^2 + 3n) + 3$ 

Since  $2(n^2 + 3n)$  is a multiple of 2 which is an even number, and that 3 is an odd number therefore  $2(n^2 + 3n) + 3$  will always be odd.

Sequence number	<i>n</i> <sup>th</sup> term
1	4 <i>n</i> +3
2	7 <i>n</i> +1
3	14 <i>n</i>
4	8 <i>n</i> -1

(b) The table below shows the  $n^{\text{th}}$  terms of 4 sequences.

For each sequence, are the numbers in the sequence always multiples of 7, sometimes multiples of 7 or never multiples of 7?

Write down the letter 'A', 'S' or 'N' to represent your answer.

- **A** Always multiples of 7
- **S** Sometimes multiples of 7
- **N** Never multiples of 7

Answer		
Sequence 1	S	
Sequence 2	N	
Sequence 3	A	
Sequence 4	S[2	]

**9** Sam is a 40-year-old man who earns a gross salary of \$8000 a month. He is a Singaporean and an employee of a Singapore firm. Sam is required to put a certain percentage of his monthly gross salary into his Central Provident Fund (CPF) account.

CPF is a mandatory (social security) savings scheme funded by contributions from employers and employees. The table below shows the CPF contribution rates by employers and employees.

Employee's age	CPF Contribution Rates from 1 January 2024 (Monthly gross salary > \$750)				
(Years)	Total (% of gross salary)	By Employer (% of gross salary)	By Employee (% of gross salary)		
55 and below	37	17	20		
Above 55 to 60	31	15	16		
Above 60 to 65	22	11.5	10.5		
Above 65 to 70	16.5	9	7.5		
Above 70	12.5	7.5	5		

(a) Find the amount of money Sam's employer must contribute to his CPF monthly.

amount of money his employer must contribute =  $\frac{17}{100} \times \$8000$ = \$1360

Answer \$..... [1]

Sam wants to plan his monthly savings and has tabulated his monthly expenditure as shown in the table. He hopes to save at least 20% of his salary each month after CPF deductions.

Expenditure	Amount (\$)
Food and Groceries	820
Transportation	90
Insurance and Healthcare	1000
Phone and Internet Subscriptions	80
Utilities	300
Housing Loan	1000
Leisure and Entertainment (movies, sports, books subscription fees, dining etc)	1580

(b) Determine whether Sam is able to achieve his saving goals by clearly showing your calculations.

amount of money he must contribute to  $CPF = \frac{20}{100} \times \$8000$ = \\$1600

> total expenditure = 820 + 90 + 1000 + 80 + 300 + 1000 + 1580= \$4870 Savings = 8000 - 1600 - 4870

Savings percentage =  $\frac{1530}{8000 - 1600} \times 100\%$ = 23.90625..% = 23.9% (3s.f.)

Since 23.9% > 20%, therefore Sam is able to save the amount he hopes to.

.....[4]

(c) After a year of saving, Sam decides to invest \$15 000 of his savings for a period of 5 years. He comes up with 2 investment plans.

Plan A						
Sam can invest his money in a	Sam can invest his money in a 5-year savings bond at the beginning of 2025. The interest					rest
rate for the 5-year term is show	vn in the tal	ble. The say	vings bond	compound	s annually.	
-			<u> </u>	*	2	
	I				[]	
Year from issue date	1	2	3	4	5	
Interest per year %	3.19	3.19	3.20	3.28	3.31	

Plan B Sam can invest his money at the beginning of 2025 into an insurance company's fixed deposit account for a period of 5 years, compounded yearly. The interest rate is fixed over the duration of the investment and is determined by the year of issuance. For example: If Sam had invested his money in 2020, over a period of 5 years, his investment plan would be at an interest of 4.3% compounded yearly over the period of 5

However, the interest rate for 2025 is not yet available. Sam finds the information about the interest rates offered by the insurance company for the last 5 years.



years.

Determine which plan Sam should choose.

Justify the decision you make and show your calculations clearly.

Answer

<u>Plan A</u>

After first 2 years	$A = 15000 \left( 1 + \frac{3.19}{100} \right)^2$
	=\$15972.26415
After 3 <sup>rd</sup> year	$A = (15972.26415)\left(1 + \frac{3.2}{100}\right)$
	= \$16483.3766
After 4 <sup>th</sup> year	$A = \left(16483.3766\right) \left(1 + \frac{3.28}{100}\right)$
	=\$17024.03136
After 5 <sup>th</sup> year	$A = (17024.03136)\left(1 + \frac{3.31}{100}\right)$
	= \$17587.52679
	= \$17587.53 (nearest cent)

<u>Plan B</u>

Average interest rate = 
$$\frac{4.3 + 3.2 + 3.5 + 3.7 + 3}{5}$$
  
= 3.54  
Amount after 5 years =15000 $\left(1 + \frac{3.54}{100}\right)^5$   
= \$17849.75 (nearest cents)

Since \$17849.75 > \$17587.53, Sam should consider Plan B.

.....[5]