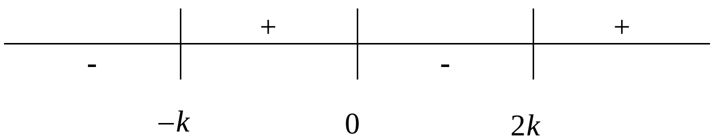
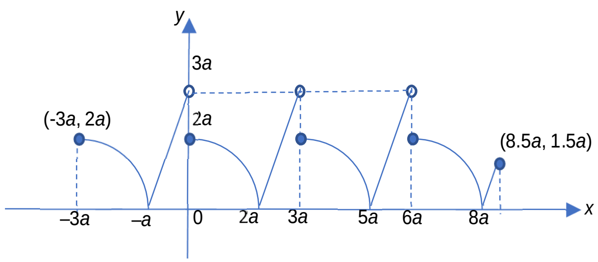
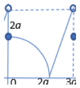
1	Suggested Solution	Marker's Comments
(a)	By GC, $(2,-1,-1)$	**Concepts and/or Skills:** • Point of intersection between 3 planes • Solving 3 linear equations using the G.C. **Learning points:** • $\begin{pmatrix}2\\-1\\-1\end{pmatrix}$ is the position vector of a point, not the coordinates of a point.
(b)	By GC, an equation of the line of intersection between p_1 and p_2 $\mathbf{r}=\begin{pmatrix}9/8\\-1/4\\0\end{pmatrix}+\mu\begin{pmatrix}-7/8\\3/4\\1\end{pmatrix},\ \mu\in\mathbb{R}$ For the line to lie on the p_3, line and normal vector are perpendicular, i.e. $\mathbf{m}\cdot\mathbf{n}=0$ $\begin{pmatrix}-7/8\\3/4\\1\end{pmatrix}\bullet\begin{pmatrix}\alpha\\-3\\4\end{pmatrix}=0$ $-\frac{7}{8}\alpha+\frac{3}{4}(-3)+4=0$ $\alpha=2$ For the line to lie on the p_3, a point on the line must lie on the plane, i.e. $\mathbf{a}\cdot\mathbf{n}=D$	**Concepts and/or Skills:** • The normal of a plane is perpendicular to the direction vector of a line that lies on the same plane. • For a line that lies on a plane, all the points on the line lies on the plane. **Common mistakes :** • Greek letter α (read as alpha) is different from English letter *a*. They should not be used interchangeably. Similarly, for the letter β (read as beta) and the letter *b*. **Learning points:** • Given that line *l* lies on a plane *p*, when the equation of *l* is substituted into equation of *p*, there will be infinitely many solution since there are infinitely many points that lie on the line and the plane. Hence students who are stuck at this working $\begin{pmatrix}9/8-7/8\,\mu\\-1/4+3/4\,\mu\\\mu\end{pmatrix}\bullet\begin{pmatrix}\alpha\\-3\\4\end{pmatrix}=\beta$ need to know that they will not be able to

	$\beta = \begin{pmatrix} 9/8 \\ -1/4 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$ $= \frac{9}{8}(2) + \left(-\frac{1}{4}\right)(-3)$ $= 3$		proceed if they do not know that for every real value substituted as μ, there will be an equation formed in terms of α and β.

2	Suggested Solution	Marker's Comments
(a)	$\dfrac{2x^2-2k^2}{x}<x+k$ $\dfrac{2\left(x^2-k^2\right)}{x}-\left(x+k\right)<0$ $\dfrac{2\left(x-k\right)\left(x+k\right)-x\left(x+k\right)}{x}<0$ $\dfrac{\left(2x-2k-x\right)\left(x+k\right)}{x}<0$ $\dfrac{\left(x-2k\right)\left(x+k\right)}{x}<0$  $x<-k$ or $0<x<2k$	**Concepts and/or Skills:** • Solving inequalities **Learning points:** • Completing the square gives more rooms for mistakes. Do factorization if possible. • For such question with unknowns, substitute a value for *k*, say $k=1$ (since question stated that *k* is a positive constant) to conduct the "sign" test easily. After which, remember to express the answer in terms of *k*.
(b)	Let $k=\sqrt{5}$ and replacing *x* with $\lvert x\rvert$. $\dfrac{2\lvert x\rvert^2-10}{\lvert x\rvert}<\lvert x\rvert+\sqrt{5}$ $\dfrac{2x^2-10}{\lvert x\rvert}<\lvert x\rvert+\sqrt{5}$ since $\lvert x\rvert^2=x^2$ Hence, $\lvert x\rvert<-\sqrt{5}$ (rejected $\because \lvert x\rvert\geq 0$) or $0<\lvert x\rvert<2\sqrt{5}$ $-2\sqrt{5}<x<0$ or $0<x<2\sqrt{5}$ (Also, $-2\sqrt{5}<x<2\sqrt{5}$ and $x\neq 0$)	**Concepts and/or Skills:** • Identify appropriate substitution to solve questions using a known result. • Solving inequality that involves modulus function. **Learning points:** • Students should sketch a simple graph of $y=\lvert x\rvert$ to solve for $0<\lvert x\rvert<2\sqrt{5}$. Please note that the skill for solving equations and inequalities is different. Please revise thoroughly. • Students need to observe from the question that $x\neq 0$ and exclude zero from the final solution.

3	Suggested Solution	Marker's Comments
(a)	$y=(1+\tan^{-1}x)^2$ Differentiating with respect to x, $\frac{\mathrm{d}y}{\mathrm{d}x}=2(1+\tan^{-1}x)\left(\frac{1}{1+x^2}\right)$ $(1+x^2)\frac{\mathrm{d}y}{\mathrm{d}x}=2(1+\tan^{-1}x)$ Differentiating with respect to x again, $(1+x^2)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2x\frac{\mathrm{d}y}{\mathrm{d}x}=2\left(\frac{1}{1+x^2}\right)$ $(1+x^2)^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2x(1+x^2)\frac{\mathrm{d}y}{\mathrm{d}x}=2$ (shown)	**Learning objectives:** - Apply $\frac{\mathrm{d}}{\mathrm{d}x}\left[\mathrm{f}^n\right]=n\,\mathrm{f}'\mathrm{f}^{n-1}$ - Apply $\frac{\mathrm{d}}{\mathrm{d}x}\left[\tan^{-1}\mathrm{f}\right]=\frac{\mathrm{f}'}{1+\mathrm{f}^2}$ - Apply implicit differentiation. Unwanted method: Many students took the inefficient way to prove from LHS to RHS, which is not the elegant way.
(b)	Differentiate $(1+x^2)^2\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+2x(1+x^2)\frac{\mathrm{d}y}{\mathrm{d}x}=2$ with respect to x, $(1+x^2)^2\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+4x(1+x^2)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+(2+6x^2)\frac{\mathrm{d}y}{\mathrm{d}x}+2x(1+x^2)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$ $(1+x^2)^2\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+6x(1+x^2)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+(2+6x^2)\frac{\mathrm{d}y}{\mathrm{d}x}=0$	**Learning objectives:** - Apply implicit differentiation. Though many students did not group the common terms together and they left the answer as $(1+x^2)^2\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+4x(1+x^2)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+(2+6x^2)\frac{\mathrm{d}y}{\mathrm{d}x}+2x(1+x^2)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$, mark is still awarded. Some students went to expand $(1+x^2)^2$ which is unnecessarily.
(c)	When $x=0$, $y=1$, $\frac{\mathrm{d}y}{\mathrm{d}x}=2$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2$, $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=-4$ Hence the Maclaurin's series for y is $y\approx1+2x+x^2-\frac{2}{3}x^3$.	**Learning objectives:** - Apply Maclaurin's series from MF26. For those who could not get the answer, mainly is due to careless calculation.
(d)	Using the Maclaurin's Series for y, $y\approx1+2x+x^2-\frac{2}{3}x^3$.	**Learning objectives:**

	It was observed that the gradient of the tangent to the curve at x = 0 is 2. Hence the gradient of normal is $-\frac{1}{2}$. From (c), $y=1$ when $x=0$. Therefore, the equation of normal to the curve at x = 0 is $y=-\frac{1}{2}x+1$.	- Deduce gradient of tangent to the curve at x = 0 from the Maclaurin's Series obtained. Unwanted method: Again, many students were not able to deduce the gradient of the normal from the Maclaurin's Series obtained in part (c). They took the longer method to find the equation of normal.

4	Suggested Solution	Marker's Comments
(a)	$y=\sqrt{4a^2-x^2} \Rightarrow y^2=4a^2-x^2 \Rightarrow x^2+y^2=(2a)^2$ $\int_0^{2a}\sqrt{4a^2-x^2}\ \mathrm{d}x$ represents the area of a quadrant of a circle, $y=\sqrt{4a^2-x^2}$, with centre at the origin and radius $2a$ units. Thus the area is $\frac{1}{4}\pi(2a)^2=\pi a^2$.	**Learning objectives:** - Relating integral to area bounded by a function and x-axis. - Identify the function Students are to **explain** in words what the integral represents.
(b)	$\mathrm{f}(8.5a)=\mathrm{f}(5.5a)=\mathrm{f}(2.5a)=3(2.5a)-6a=1.5a$	**Learning objectives:** - Understand and apply what is meant by a periodic function i.e. $\mathrm{f}(x+3a)=\mathrm{f}(x)$ for all real values of x. This part is well done.
(c)		**Learning objectives:** - Sketch of a periodic and piecewise function. This part was badly performed as there are students who do not know how to sketch a piecewise and periodic function despite that such common questions are found in tutorial and revision packages. It appeared more than once in A Level TYS. Many students forgot to indicate whether the endpoints are inclusive or exclusive, resulting in mark deduction.
(d)	$\int_0^{\frac{599}{2}a}\mathrm{f}(x)\ \mathrm{d}x=99\left[\pi a^2+\frac{1}{2}a(3a)\right]+\pi a^2+\frac{1}{2}(0.5a)(1.5a)$ $=100\pi a^2+\frac{297}{2}a^2+\frac{3}{8}a^2$ $=a^2\left(100\pi+\frac{1191}{8}\right)$	**Learning objectives:** - To identify and apply formula for finding area of a right-angle triangle. - To deduce that the area of  can be found be adding the area of the quadrant from (a) and area of a right-angle triangle without doing any integration. This part was badly performed. Many students were not able to infer.

<table>
<tr><th>5</th><th>Suggested Solution</th><th>Marker's Comments</th></tr>
<tr><td>(a)</td><td>

$y=\dfrac{4x-9}{5-2x}=-2+\dfrac{1}{5-2x}$

$y+2=\dfrac{1}{5-2x}$

Replace y with $(y-2)$, we have $y=\dfrac{1}{5-2x}$.

Replace x with $\dfrac{1}{2}x$, we have $y=\dfrac{1}{5-x}$.

Replace x with $(x+5)$, we have $y=\dfrac{1}{5-(x+5)}=-\dfrac{1}{x}$.

(1) Translate the graph in the positive *y*-direction by 2 units.
(2) Scale the graph parallel to the *x-axis* by a factor 2.
(3) Translate the graph in the negative *x*-direction by 5 units.

Alternatively,

Replace y with $(y-2)$, we have $y=\dfrac{1}{5-2x}$.

Replace x with $(x+2.5)$, we have

$y=\dfrac{1}{5-2(x+2.5)}=\dfrac{1}{-2x} \quad \Rightarrow \quad 2y=-\dfrac{1}{x}$.

Replace y with $\dfrac{1}{2}y$, we have $y=-\dfrac{1}{x}$.

(4) Translate the graph in the positive *y*-direction by 2 units.
(5) Translate the graph in the negative *x*-direction by 2.5 units.
(6) Scale the graph parallel to the *y-axis* by a factor 2.

</td><td>

Concepts and/or Skills:

- Transformations of Graphs (Scaling and Translation)

Common mistakes :

- Associate the replacements with the wrong transformations.
- Fail to use the terms accurately.
- Read the question wrongly.

Learning points:

1. When we are dealing with an improper rational function for transformation of graphs, usually we should perform a long division first.

2. Spend some time revising how to associate different replacements with their corresponding transformations. We should special attention to the direction in translation and the factor in scaling.

3. Spend some time learning the technical terms, such as "translate in the positive/negative x/y-direction by _ units", "scale parallel to the x/y-axis by a factor _" . Avoid using layman words such as "shift", "move", "stretch", "compress" and so on.

4. Read the question carefully. Misreading usually will result in losing all or most of the marks.

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<table>
<tr>
<td>(b)</td>
<td>

$$y=\sqrt{1+\frac{x^2}{q^2}}$$

$$y^2=1+\frac{x^2}{q^2}$$

$$\frac{y^2}{1^2}-\frac{x^2}{q^2}=1$$

It is the upper half of a hyperbola (or the part of a hyperbola above the y-axis)
</td>
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Concepts and/or Skills:

- Conics

Common mistakes :

- Associate the equation of a conics to a wrong name.
- Fail to recognize the impact of positive square root in manipulating the equation to the standard form.
- Classification of curve is not determined how it looks like, but its equation.
- Do not know the proper name.
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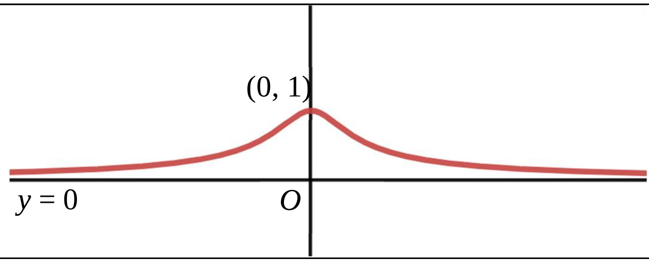
<tr>
<td>(b)(i)</td>
<td>



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Concepts and/or Skills:

- Transformations of Graphs (Reciprocal and derivative graphs)

Common mistakes :

- Fail to label the coordinates and the equations of asymptotes.
- Use arbitrary values instead of q in answering the question.
- Fail to identify the horizontal asymptotes on the derivative graph of a hyperbola.
- Read the question wrongly.

Learning points:

1. Hyperbola has oblique asymptotes, so its derivative graph has horizontal asymptotes whose equations can be obtained by differentiating the equations of oblique asymptotes.

2. A stationary point (not on the x-axis) will remain as a stationary point on the reciprocal graph. (can be shown by differentiating $\frac{1}{\mathrm{f}(x)}$)

3. GC can give great help in answering both parts by choosing a value for q. Learn how to di it if you have no idea. But remember to use back the unknown constant given in the question in your answer
</td>
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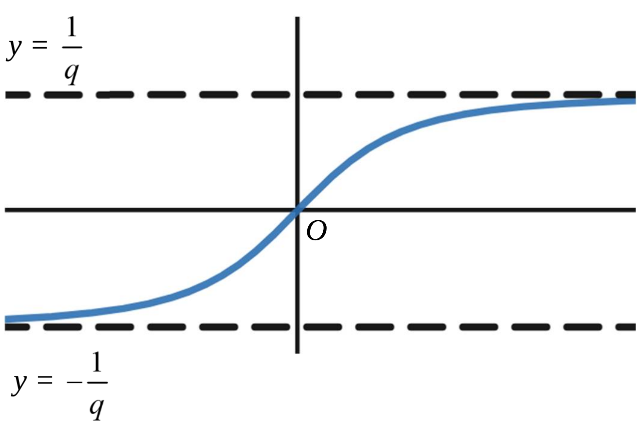
<td>(b)(ii)</td>
<td>



The original hyperbola has asymptotes $y=\pm\frac{x}{q}$.
</td>
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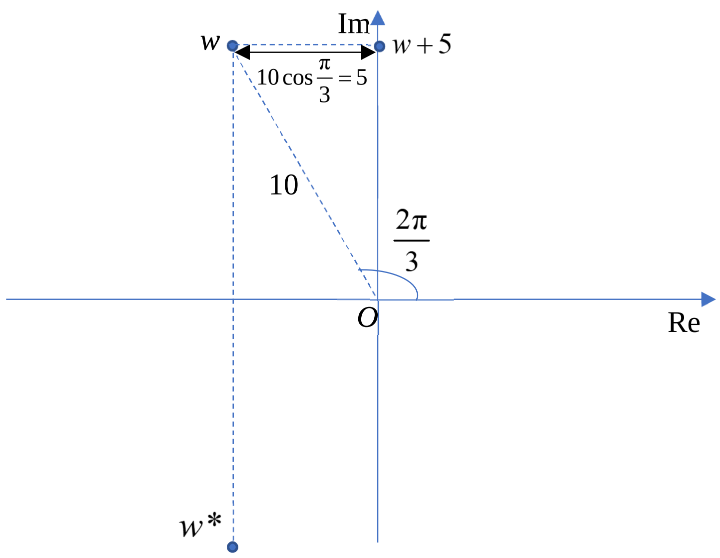
6	Marker's Comments	Marker's Comments
(a)	$u_{n+1}=0.1u_n+k$ $u_2=0.1u_1+k=0.1(-3)+k=k-0.3$ $u_3=0.1u_2+k=0.1(k-0.3)+k=1.1k-0.03$	Well-done.
(b)	We are given $u_n=\beta+\alpha\left(\frac{1}{10}\right)^n$ When $n=1$, $-3=\beta+\alpha\left(\frac{1}{10}\right)^1$ $-3=\beta+0.1\alpha \quad ...(1)$ When $n=2$, $k-0.3=\beta+\alpha\left(\frac{1}{10}\right)^2$ $k-0.3=\beta+0.01\alpha \quad ...(2)$ (1) – (2), $-3-(k-0.3)=(\beta+0.1\alpha)-(\beta+0.01\alpha)$ $-2.7-k=0.09\alpha$ $\alpha=\frac{100}{9}(-2.7-k)$ $=-\frac{100}{9}k-30$ Sub into (1), $-3=\beta+\frac{1}{10}\left(-\frac{100}{9}k-30\right)$ $-3=\beta-\frac{10}{9}k-3$ $\beta=\frac{10}{9}k$	**Concepts and/or Skills:** Understanding the expression for each term in a sequence and solving for the unknowns using simultaneous equations. **Learning points:** Some of you were not able to work out the correct expression for α, but smartly used $\alpha=-\frac{100}{9}k-30$ to calculate the expression for β.

(c)	$\lim_{n\to\infty} u_n = \lim_{n\to\infty}\left(\frac{10}{9}k + \alpha\left(\frac{1}{10}\right)^n\right) = \frac{10}{9}k$	**Concepts and/or Skills:** Taking the limit of a sequence of terms whose absolute value is less than 1. **Learning points:** Note that the question asked "$\lim_{n\to\infty} u_n$ in terms of k". So stating "β" will not give you credit.
(d)	When $k=9$, $\alpha = -\frac{100}{9}(9) - 30 = -130$. $u_n = 10 - 130\left(\frac{1}{10}\right)^n$ $\sum_{i=1}^{n} u_i = \sum_{i=1}^{n}\left[10 - 130\left(\frac{1}{10}\right)^i\right]$ $= \sum_{i=1}^{n} 10 - 130\sum_{i=1}^{n}\left(\frac{1}{10}\right)^i$ $= 10n - (130)\frac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{1-\frac{1}{10}}$ $= 10n - \frac{130}{9}\left(1-\frac{1}{10^n}\right)$ $= 10n - \frac{130}{9} + \frac{130}{9\left(10^n\right)}$	**Concepts and/or Skills:** Application of the APGP formula and understanding of the sigma notation. **Learning points:** The application of the geometric series formula is poor. Please see that $\sum_{i=1}^{n}\left(\frac{1}{10}\right)^i = \frac{1}{10} + \left(\frac{1}{10}\right)^2 + \left(\frac{1}{10}\right)^3 + \ldots + \left(\frac{1}{10}\right)^n$. There are n terms, common ratio r is $\frac{1}{10}$ and the first term a is $\frac{1}{10}$. As such, the GP series formula when applied correctly, should be: $\frac{\frac{1}{10}\left(1-\frac{1}{10^n}\right)}{1-\frac{1}{10}}$.

7	Suggested Solution	Marker's Comments
(a)	$z_1=\sqrt{2}\left(1-\mathrm{i}\right)=2\mathrm{e}^{-\mathrm{i}\frac{\pi}{4}}$ $z_2=2+\mathrm{i}2\sqrt{3}=4\mathrm{e}^{\mathrm{i}\frac{\pi}{3}}$ $z_3=\dfrac{\mathrm{e}^{\mathrm{i}\frac{\pi}{12}}}{5}\Rightarrow z_3{}^*=\dfrac{\mathrm{e}^{-\mathrm{i}\frac{\pi}{12}}}{5}$ $w=\dfrac{z_2}{\left(z_3{}^*\right)z_1}$ $=\dfrac{4\mathrm{e}^{\mathrm{i}\frac{\pi}{3}}}{\left(\dfrac{\mathrm{e}^{-\mathrm{i}\frac{\pi}{12}}}{5}\right)\left(2\mathrm{e}^{-\mathrm{i}\frac{\pi}{4}}\right)}=10\mathrm{e}^{\mathrm{i}\frac{2\pi}{3}}$	**Concepts and/or Skills:** • Convert complex numbers in cartesian form to exponential form. • Application of laws of modulus and argument. **Learning points:** • Determine the quadrant that the complex number lies in before calculating the argument of a complex number.
(b)		**Concepts and/or Skills:** • Represent complex numbers on argand diagram. **Common mistakes :** • Not knowing the size of $\dfrac{2\pi}{3}$ radians and mark the complex number on the wrong quadrant. **Learning points:** • Complex number is represented as a point on the diagram, not a line. Any working lines should be drawn as dotted lines instead of solid lines. • The modulus and argument of a complex number must always be labelled clearly on the diagram. • When more than 1 complex number needs to be plotted on the same diagram, look out for any possible geometrical relationships between the complex numbers. In this case, many students are unable to figure out the relative position of w and $w+5$.

<table>
<tr>
<td>(c)</td>
<td>
Since c is a real number, $(v-c)^*=v^*-c^*=v^*-c$
$$\frac{v+c}{v-c}\times\frac{(v-c)^*}{(v-c)^*}=\frac{(v+c)(v^*-c)}{|v-c|^2}$$
$$=\frac{vv^*+cv^*-cv-c^2}{|v-c|^2}$$
$$=\frac{|v|^2-c^2-c(v-v^*)}{|v-c|^2}$$
$$=\frac{|v|^2-c^2-2c\,\mathrm{Im}(v)\mathrm{i}}{|v-c|^2}$$
Since $\dfrac{v+c}{v-c}$ is purely imaginary,
$|v|^2-c^2=0$
$|v|=c^2$
$|v|=c$ (Reject $|v|=-c$)
<hr>
Alternatively,
Let $\dfrac{v+c}{v-c}=k\mathrm{i}$ and $v=a+b\mathrm{i}$ where $a,b,k\in\mathbb{R},k\neq 0$.
$$\frac{a+b\mathrm{i}+c}{a+b\mathrm{i}-c}=k\mathrm{i}$$
$(a+c)+b\mathrm{i}=-bk+(a-c)k\mathrm{i}$

Comparing real and imaginary parts,
$$\begin{cases} a+c=-bk & \ldots(1) \\ b=(a-c)k & \ldots(2) \end{cases}$$
$(1)\div(2)$,
</td>
<td>
Concepts and/or Skills:
• Modulus of complex number.
• How to separate the real and imaginary parts.
• Realise the denominator of a complex number.

Common mistakes :
• Not knowing that v has got real and imaginary parts and treated v as purely imaginary.

Learning points:
• Realise the denominator of a complex number to separate the real and imaginary parts.
</td>
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$$\frac{a+c}{b}=\frac{-b}{a-c}$$

$$a^2-c^2=-b^2$$

$$a^2+b^2=c^2$$

$$|v|=c^2$$

$$|v|=c \text{ (Reject } |v|=-c\text{)}$$

Alternatively,

Let $\dfrac{v+c}{v-c}=k\mathrm{i}$ where $k\in\mathbb{R}, k\neq 0$.

$$\frac{v+c}{v-c}=k\mathrm{i}$$

$$v+c=v\left(k\mathrm{i}\right)-ck\mathrm{i}$$

$$v\left(1+ki\right)=-c-ck\mathrm{i}$$

$$v=\frac{-c-ck\mathrm{i}}{1+k\mathrm{i}}$$

$$|v|=\left|\frac{-c-ck\mathrm{i}}{1+k\mathrm{i}}\right|$$

$$=\frac{|-c-ck\mathrm{i}|}{|1+k\mathrm{i}|}$$

$$=\frac{\sqrt{\left(-c\right)^2+\left(-ck\right)^2}}{\sqrt{1^2+k^2}}$$

$$=\frac{\sqrt{c^2\left(1+k^2\right)}}{\sqrt{1+k^2}}$$

$$=\frac{c\sqrt{1+k^2}}{\sqrt{1+k^2}} \quad \text{(since } c>0\text{)}$$

$$=c$$

<table>
<tr><th>8</th><th>Suggested Solution</th><th>Marker’s Comments</th></tr>
<tr><td>(a)</td><td>$\overrightarrow{AB}=\begin{pmatrix}-9\\2\\0\end{pmatrix}$ and $\overrightarrow{AC}=\begin{pmatrix}-9\\0\\3\end{pmatrix}$.
$\begin{pmatrix}-9\\2\\0\end{pmatrix}\times\begin{pmatrix}-9\\0\\3\end{pmatrix}=\begin{pmatrix}6\\27\\18\end{pmatrix}=3\begin{pmatrix}2\\9\\6\end{pmatrix}$.
Substitute $(9,0,0)$ to $2x+9y+6z=D$, we have $D=18$
The cartesian equation of ABC is $2x+9y+6z=18$</td><td>Concepts and/or Skills:
• Finding an equation of a plane passing through 3 given points.

Common mistakes :
• Do not understand the term “cartesian” or ignore the requirement of the question.
• Poor accuracy in computing the cross product of two vectors.</td></tr>
<tr><td>(b)</td><td>The required angle $=\sin^{-1}\dfrac{\left|\begin{pmatrix}2\\9\\6\end{pmatrix}\cdot\begin{pmatrix}0\\0\\1\end{pmatrix}\right|}{\sqrt{2^2+9^2+6^2}\times 1}$
$=\sin^{-1}\dfrac{6}{11}$
$=33.1°$ (1 d.p.)</td><td>Concepts and/or Skills:
• Finding the angle between a line and a plane.

Common mistakes :
• Do not know when to use sine or cosine functions.

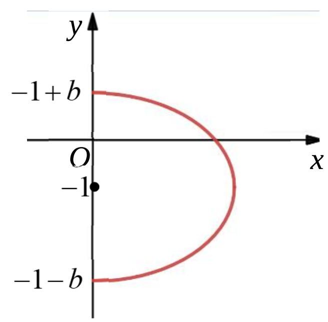
Learning points:
You should revise by summarising the formulae in finding angles in different scenario in this topic. (Really free marks)</td></tr>
<tr><td>(c)</td><td><u>Method 1 (intersection)</u>
Let the foot of perpendicular be N. Then $\overrightarrow{ON}=\lambda\begin{pmatrix}2\\9\\6\end{pmatrix}$.
Substitute $(2\lambda, 9\lambda, 6\lambda)$ to $2x+9y+6z=18$,
$4\lambda+81\lambda+36\lambda=18$
$121\lambda=18$
$\lambda=\dfrac{18}{121}$
N has coordinates $\left(\dfrac{36}{121},\dfrac{162}{121},\dfrac{108}{121}\right)$.</td><td>Concepts and/or Skills:
• Finding the foot of perpendicular from a point to a plane.

Common mistakes :
• Low precision in recalling and/applying the projection formula.
• Do not understand the difference between position vector and coordinates or ignore the requirement.

Learning points:

Revise the following thoroughly. (Really free marks)
1. Foot of perpendicular from a point to a line.
2. Shortest distance from a point to a line.</td></tr>
</table>

	<u>Method 2 (projection)</u> Let the foot of perpendicular be N. $\overrightarrow{ON}$ is project vector of $\overrightarrow{OA}$ on the normal. $\overrightarrow{ON} = \left(\overrightarrow{OA}\cdot\hat{\mathbf{n}}\right)\hat{\mathbf{n}}$ $= \left[\dfrac{\begin{pmatrix}9\\0\\0\end{pmatrix}\cdot\begin{pmatrix}2\\9\\6\end{pmatrix}}{11^2}\right]\begin{pmatrix}2\\9\\6\end{pmatrix}$ $= \dfrac{18}{121}\begin{pmatrix}2\\9\\6\end{pmatrix}$ N has coordinates $\left(\dfrac{36}{121}, \dfrac{162}{121}, \dfrac{108}{121}\right)$.	3. Foot of perpendicular from a point to a plane. 4. Shortest distance from a point to a plane.
(d)	The midpoint of AC is $\left(\dfrac{9}{2}, 0, \dfrac{3}{2}\right)$. The reflection of O in ABC is $\left(\dfrac{72}{121}, \dfrac{324}{121}, \dfrac{216}{121}\right)$. For direction vector for the reflection line: $\begin{pmatrix}\frac{9}{2}\\0\\\frac{3}{2}\end{pmatrix} - \begin{pmatrix}\frac{72}{121}\\\frac{324}{121}\\\frac{216}{121}\end{pmatrix} = \begin{pmatrix}\frac{945}{242}\\-\frac{324}{121}\\-\frac{69}{242}\end{pmatrix} = \dfrac{3}{242}\begin{pmatrix}315\\-216\\-23\end{pmatrix}$. A vector equation is $\mathbf{r} = \begin{pmatrix}\frac{9}{2}\\0\\\frac{3}{2}\end{pmatrix} + \mu\begin{pmatrix}315\\-216\\-23\end{pmatrix}, \mu \in \mathbb{R}$.	**Concepts and/or Skills:** • Finding the reflection of a line in a plane. **Common mistakes :** • Wrong understanding of the scenario with a wrong diagram • Apply a very tedious (though more general) method and make mistakes in manipulations, likely due to memorizing the steps without understanding. **Learning points:** 1. The midpoint of AC (say D) is on the plane ABC, since the line AC is in this plane. We do not need to perform reflection on this point. 2. We only need to reflect O about ABC. Its reflection O' has position vector $\overrightarrow{OO'} = 2\overrightarrow{ON}$ where N is the foot of perpendicular found in the previous part. Then we just need to find a direction vector $\overrightarrow{O'D}$ and manipulate to the required form.

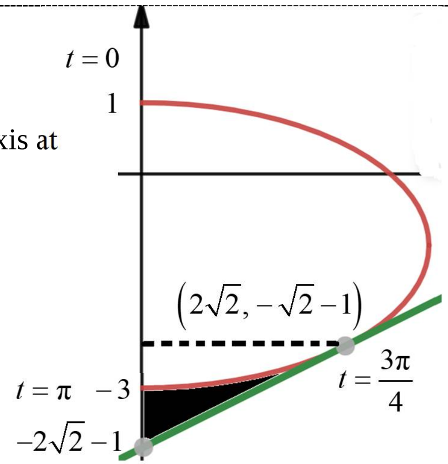
9	Suggested Solution	Marker's Comments
(a)	When $x=0$, $4\sin t=0$ $t=0$ or $t=\pi$. When $t=0$, $y=-1+b\cos 0=-1+b$ When $t=\pi$, $y=-1+b\cos\pi=-1-b$ 	**Concepts and/or Skills:** • Sketch a parametric curve. **Common mistakes :** • Fail to consider the given range of values of the parameter in using a GC. • Fail to recognize that the curve is NOT symmetrical about *x*-axis.
(b)	$\frac{dx}{dt}=4\cos t$ and $\frac{dy}{dt}=-b\sin t \Rightarrow \frac{dy}{dx}=\frac{-b\sin t}{4\cos t}=\frac{-b\tan t}{4}$ When $\frac{-b\tan t}{4}=\frac{b}{4}$, $\tan t=-1$ basic angle $=\frac{\pi}{4}$ $t=\pi-\frac{\pi}{4}=\frac{3\pi}{4}$ When $t=\frac{3\pi}{4}$, $x=4\sin\frac{3\pi}{4}=2\sqrt{2}$, $y=b\cos\frac{3\pi}{4}-1=-\frac{b\sqrt{2}}{2}-1$ The equation of the tangent is $y-\left(-\frac{b\sqrt{2}}{2}-1\right)=\frac{b}{4}\left(x-2\sqrt{2}\right)$ $y=\frac{bx}{4}-\frac{b\sqrt{2}}{2}-\frac{b\sqrt{2}}{2}-1$ $y=\frac{bx}{4}-\sqrt{2}b-1$	**Concepts and/or Skills:** • Find the equation of a tangent to the parametric curve. **Common mistakes :** • Cannot solve $\tan t=-1$ correctly within the given range of values of *t*. **Learning points:** 1. Learn how to solve a trigonometric equation properly (A-Maths, but still relevant in many H2 Mathematics topics) 2. In a "show" part, if you notice the equation obtained is slightly different from the one given in the question, look for a positive mistake in your working instead of trying to bluff the marker.

(c)

<u>Method 1 (y-axis)</u>

Now $b = 2$.

The tangent touches the curve at $\left(2\sqrt{2}, -\sqrt{2}-1\right)$ and cuts the y-axis at $-2\sqrt{2}-1$.



The area of triangle is

$$\frac{1}{2}\sqrt{2}\left[\left(-\sqrt{2}-1\right)-\left(-2\sqrt{2}-1\right)\right]$$

$$=\frac{1}{2}\sqrt{2}\left(\sqrt{2}\right)$$

$$=2$$

The area under C is

$$\int_{\pi}^{\frac{3\pi}{4}} x\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)\,\mathrm{d}t=\int_{\pi}^{\frac{3\pi}{4}}\left(4\sin t\right)\left(-2\sin t\right)\,\mathrm{d}t$$

$$=8\int_{\frac{3\pi}{4}}^{\pi}\sin^2 t\,\mathrm{d}t$$

$$=8\int_{\frac{3\pi}{4}}^{\pi}\frac{1-\cos 2t}{2}\,\mathrm{d}t$$

$$=4\left(t-\frac{\sin 2t}{2}\right)_{\frac{3\pi}{4}}^{\pi}$$

$$=4\left(\pi-\frac{\sin 2\pi}{2}-\frac{3\pi}{4}+\frac{\sin\frac{3}{2}\pi}{2}\right)$$

$$=4\left(\frac{\pi}{4}-\frac{1}{2}\right)$$

$$=\pi-2$$

The area of the shaded region is $2-\left(\pi-2\right)=4-\pi$

Concepts and/or Skills:

- Find the area bounded by a parametric curve by integration.
- Integration Techniques (double-angle formula)

Common mistakes :

- Attempt to answer this part without a diagram or with a wrong diagram.
- Do not know how to work on parametric equations to find area.
- Use wrong limits in the integration.
- Attempt to convert to a cartesian equation which does not help in finding the exact value.

Learning points:

1. Always draw a diagram to aim visualization. When labelling points on a parametric curves, the values of the parameter help usually.

2. A tangent to the curve should be touching the curve at a point. If you find a different (so wrong) equation as required in the previous part. Use the given (so correct) equation to solve this part.

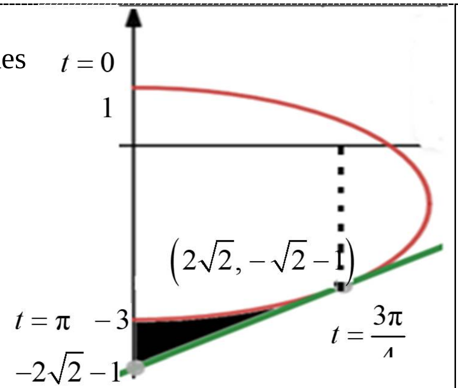
3. Spend some time revising how to find area under a parametric curve. DO NOT assume/bet on that converting the parametric curve to its cartesian form will always work (though it is important to know how and sometimes it works).

4. Whenever doing an integration, always use GC to check your answer (for example, some of you got $\pi+2$ instead of $\pi-2$ in evaluating the integral and this mistake can be easily identified or even rectified, as you were getting 5.14 instead of 1.14)

Even for indefinite integrals, you may still choose arbitrary limits to help you check answer.

Method 2 (x-axis)

Now $b=2$, the tangent touches the curve at $\left(2\sqrt{2},-\sqrt{2}-1\right)$ and cuts the y-axis at $-2\sqrt{2}-1$.



The area of the trapezium is

$$\frac{1}{2}\left(\sqrt{2}+1+2\sqrt{2}+1\right)\left(2\sqrt{2}\right).$$

$$=6+2\sqrt{2}$$

The area under C is

$$-\int_{\pi}^{\frac{3\pi}{4}} y\left(\frac{dx}{dt}\right)\, dt=\int_{\frac{3\pi}{4}}^{\pi}\left(2\cos t-1\right)\left(4\cos t\right)\, dt$$

$$=\int_{\frac{3\pi}{4}}^{\pi} 8\cos^2 t-4\cos t\, dt$$

$$=\int_{\frac{3\pi}{4}}^{\pi} 8\cos^2 t-4\cos t\, dt$$

$$=\int_{\frac{3\pi}{4}}^{\pi} 8\left(\frac{\cos 2t+1}{2}\right)-4\cos t\, dt$$

$$=\int_{\frac{3\pi}{4}}^{\pi} 4\cos 2t-4\cos t+4\, dt$$

$$=\left(2\sin 2t-4\sin t+4t\right)_{\frac{3\pi}{4}}^{\pi}$$

$$=\left(2\sin 2\pi-4\sin\pi+4\pi\right)-\left(2\sin\frac{3\pi}{2}-4\sin\frac{3\pi}{4}+3\pi\right)$$

$$=0-0+4\pi+2+2\sqrt{2}-3\pi$$

$$=\pi+2+2\sqrt{2}$$

The area of the shaded region is $6+2\sqrt{2}-\left(\pi+2+2\sqrt{2}\right)=4-\pi$

10	Suggested Solution	Marker's Comments
(a)	the amount of money Jeremy will have at the end of the year 2025 is $(100000\times1.032+X)\times1.032=100000\times1.032^2+1.032X$ $=106502.40+1.032X$ The amount is $\$(106502.40+1.032X)$.	**Concepts and/or Skills:** Interpret and formulate a real-world problem mathematically Compound interest model. Find the amount of money at the end of 2 years. To help students to see pattern and continue with (b) **Common mistakes :** - Use 3.2 or 0.032 and did not take out common factor to get 1.032. - write in % - Did not simplify final answer. - Did not write down unit '\$' **Learning points:** To find the amount of money after interest are added. Should add the principal amount. If the interest rate is *r*%, need to multiply by $\left(1+\frac{r}{100}\right)$. If using $\frac{r}{100}$, then have to add the principal amount and simplify first, before moving to the next year.
(b)	$100000(1.032^n)+(1.032^{n-1})X+(1.032^{n-2})X+...+1.032X$ $=100000(1.032^n)+\frac{(1.032)X(1-1.032^{n-1})}{1-1.032}$ $=100000(1.032^n)+32.25X(1.032^{n-1}-1)$ The amount of money Jeremy will have at the end of *n* years is $\$\left[100000(1.032^n)+32.25X(1.032^{n-1}-1)\right]$	**Concepts and/or Skills:** Interpret and formulate a real-world problem mathematically Compound interest model. Apply Sum of first *n* terms of a geometric Series **Common mistakes :** -not able to write out the expression of the amount of money at the end of *n* years correct. Some students are not aware that the 2nd term onwards is GP, - For those who knew it is GP, did not write out the terms fully, and apply the GP sum formula with *n* terms or the first term as *X*. - did not simplify expression though question asked for it.

<table>
<tr><td></td><td></td><td>Learning points :
There is a need to show clear GP pattern before the application of formula. Write out for 3rd year, 4 th year to see the pattern. Then write out $100000\left(1.032^n\right)+\left(1.032^{n-1}\right)X+\left(1.032^{n-2}\right)X+...+1.032X$.
So that we can see that GP for
$\left(1.032^{n-1}\right)X+\left(1.032^{n-2}\right)X+...+1.032X$ with first term $1.032X$ and common ratio 1.032 and number of terms is $n-1$</td></tr>
<tr><td>(c)</td><td>To fully pay the car loan

$100000\left(1.032^n\right)+32.25(8000)\left(1.032^{n-1}-1\right)\geq 170000$
$100000\left(1.032^n\right)+258000\left(1.032^{n-1}-1\right)\geq 170000$

Using GC,
<table><tr><td>n</td><td>Amount</td></tr><tr><td>6</td><td>164811</td></tr><tr><td>7</td><td>178341</td></tr></table>$n = 7$
164811 + 8000 = 172811 > 170000
Hence he will have sufficient amount in the account on the first day of 2030.</td><td>Concepts and/or Skills:
Solve inequality to find least n, using part (b)

Common mistakes :
- Did not use inequality ≥ 170000
- Did not refer to question that start form 2024. Only wrote answer as start of 7th year.
- 7 years from 2024 should be year 2030. Not 2029.
- Did not check correctly at the start or end of 2030.

Learning points: For solving inequality involving whole numberm should show table of 2 rows of values when using table in GC, if using graph in GC, should write down the answer in equality, then write down least/greatest $n =$.</td></tr>
<tr><td>(d)</td><td>Total amount = $170000+170000\times 0.028m$
$=170000+4760m$
Thus the annual instalment is $\$\frac{1}{m}(170000+4760m)$.</td><td>Concepts and/or Skills:
Interpret and formulate a real-world problem mathematically
Simple interest model.

Common mistakes :
- A few students try to use AP to prove the expression.
- Missed out m in 4760m

Learning points : For show question, answer is given, must explain clearly how to get the given answer..</td></tr>
</table>

	When $m=10$, the annual instalment is $\frac{1}{10}(170000+4760\times 10)=21760$, he needs to withdraw $21760-8000=13760$ from his current account every year. Take $X=-13760$ for **(b)** answer, the amount in the current account at the end of *n* years is $100000(1.032^n)+32.25(-13760)(1.032^{n-1}-1)$ $=100000(1.032^n)-443760(1.032^{n-1}-1)$ By GC, <table><tr><td>*n*</td><td>Amount in the current account</td></tr><tr><td>9</td><td>5601.5</td></tr><tr><td>10</td><td>– 8420</td></tr></table> Jeremy will not have sufficient money in his current account at the end of 10^{th} year, so the arrangement is not possible.	**Common mistakes :** - Did not understand the question and leave blank. - Miss out the information that \$8000 is used to pay the monthly car installment. - Confuse with the years. Start to pay car instalment in 2025. 10000- is in the bank current account at the start of 2024. - Did not consider the monthly instalments of \$13760 is to be deducted yearly. It is not appropriate to consider total instalment in 10 years and compare with 100000 in the current account for 10 years. **Learning points:** Need to be conscious that the questions are linked. Should try to use answer from previous parts, instead of writing out the expressions again.

11	Suggested Solution	Marker's Comments
(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}$ gives the gradient of the tangent of C. At W_1 with coordinates (x, y), absolute value of the gradient of the tangent is $\frac{y}{\sqrt{6^2 - y^2}}$ by Pythagoras theorem. Since gradient is negative from the diagram/context, $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{y}{\sqrt{36 - y^2}}$	**Concepts and/or Skills:** Formulating of a differential equation when given the context. **Learning points:** Students should see that there is a right-angled triangle and also that there is no x term in the differential equation. This should guide their thinking to think in terms of using y terms together with the Pythagoras Theorem.
(b)	$w^2 = 36 - y^2$ $2w\frac{\mathrm{d}w}{\mathrm{d}x} = -2y\frac{\mathrm{d}y}{\mathrm{d}x}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{w}{y}\frac{\mathrm{d}w}{\mathrm{d}x}$ Sub $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{w}{y}\frac{\mathrm{d}w}{\mathrm{d}x}$ into $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-y}{\sqrt{36 - y^2}}$: $-\frac{w}{y}\frac{\mathrm{d}w}{\mathrm{d}x} = \frac{-y}{\sqrt{w^2}}$ $\frac{\mathrm{d}w}{\mathrm{d}x} = \frac{y^2}{w\sqrt{w^2}}$ $= \frac{y^2}{w\lvert w\rvert} \qquad \left(\sqrt{w^2} = \lvert w\rvert\right)$ $= \frac{y^2}{w^2} \qquad \left(\lvert w\rvert = w, \text{since } w \ge 0\right)$ $= \frac{36 - w^2}{w^2}$	**Concepts and/or Skills:** Simplifying a differential equation by substitution. **Learning points:** Students can also use the chain rule $\frac{\mathrm{d}y}{\mathrm{d}x}\frac{\mathrm{d}w}{\mathrm{d}y} = \frac{\mathrm{d}w}{\mathrm{d}x}$ to reduce the differential equation. Doing the differentiation in this manner $2w\frac{\mathrm{d}w}{\mathrm{d}x} = -2y\frac{\mathrm{d}y}{\mathrm{d}x}$, students need to do the implicit differentiation correctly, bearing in mind that we have to differentiate w and y correctly with respect to x.

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<td>(c)</td>
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$\frac{w^2}{36-w^2}\frac{\mathrm{d}w}{\mathrm{d}x}=1$

$\int \frac{w^2}{36-w^2}\,\mathrm{d}w=\int 1\,\mathrm{d}x$

$\int -1+\frac{36}{36-w^2}\,\mathrm{d}w = x+c$

$\int -1+\frac{36}{36-w^2}\,\mathrm{d}w = x+c$

$\int -1\,\mathrm{d}w + 36\int \frac{1}{6^2-w^2}\,\mathrm{d}w = x+c$

$-w+3\ln\frac{6+w}{6-w} = x+c$ since $0<w<6$

$w^2 = 36-y^2 \Rightarrow w=\sqrt{36-y^2}$ (since $w \geq 0$)

Sub $w=\sqrt{36-y^2}$:

$x=-\sqrt{36-y^2}+3\ln\frac{6+\sqrt{36-y^2}}{6-\sqrt{36-y^2}}-c$

From $W_0(0,6)$:

$0=-\sqrt{36-6^2}+3\ln\frac{6+\sqrt{36-6^2}}{6-\sqrt{36-6^2}}-c$

$0=0+3\ln 1+c$

$c=0$

Final equation:

$x=-\sqrt{36-y^2}+3\ln\frac{6+\sqrt{36-y^2}}{6-\sqrt{36-y^2}}$

or

$x=-\sqrt{36-y^2}+3\ln\left(6+\sqrt{36-y^2}\right)-3\ln\left(6-\sqrt{36-y^2}\right)$

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Concepts and/or Skills:
Solving a differential equation

Learning points/common mistakes:

$\frac{w^2}{36-w^2}=-1+\frac{36}{36-w^2}$ via long division posed many challenges to students.

$\int \frac{36}{36-w^2}\,\mathrm{d}w$ was confused with $\int \frac{1}{\sqrt{1-x^2}}\,\mathrm{d}x$.

Note that modulus was not necessary.

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(d)	As $y \to 0$, $-\sqrt{36-y^2} \to -6$ $3\ln\left(6+\sqrt{36-y^2}\right) \to 3\ln 12$ $-3\ln\left(6-\sqrt{36-y^2}\right) \to -3\ln 0 \to \infty$ Thus $x = -\sqrt{36-y^2} + 3\ln\left(6+\sqrt{36-y^2}\right) - 3\ln\left(6-\sqrt{36-y^2}\right)$ increases to infinity. (or $\dfrac{6+\sqrt{36-y^2}}{6-\sqrt{36-y^2}}$ tends to infinity)	**Concepts and/or Skills:** Find the limit of an expression. **Learning points/common mistakes:** Good to say that *x* <u>increases</u> to infinity.