02 Forces and Moments

PROBLEM SET (SUGGESTED SOLUTIONS)

1 **(a)** uneven distribution of mass in the boat (although boat looks symmetrical from the side view)

(b) zero resultant force on the boat
zero resultant torque (sum of the moments about the same axis is zero) on the boat

(c) take moments at rope 1,

sum of clockwise moments = sum of anticlockwise moments

$$15000 \times 0.75 = T_2 \times 2.00$$

$$T_2 = 5630 \text{ N}$$

take moments at rope 2,

sum of anticlockwise moments = sum of clockwise moments

$$15000 \times (2.00 - 0.75) = T_1 \times 2.00$$

$$T_1 = 9380 \text{ N}$$

2 **(a)** For an object in rotational equilibrium, the sum of clockwise moment about a point is equal to the sum of anticlockwise moment about the same point.

(b) **(i)** Since centre of mass is nearer to A, it must support a larger magnitude of the weight.

$$W = mg = (4.5)(9.81) = 44.145 \text{ N}$$

$$T_A = \frac{7}{10} \times 44.145 = \ = 30.9015 = 31 \text{ N (2 s.f.)}$$

$$T_B = \frac{3}{10} \times 44.145 = 13.2435 = 13 \text{ N (2 s.f.)}$$

(ii) Sign is in rotational equilibrium. Taking moment about the top left corner of the sign,

sum of clockwise moments = sum of anticlockwise moments

$$4.5 \times 9.81 \times d = T_A \times 0.2 + T_B \times 0.8$$

$$4.5 \times 9.81 \times d = 30.9015 \times 0.2 + 13.2435 \times 0.8$$

$$d = 0.38 \text{ m}$$

(c) Sign is in equilibrium, the total horizontal component of the forces exerted by the rigid support at the two supports must be equal in magnitude and opposite in direction to the force exerted by the wind.

3 **(a)** Bar AB is in rotational equilibrium. Taking moment about point A,

sum of clockwise moments = sum of anticlockwise moments

$$36 \times (0.45 \cos 60^o) = (X \cos 70^o) \times 1.2 \sin 60^o + (X \sin 70^o) \times 1.2 \cos 60^o$$

$$X = 8.8 \text{ N (2 s.f.)}$$

(b) **(i)** Resultant force on bar is zero. X has a leftward component, so there must be a rightward horizontal component of *F* for zero resultant force in the horizontal direction.

Resultant moment about any points on the bar must be zero. Taking moment about B, *F* have a vertical component to provide a clockwise moment that balances the anticlockwise moment of the weight and the horizontal component of *F*.

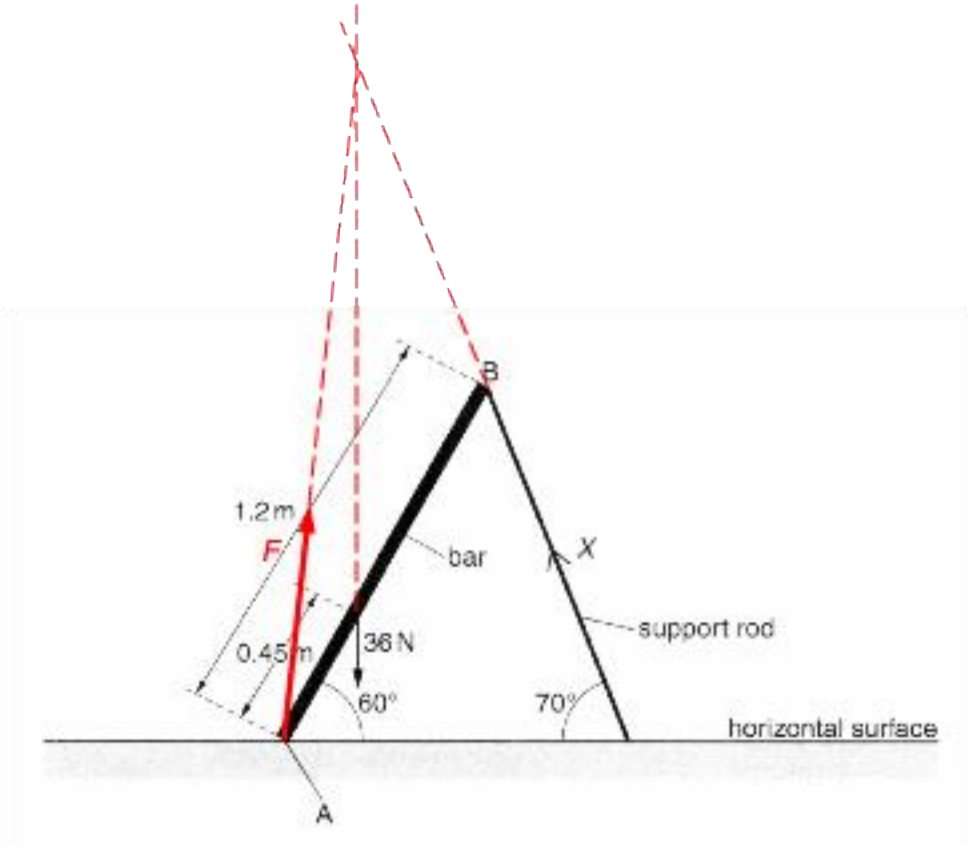
(ii) Horizontal component of *F* = horizontal component of X = $8.8 \cos 70^o$

Vertical component of *F* = $36 - 8.8 \sin 70^o$

Magnitude of *F* = $[(8.8 \cos 70^o)^2 + (36 - 8.8 \sin 70^o)^2]^{1/2} = 27.9$ N

*Note: You must use the value of X shown in **(a)** instead of the raw value.*

(iii)



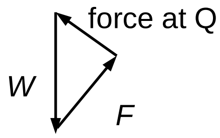
4 **(a)** Taking moments about point Q, clockwise moment = anticlockwise moment

$$F\sin 43^\circ \times \frac{2}{3}\mathrm{PQ} = mg \times \left(\frac{1}{2}\mathrm{PQ}\times\cos 58^\circ\right)$$
$$F = \frac{2.3\times 9.81\times\cos 58^\circ}{\frac{4}{3}\sin 43^\circ} = 13.149 = 13\ \mathrm{N}\ (2\ \mathrm{s.f.})$$

(b) *F* has a horizontal component. For sign PQ to be in equilibrium, another horizontal force must exist at the other contact point Q, and should be acting in the opposite direction as that of the horizontal component of F.

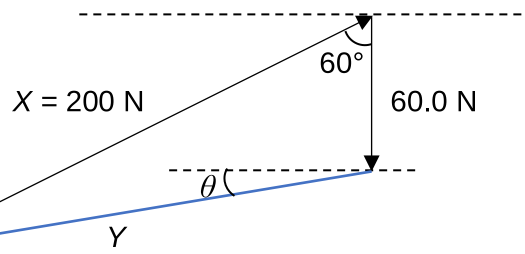
OR

The three forces acting on sign must form a close vector triangle. Based on the direction of *F* and weight, the force at Q can never point vertically.



5 **(a)** **(i)** If magnitude of force X equal to zero, then **there will only be forces Y and 60.0 N** acting on S. For S to remain in equilibrium, *Y* should be pointing upwards and be equal in magnitude and opposite in direction to the weight.
Therefore, *Y* = 60.0 N upwards

(ii) The vector triangle is



Use cosine rule,

$$Y^2 = 200^2 + 60^2 - 2(200)(60)\cos 60^\circ$$
$$Y = 177.76 = 178\ \mathrm{N}$$

Use sine rule,

$$\frac{177.76}{\sin 60^\circ} = \frac{200}{\sin(90^\circ+\theta)}$$
$$90^\circ + \theta = 103.00^\circ$$
$$\theta = 13.0^\circ$$

Force is 178 N at 13.0° below horizontal.

(b) There was always a horizontal component from the force X.

There is no horizontal component by force Y if rope B is parallel to the weight of S. The horizontal forces on S cannot be balanced and it is not in equilibrium.

6 **(a)** zero resultant force on the body
zero resultant torque (sum of the moments <u>about the same axis</u> is zero) on the body

(b) Resultant force in the horizontal direction is zero, so

$$T_2 \cos 10° - T_1 \cos 20° = 0 \Rightarrow T_2 = T_1 \frac{\cos 20°}{\cos 10°} \ldots (1)$$

Resultant force in the vertical direction is zero, using equation (1), so

$$T_1 \sin 20° + T_2 \sin 10° - 700 = 0$$
$$T_1 \sin 20° + \left(T_1 \frac{\cos 20°}{\cos 10°}\right) \sin 10° - 700 = 0$$
$$T_1 = 1378.7 = 1400 \text{ N (2 s.f.)}$$

$$T_2 = (1378.7) \frac{\cos 20°}{\cos 10°} = 1315.5 = 1300 \text{ N (2 s.f.)}$$

(c) Pole is in rotational equilibrium. Take moments about base of pole,

sum of clockwise moment = sum of anticlockwise moment

$$T \sin\left(\tan^{-1} \frac{1.6}{1.2}\right) \times 1.2 = 150 \cos 10° \times 1.8$$
$$T = 276.98 = 280 \text{ N}$$

7 **(a)** extension = 10.8 – 8.0 = 2.8 cm

load = 0.140 × 9.81

force constant = load / extension = 0.140 × 9.81 / 0.028 = 49.05 = 49 N m^{-1} (2 s.f.)

(b) **(i)** $k = F / x = mg / x$

$x = L_2 - L_1 \Rightarrow \Delta x = \Delta L_2 + \Delta L_1 = 2 \text{ mm}$

$$\frac{\Delta k}{k} = \frac{\Delta m}{m} + \frac{\Delta g}{g} + \frac{\Delta x}{x} = 0.01 + 0 + \frac{0.2}{2.8} = 0.08 \text{ (1 s.f.)}$$

percentage uncertainty in the force constant = 8%

(ii) $\Delta k = 8.1429\% \times 49.05 = 4 \text{ N m}^{-1} \text{ (1 s.f.)}$

$k = (49 \pm 4) \text{ N m}^{-1}$