1 Find the range of values of the constant m for which the curve $y = (m - 6)x^2 - 8x + m$ lies completely above the x-axis. [4]

2 Show that the line $x + y = m$ will intersect the curve $x^2 + 2y^2 = 2x + 3$ if $m^2 \leq 2m + 5$. [4]

3 It is given that $f(x) = (b - 3x)e^{2-3x}$. Find the value of the constant b if $f(x)$ is a decreasing function when $x < \frac{4}{3}$. [4]

4 Prove that $\frac{2-\operatorname{cosec}^2\theta}{\operatorname{cosec}^2\theta+2\cot\theta}=\frac{1-\cot\theta}{1+\cot\theta}$. [4]

5 Express $\frac{4x^2+5x-32}{(x+2)(x^2-9x-22)}$ in partial fractions. [5]

6 (a) Show that $3\cos x = 2\operatorname{cosec} x \cot x$ can be written as $\cos x\,(3\sin^2 x - 2) = 0$. [3]

(b) Hence, solve the equation $3\cos(0.6y - 1.4) = 2\operatorname{cosec}(0.6y - 1.4)\cot(0.6y - 1.4)$ for values of y between -3 and 4. [5]

7 (a) Given that $y = \frac{1+\sin x}{\cos x}$, find $\frac{dy}{dx}$. [2]

(b) Hence, without using a calculator, find the value of each of the constants p and q for which $\int_0^{\frac{\pi}{3}} \frac{3 + 3\sin x - 10\cos^3 x}{5\cos^2 x}\, dx = p + q\sqrt{3}$. [6]

8 The height of Jeremiah above the surface of the water, h metres, can be modelled by the equation $h = -4.9t^2 + 8t + 5$, where t is the time in seconds after he leaves the diving board.

(a) State the height of the diving board above the surface of the water. [1]

(b) Express h in the form $k - a(x - b)^2$, where k, a and b are constants to be determined. [3]

(c) State the greatest height reached by Jeremiah and the corresponding time when the greatest height occurs. [2]

(d) Using your answer obtained in **(b)**, calculate the duration which Jeremiah stay in the air. [2]

9 Solve the simultaneous equations.

$$\frac{5^p}{25} = 125^q$$

$$\log_3 7 = 1 + \log_3(11q - 2p)$$ [5]

10 The equation of a curve is $y = x^3 \ln x$.

(a) Find the exact coordinates of the stationary point(s) of the curve. [4]

(b) Determine the nature of the stationary point(s) of the curve. [2]

A particle moves along the curve $y = x^3 \ln x$. At point *M*, the *x*-coordinate of the particle is increasing at a rate of 0.06 units/s and the *y*-coordinate is increasing at a rate of $0.3x^2$ units/s.

(c) Find the exact coordinate of *M*. [4]

11 A circle, C_1, has equation $x^2 + y^2 + kx - 6y = h$, where k and h are constants.

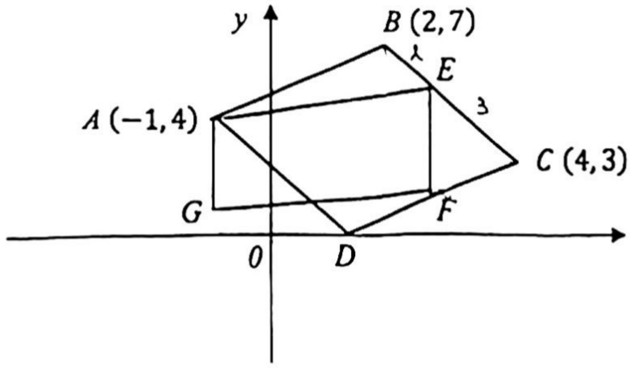
(a) Given that the radius C_1 of is 7 units and the coordinates of the centre is $(-2, m)$, find the values of k, m and h. [4]

(b) Another circle, C_2, has diameter PQ. The point P is $(3, n)$ and Q is $(-5, 5)$. The equation PQ is $4y + 3x = 5$.

(i) Find the equation of C_2. [4]

(ii) Explain, with appropriate working, why the point $S\ (4, 5)$ only lies inside the circle C_1 but not C_2. [2]

12 The diagram shows a parallelogram $ABCD$ in which D lies on the x-axis. Point A is $(-1, 4)$, $B\ (2, 7)$ and $C\ (4, 3)$. The point E lies on BC such that $5BE = 2BC$.



(a) Find the coordinates of D, E and F. [8]

Continuation of working space for question **12(a)**.

(b) *AG* and *EF* are two vertical lines. The *y*-coordinate of *G* is $\frac{2}{5}$. *E* and *F* lie on *BC* and *DC* respectively. Explain, with an appropriate working, what is the name of the special quadrilateral *AEFG*. [2]

13 A container in the shape of a right pyramid, has a height of 45 cm and a square base of side 20 cm, was initially empty. Sand is then allowed to flow into the container through a small hole at the top. After t seconds, the height of the sand in the container is $(45 - x)$ cm and the volume of the sand in the container is V cm^3.

(a) Show that $V = 6000 - \frac{16}{243}x^3$. [3]

(b) Given that the rate of flow of the sand into the container is bx^2 cm^3/s, where b is a constant. Find the **numerical** value of the rate of change of x if the height of the sand in the container is 36 cm after 24 seconds. [7]

End of paper