Dunman High School (Senior High Physics Department)

H2 Topic 6B

Thermal Physics (II)

Content

- Kinetic Theory of Matter
- Internal Energy
- Ideal Gases and the Ideal Gas Law
- Internal Energy of an Ideal Gas
- The First Law of Thermodynamics
- P-V Diagrams

Learning Outcomes

Candidates should be able to

(a) show an understanding that internal energy is determined by the state of the system and that it can be expressed as the sum of a random distribution of kinetic and potential energies associated with the molecules of a system.

(b) relate a rise in temperature of a body to an increase in its internal energy.

(c) explain using a simple kinetic model for matter why

i. melting and boiling take place without a change in temperature,

ii. the specific latent heat of vaporisation is higher than specific latent heat of fusion for the same substance,

iii. cooling effect accompanies evaporation.

(d) recall and use the first law of thermodynamics expressed in terms of the change in internal energy, the heating of the system and the work done on the system.

(e) recall and use the ideal gas equation $pV = nRT$, where n is the amount of gas in moles.

(f) show an understanding of the significance of the Avogadro constant as the number of atoms in 0.012 kg of carbon-12.

(g) use molar quantities where one mole of any substance is the amount containing a number of particles equal to the Avogadro constant.

(h) recall and apply the relationship that the mean kinetic energy of a molecule of an ideal gas is proportional to the thermodynamic temperature to new situations or to solve related problems.

INTRODUCTION

Now that we have dealt with how we perceive heat and thermal energy at a macroscopic level, let us deal with how heat manifests at a microscopic level. In this set of lecture notes, we'll be dealing with the kinetic theory of matter, and thermodynamics.

6.3 KINETIC THEORY

6.3.1 The Kinetic Model of Matter

Learning Outcome

(a) show an understanding that internal energy is determined by the state of the system and that it can be expressed as the sum of a random distribution of kinetic and potential energies associated with the molecules of a system.

The kinetic model of matter refers to the theory that attempts to explain the macroscopic behaviour of matter in terms of their microscopic properties. Key to this model is the simple assumption that **all matter is made of atoms/molecules**.

Any atom in a substance possesses two different kinds of energies, that being kinetic, and potential energy.

Random kinetic energy refers to the energy associated with how fast the particle is moving in the frame of the object. **The average random kinetic energy of all atoms in a substance is associated with an object's temperature.** If a substance has experienced an increase in temperature, we know that on average, the particles are moving faster.

Note:
The random kinetic energy of the particle must be contrasted with the bulk translational kinetic energy. The bulk translational kinetic energy refers to the energy that is associated to motion of the entire object in the lab reference frame.

Potential energy refers to the energy associated with the intermolecular forces of attraction. **The potential energy is associated with the phase of a substance, and how far apart the atoms are in the substance.** If a substance has increased its potential energy, we know that the intermolecular spacing between particles has increased, and/or a change of phase (from solid to liquid or from liquid to gas) has taken place.

Note:
The value of potential energy is a negative value. This is because energy has to be supplied to the atoms in order to free them from the influence of the other atoms. If for example, the potential energy of a particular atom is -5 J, this means that 5 J has to be supplied in order to free it from the force of attraction due to the other atoms. A potential energy of 0 J implies that the atom is completely free.

The internal energy of a substance is the sum of kinetic energy due to the random motion of all particles, and the potential energy due to the intermolecular forces of attraction.

$$\boxed{U = KE_{total} + PE_{total}}$$

Learning outcome

(c) explain using a simple kinetic model for matter why

i. melting and boiling take place without a change in temperature.

Solution:

When melting and boiling takes place,

- energy is being used to break the intermolecular bonds and the increase the potential energy of the substance.
- As the average kinetic energy of the system does not increase, the temperature remains the same.

ii. the specific latent heat of vaporisation is higher than specific latent heat of fusion for the same substance,

Solution:

When a substance is changing from solid to liquid,

- the intermolecular bonds are being partially broken and
- the intermolecular spacing does not increase much.

When a substance is changing from liquid to gas,

- the intermolecular bonds are being broken further and
- the intermolecular spacing between molecules increases significantly.
- Furthermore, work has to be done to overcome atmospheric pressure.

As such, the specific latent heat vaporisation is larger than the specific latent heat of fusion.

iii. Cooling effect accompanies evaporation.

Solution

The molecules of a liquid have varying kinetic energies.

- The most energetic molecules at the surface obtain energy from the environment (through collision with air particles, or radiant sunlight)
- These molecules thus obtain enough energy to break free of the intermolecular bonds of attraction and leave the system.
- As the most energetic molecules are leaving the system, the average kinetic energy of the system decreases.
- This causes the temperature of the system to fall.
- Heat is drawn from the external environment and cooling effect accompanies.

6.3.2 Kinetic Theory of Gases

The kinetic theory of gases makes several assumptions.

(a) A gas consists of particles called atoms/molecules.
(b) The total number of molecules is very large.
(c) The molecules are in constant, random motion and obey Newton's laws of motion.
(d) The pressure on the container is the result of collisions with the walls as the molecules strike and rebound.

For gases, four macroscopic variables are of utmost importance. They are, namely,

1. Amount of gas (number of molecules/moles)
2. Volume
3. Pressure
4. Temperature

We shall collectively refer to these four as the "state variables" of the gas. From now on, whenever we talk about the "state" of a gas, we are really talking about these four quantities. With knowledge of the state variables, we are also able to determine the internal energy of the system. You would do well to remember the following statement.

The internal energy of a substance is a function of its state.

6.3.3 Gas Laws and The Equation of State

Learning outcomes:

(e) *recall and use the ideal gas equation pV = nRT, where n is the amount of gas in moles.*

(f) *show an understanding of the significance of the Avogadro constant as the number of atoms in 0.012 kg of carbon-12.*

(g) *use molar quantities where one mole of any substance is the amount containing a number of particles equal to the Avogadro constant.*

When the pressure of gas in a container is sufficiently low and temperature sufficiently high, the state variables are found to obey three simple laws. They are, respectively,

Charles' Law	:	$V \propto T$	(P constant)
Boyle's Law	:	$P \propto \frac{1}{V}$	(T constant)
Gay-Lussac's Law	:	$P \propto T$	(V constant)

Useful form

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$P_1V_1 = P_2V_2$$

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Note: <u>*Temperature must be in Kelvins*</u>

Putting all three together, we get what we call the **equation of state of an ideal gas**.

$$\boxed{PV = nRT}$$

Where *n* represents the amount of gas in moles and R is known as the molar gas constant.

If instead, we choose to work with the number of molecules instead, we ought to use the form

$$\boxed{PV = NkT}$$

where *N* represents the amount of gas in terms of the number of atoms/molecules and *k* is known as the Boltzmann's constant.

From Junior High chemistry, let us recall the definition of a mole.

One mole is the amount of substance that contains the same number of particles equal to the Avogadro's constant.

Let's look at the mathematical relationship between *n* and *N*.

$$\boxed{N_A n = N}$$

N_A is known as Avogadro's number, which is the number of atoms in 0.012kg of carbon-12. This is roughly 6.022×10^{23} molecules. Basically think of 1 mole as just another way to say 6.022×10^{23} molecules.

If we compare the two forms for the equation of state closely, we find that

$$nR = Nk$$

$$nR = N_A nk$$

$$\boxed{R = N_A k}$$

6.3.4 The Ideal Gas

An ideal gas is a hypothetical gas that obeys the equation of state of an ideal gas, PV=nRT, at all pressures, volumes and temperatures for the same amount of gas.

As such, the equation, PV=nRT, is also referred to as the **ideal gas law**. This hypothetical gas is assumed to have the following properties.

1. The volume of the molecules is a negligibly small fraction of the total volume occupied by the gas.
2. The intermolecular forces are negligible.
3. The collisions are elastic and are of negligible duration.
4. Between collisions, a molecule moves with uniform velocity.

Example 1:

Find the volume occupied by one mole of an ideal gas at a temperature of 273 K and a pressure of 1.01×10^5 Pa.

Solution:

$$PV = nRT$$

$$V = \frac{nRT}{P} = \frac{1 \times 8.31 \times 273}{1.01 \times 10^5} = 0.0225 \text{ m}^3$$

Example 2:

A bubble at the bottom of the lake which has a depth of 15 m has a volume of 3 cm^3. What is the volume of the bubble at the top of the pond given that the hydrostatic pressure of water for every 10 m is 1 atm. Assume that the temperatures at the top and bottom of the pond are the same and that no air leaks out of the bubble.

Solution:

$$\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}$$

$$P_1V_1 = P_2V_2$$

$$V_2 = \frac{P_1}{P_2}V_1 = 2.5 \times 3 = 7.5 \text{ cm}^3$$

Example 3:

A gas cylinder is fitted with a safety valve which releases a gas when the pressure inside the cylinder reaches 2.0× 10^6 Pa. Given that the maximum mass of this gas the cylinder can hold at 10^{o}C is 15 kg, what would be the maximum mass at 30^{o}C?

Solution:

Let us recall that a mass of a sample of gas is related to the number of moles by

$$\boxed{M_{gas} = M_{molar} n}$$

Note that the pressure and volume of the cylinder remains the same. Thus, making the necessary substitutions into the equation of state,

$$\frac{P_1V_1}{n_1T_1} = \frac{P_2V_2}{n_2T_2}$$

$$\frac{PVM_{molar}}{M_1T_1} = \frac{PVM_{molar}}{M_2T_2}$$

$$M_2 = \frac{T_1}{T_2}M_1 = 14.0 \text{ kg}$$

Example 4:

An ideal gas exerts a pressure of 60Pa when its temperature is 400 K and the number of molecules present per unit volume is N.

Another sample of the same gas exerts a pressure of 30 Pa when its temperature is 300 K. How many molecules are present in unit volume of this second sample?

Solution:

Note that the question says *molecules present per unit volume*. Thus the number $\frac{N_1}{V_1} = N$.

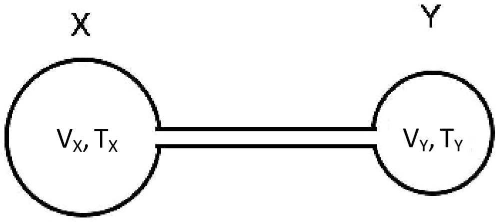
$$\frac{P_1V_1}{N_1T_1} = \frac{P_2V_2}{N_2T_2}$$

$$\frac{60}{N\times400} = \frac{30\times V_2}{N_2\times300}$$

$$\frac{N_2}{V_2} = \frac{30\times400\times N}{300\times60} = 0.667N \text{ molecules.m}^{-3}$$

Example 5:

Two bulbs, X of volume 100 cm^3 and Y of volume 50 cm^3, are connected with a tube of negligible volume. A valve prevents gas to flow between the two bulbs. Initially bulb X is filled with an ideal gas at 10 ˚C to a pressure of 3×10^5 Pa. Bulb Y is filled with the same ideal gas at 100 ˚C to a pressure of 1×10^5 Pa. The valve is opened and the temperature of A and B are maintained at their initial temperatures. Determine the new equilibrium pressure of the system.



Solution:

Recognise that the final pressure, P_f, of both vessels must be the same. Furthermore, the total number of particles in vessel X and Y is conserved. Therefore,

$$n_{X1} + n_{Y1} = n_{X2} + n_{Y2}$$

$$\frac{P_{X1}V_X}{RT_X} + \frac{P_{Y1}V_Y}{RT_Y} = \frac{P_fV_X}{RT_X} + \frac{P_fV_Y}{RT_Y}$$

$$P_f = \frac{\left(\frac{P_{X1}V_X}{T_X} + \frac{P_{Y1}V_Y}{T_Y}\right)}{\left(\frac{V_X}{T_X} + \frac{V_Y}{T_Y}\right)} = 2.45 \times 10^5 \text{ Pa}$$

6.3.4 Internal Energy of an Ideal Gas

Learning Outcome:

(b) relate a rise in temperature of a body to an increase in its internal energy.

(i) recall and apply the relationship that the mean kinetic energy of a molecule of an ideal gas is proportional to the thermodynamic temperature to new situations or to solve related problems.

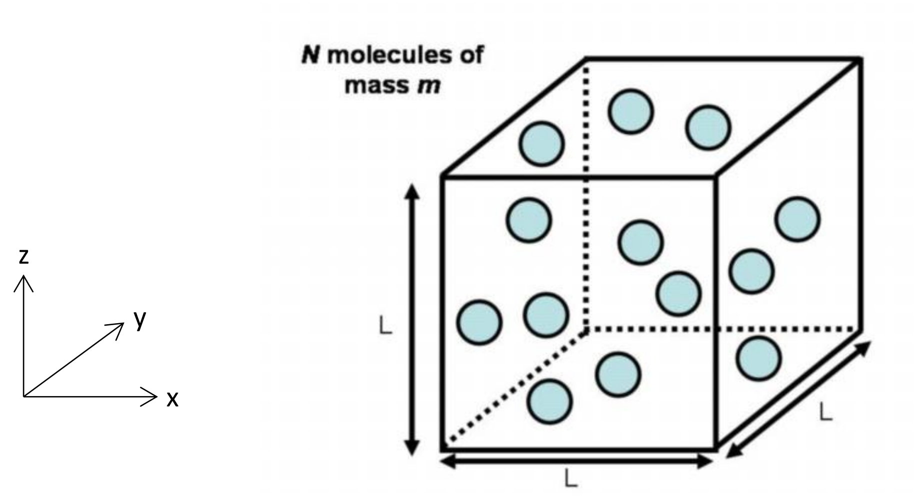
Assumption 2 implies that the molecules exert no forces of attraction or repulsion on each other until they collide. This implies that for an ideal gas,

$$\text{PE} = 0$$

Therefore,

$$\boxed{U_{\text{ideal gas}} = KE_{\text{total}}}$$

Consider a box filled with an ideal gas containing N molecules. Remember that all particles are running around at different velocities.



Let us focus on a single particle in the box which happens to be moving with a velocity v in some given direction. The velocity v may be resolved into 3 different components, v_x, v_y and v_z. For now, let us focus on just the x-component of the velocity. Thus, for the all intents and purposes, the particle is travelling along the x-axis.

Let us assume that the particle starts from one end of the box, collides with the surface and runs back to the other end. The time taken for the particle to do so is given by,

$$\Delta t = \frac{2L}{v_x}$$

The impulse delivered to the particle in that collision is thus,

$$\Delta p_x = p_{x,\,final} - p_{x,\,initial} = (-mv_x - mv_x) = -2mv_x$$

However, by the conservation of momentum, the impulse delivered to the wall must be equal in magnitude and opposite in direction. Thus,

$$\Delta p_{wall} = 2mv_x$$

The force experienced by the wall is thus

$$F = \frac{\Delta p_{wall}}{\Delta t} = 2mv_x \div \frac{2L}{v_x} = \frac{mv_x^2}{L}$$

The pressure exerted on the wall in that single collision is thus,

$$P = \frac{F}{A} = \frac{mv_x^2}{L \times L^2} = \frac{mv_x^2}{V}$$

Bringing the volume over, we obtain,

$$PV = mv_x^2$$

Substituting the ideal gas equation where N = 1, and multiplying ½ on both sides, we get,

$$\frac{1}{2}kT = \frac{1}{2}mv_x^2 = KE_x$$

However, this is just the energy considering the molecule moves solely in the *x* direction. As the steps above may be repeated for the *y* and *z* directions, $KE_y = KE_z = \frac{3}{2}kT$. The *KE* for one molecule is thus,

$$KE_{\text{one molecule}} = \frac{1}{2}mv^2 = \frac{1}{2}m\left(v_x^2 + v_y^2 + v_z^2\right) = \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2$$

$$KE_x + KE_y + KE_z = \frac{3}{2}kT$$

As we make no distinction between the molecules. This is also the average *KE*. This relation shows that temperature is a measure of the average *KE* of a gas.

The total *KE* and thus the internal energy, *U*, for a gas of *N* molecules is thus,

$$\boxed{KE_{\text{total}} = U = \frac{3}{2}NkT = \frac{3}{2}nRT = \frac{3}{2}PV}$$

It is important to realise from this equation that the internal energy of an ideal gas is related to its temperature*. For a constant amount of ideal gas, if there is no change in U, there will be no change in temperature. Conversely, no change in temperature implies no change in U.*

6.3.5 Root Mean Square Velocities

From the previous section, we have determined for a gas of N particles,

$$\mathrm{KE_{total}} = \frac{3}{2}\mathrm{NkT}$$

Thus, the average kinetic energy per particle is given by,

$$\langle \mathrm{KE} \rangle = \frac{\mathrm{KE_{total}}}{\mathrm{N}} = \frac{3}{2}\mathrm{kT}$$

$$\frac{1}{2}\mathrm{m}\frac{\left(v_1^2 + v_2^2 + v_3^2 + \ldots + v_N^2\right)}{\mathrm{N}} = \frac{3}{2}\mathrm{kT}$$

where *m* is the mass of a single particle. From the above, we obtain,

$$\frac{1}{2}\mathrm{m}\langle v^2 \rangle = \frac{3}{2}\mathrm{kT}$$

Where

$$\langle v^2 \rangle = \frac{\left(v_1^2 + v_2^2 + v_3^2 + \ldots + v_N^2\right)}{\mathrm{N}}$$

This quantity which is the mean of the squared velocities is known as the **mean square speed**. This quantity is important because it gives us a feeling for the average KE of the system.

Square rooting both sides.

$$\boxed{v_{\mathrm{RMS}} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{\left(v_1^2 + v_2^2 + v_3^2 + \ldots + v_N^2\right)}{\mathrm{N}}}}$$

This quantity, v_{RMS} is known as the **root mean square velocity**.

The root mean square speed is defined as the square root of the mean value of the molecular speed squared.

We can understand it as being the velocity that averages the KE of the gas over all particles

Example 6:

Ten particles have the following speeds in m s^{-1}:

1.0 ; 7.0 ; 9.0 ; 5.0 ; 5.0 ; 6.0 ; 6.0 ; 5.0 ; 3.0 ; 4.0

Calculate

(a) the mean speed, and

Solution:

The mean speed is given by

$$\langle v \rangle = \frac{1+7+9+5+5+6+6+5+3+4}{10} = 5.1 \text{ m.s}^{-1}$$

(b) the r.m.s. speed.

Solution:

The r.m.s speed is given by

$$v_{RMS} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{1^2+7^2+9^2+5^2+5^2+6^2+6^2+5^2+3^2+4^2}{10}} = 5.5 \text{ m.s}^{-1}$$

Example 7

Helium gas occupies a volume of 0.0400 m^3 at a pressure of 200 kPa and temperature 300 K. Calculate the RMS speed of its molecules. (Molar mass of helium is 4)

Solution:

$$U = \frac{1}{2} m v_{total}^2 = \frac{3}{2} PV$$

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} \frac{PV}{N}$$

$$\langle v^2 \rangle = \frac{3PV}{Nm} = \frac{3nRT}{M_{gas}} = \frac{3nRT}{nM_{molar}} = \frac{3RT}{M_{molar}}$$

$$v_{RMS} = \sqrt{\frac{3RT}{M_{molar}}} = \sqrt{\frac{3 \times 8.31 \times 300}{0.004}} = 1370 \text{ m.s}^{-1}$$

Example 8

An ideal gas is contained in a vessel at 300K. If the temperature is increased to 900K, by what factor does each one of the following change?

(a) the average kinetic energy of the molecules,

Solution:

From above, we saw that

$$\langle KE \rangle = \frac{3}{2} kT$$

Hence, if T increases three fold, so will $\langle KE \rangle$.

(b) the root mean square molecular speed,

Solution:

From the equation $\frac{1}{2} m v_{RMS}^2 = \frac{3}{2} kT$,

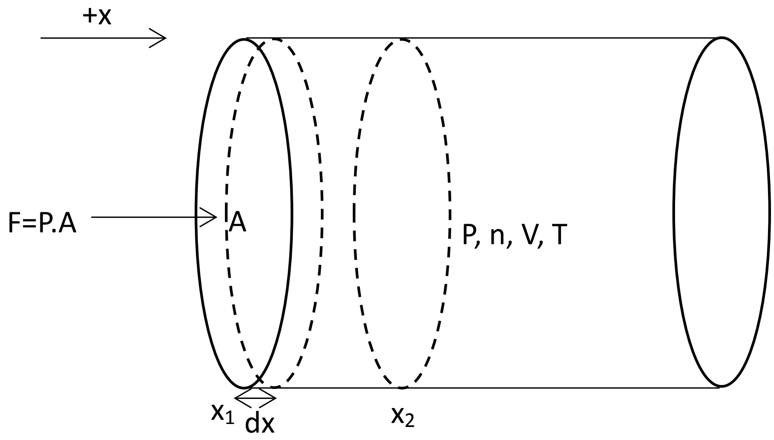
$$v_{RMS} \propto \sqrt{T}$$

Hence, if T increases three fold, v_{RMS} will increase by a factor of $\sqrt{3}$.

6.4 THERMODYNAMICS

6.4.1 Work

Consider a piston filled with some gas (may or may not be ideal). The cross section is uniform with an area *A*.



The gas exerts a pressure of *P* on the walls. Hence, if we wish to compress the piston, we have to exert a force of at least,

$$F = P.A$$

If we shift the piston from a position of x_1 to x_2, the amount of <u>*work done on the gas*</u> is given by

$$W = \int_{x_1}^{x_2} F.dx = \int_{x_1}^{x_2} PA.dx$$

However, note that with every shift of the piston in the positive *x* direction, the gas is compressed and the volume of the piston decreases. Hence,

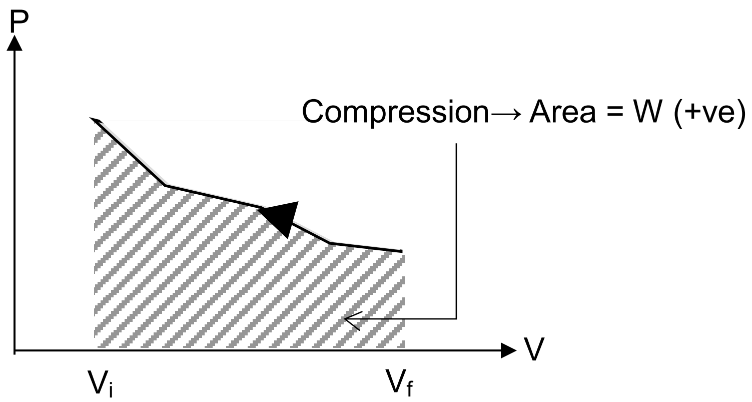
$$dV = -A.dx$$

Substituting this into the equation for work,

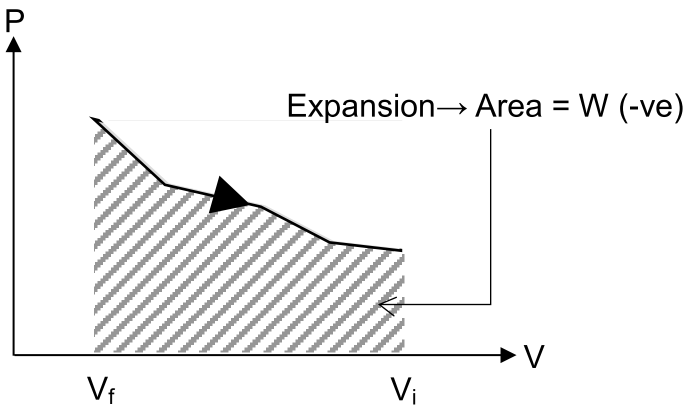
$$\boxed{W = -\int_{V_1}^{V_2} P.dV}$$

This formula tells us that the amount of work done may be determined by the area under the P-V graph.

Physically, the negative sign indicates that if *volume decreases*, *work is done **on*** the piston. Hence *energy goes into the gas*. *W is positive*.



Conversely, if the *volume increases*, *work is done **by*** **t**he piston. Hence, *energy goes out of the gas*. *W is negative*.



Note that if the question asks for work done *by* the gas, simply reverse the sign of *W*.

6.4.2 The First Law of Thermodynamics

Learning Outcome:

(d) recall and use the first law of thermodynamics expressed in terms of the change in internal energy, the heating of the system and the work done on the system.

The First Law of Thermodynamics states that the ***increase*** in internal energy **ΔU** of a system is the **sum of the heat supplied to the system *Q*, and the work done on the system, *W***.

Mathematically, this is expressed as

$$\boxed{\Delta U = Q + W}$$

Q (heat/thermal energy) refers to the amount of energy gained from the external environment. *Q is positive when thermal energy is supplied to the system*. Conversely, *Q is negative when thermal energy is removed from the system.*

Note:
If *Q* is supplied or removed, it need not necessarily bring about an increase/decrease in temperature of the substance since internal energy depends on the work done. Also, if it is stated that the system is thermally insulated, $Q=0$.

Example 9:

(a) Find the change in internal energy when 5 J of heat is supplied to the system and 3 J of work is done on the system.

Solution:

$$\Delta U = Q + W = 5 + 3 = 8 \text{ J}$$

The positive sign on ΔU means that the internal energy of the system has increased.

(b) 10 J of heat is given off by the system while 8 J of work is done on the system. What is the change in internal energy of the system?

Solution:

$$\Delta U = Q + W = -10 + 8 = -2\text{ J}$$

The positive sign on ΔU means that the internal energy of the system has decreased.

(c) The system absorbs 5 J of heat while doing 7 J of work. What is the loss in internal energy?

Solution:

Note that work is done BY the system. Therefore work done ON system (W) is given by W = -7 J

$$\Delta U = Q + W = 5 - 7 = -2\text{ J}$$

(d) The internal energy of a system increases by 10 J when 14 J of work is done on it. How much heat is lost by the system?

Solution:

$$\Delta U = Q + W$$

$$10 = Q + 14$$

$$Q = -4\text{ J}$$

As question asks for heat lost by the system, answer is 4 J.

(e) A system loses 5 J of heat and the internal energy decreases by 8 J. How much work is done by the system?

Solution:

$$\Delta U = Q + W$$

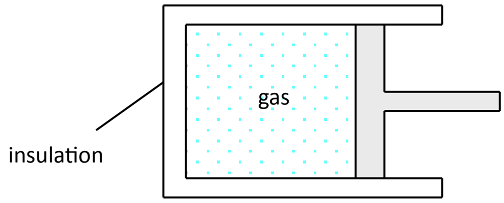
$$-8 = -5 + W$$

$$W = -3\text{ J}$$

As the question asks for work done by the system, answer is 3 J.

Example 10

Some gas, assumed to behave ideally, is contained within a cylinder that is surrounded by insulation to prevent loss of heat, as shown in the diagram.



Iniitially, the volume of gas is 2.9×10^{-4} m^3, its pressure is 1.04×10^5 Pa and its temperature is 314 K.

(a) Use the equation of state for an ideal gas to find the amount, in moles, of gas in the cylinder.

Solution:

$$PV = nRT$$

$$n = \frac{PV}{RT} = \frac{(1.04\times10^5)(2.9\times10^{-4})}{8.31\times314} = 0.0116 \text{ mol}$$

(b) The gas is then compressed to a volume of 2.9×10^{-5} m^3 and its temperature rises to 790 K. Calculate the pressure of the gas after this compression.

Solution:

$$PV = nRT$$

$$P = \frac{nRT}{V} = \frac{(0.0116)(8.31)(790)}{2.9\times10^{-5}} = 2.63\times10^5 \text{ Pa}$$

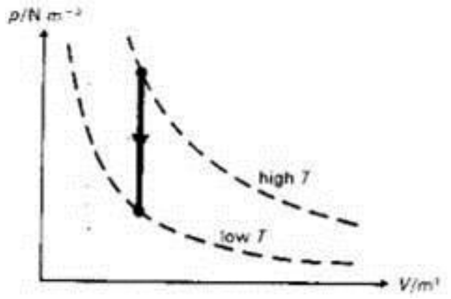
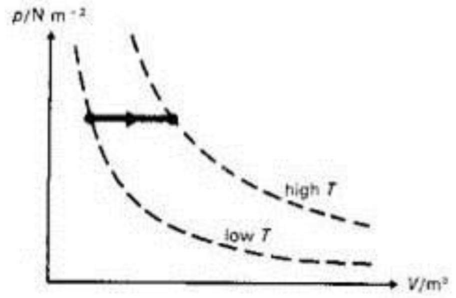
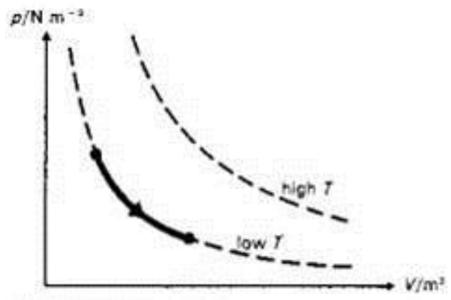
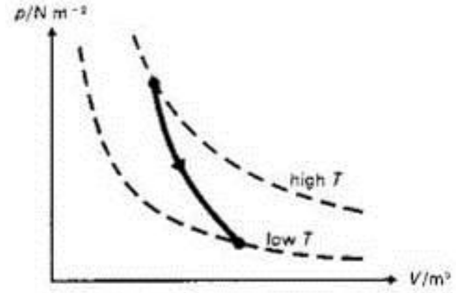
(c) The work done on the gas during the compression is 91 J. Use the first law of thermodynamics to find the change in internal energy of the gas.

Solution:

$$\Delta U = Q + W = 0 + 91 = 91 \text{ J}$$

6.4.3 Thermal-Processes & P-V Diagrams

The energy changes of a gas brought about by heat and work may cause the state variables to change. This is most effectively captured by drawing a graph of P vs V. The following 4 thermal processes for an ideal gas are most important for the A-levels.

Process	Characteristics	Graph
Isochoric/ Isovolumetric	• V = constant • W = 0 J • Therefore • $\Delta U = Q$	
Isobaric	• P = constant • $W = -P\Delta V$	
Isothermal	• T = constant • $P = \frac{\text{constant}}{V}$ • $W = \pm area\ under\ graph$ • $\Delta U=0$; $-Q = W$ • Slow process.	
Adiabatic/ Isolated/ Insulated	• All variables change. • Q = 0 J • W = ±area under graph • $\Delta U = W$ • Fast process.	

**Assign the + or – based on whether the arrow points left or right.

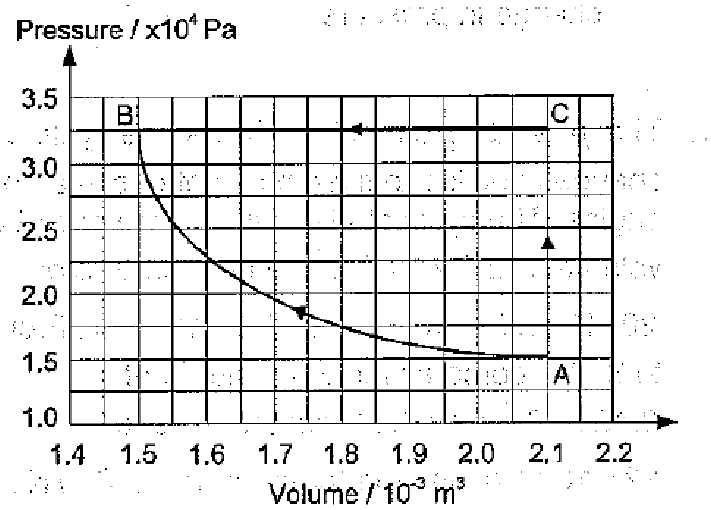
It must be emphasized that for a fixed amount of ideal gas, each point on the P-V graph represents a unique state that the gas is in, with its own internal energy. Recall,

$$U = \frac{3}{2} = nRT = \frac{3}{2}PV$$

Thus, if a gas moves from one point to another, no matter what the path, the ΔU is always the same.

Example 11

The graph below shows the variation of pressure with wolume of an ideal gas that is confined in a container. The gas is allowed to changes its volume along the curved line between A and B, and at the same time, 3.50 J of heat is supplied to the container.



Use the graph to

(a) Explain whether or not the change is isothermal.

Solution:

If the process is isothermal,

$$P_1V_1 = \text{constant} = P_2V_2$$

Choosing any two points,

$$P_1V_1 = (3.25 \times 10^4)(1.5 \times 10^{-3}) = 48.75 \text{ Pa.m}^3$$

$$P_2V_2 = \left(1.5 \times 10^4\right)\left(2.1 \times 10^{-3}\right) = 31.5 \text{ Pa.m}^3$$

As $P_1V_1 \neq P_2V_2$, process is not isothermal.

(b) Estimate the work done for the process A to B along the curved line.

Solution:

Splitting the graph into 3 trapeziums,

$$|W| = \frac{(3.25 \times 10^4 + 2.25 \times 10^4)(0.1 \times 10^{-3})}{2} + \frac{(2.25 \times 10^4 + 1.75 \times 10^4)(0.2 \times 10^{-3})}{2} + \frac{(1.75 \times 10^4 + 1.50 \times 10^4)(0.3 \times 10^{-3})}{2} = 11.625 \text{ J}$$

As the arrow points left, W is positive. Therefore,

$$W = 11.6 \text{ J}$$

(c) If the same gases begins at A and is changed along the vertical line from A to C and then along a horizontal line from C to B, determine the heat flow for the process ACB, stating whether heat flows into or out of the gas.

Solution:

In moving from state A to state B, the change in internal energy is,

$$\Delta U = Q + W = 11.625 + 3.5 = 15.125 \text{ J}$$

As no work is done along AC, along the path ACB, all work is done in process CB. As arrow points left, W is positive.

$$W_{CB} = \left(0.6 \times 10^{-3}\right)\left(3.25 \times 10^4\right) = 19.5 \text{ J}$$

Therefore,

$$\Delta U = Q_{AC} + Q_{CB} + W_{CB} = Q_{ACB} + W_{CB}$$

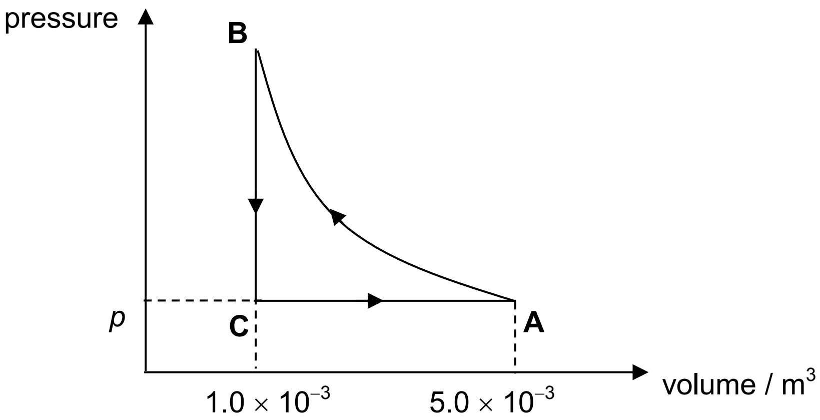
$$Q_{ACB} = \Delta U - W_{CB} = 15.125 - 19.5 = -4.375 \text{ J}$$

As Q is negative, heat has flown out of the gas.

When more than one thermal process takes place such that the gas is in its original state, this is known as a *cyclic process*. *As the gas has returned to its original state, its internal energy must be the same and hence ΔU must be 0 J. The area enclosed by the graph is the total (net) work done.*

Example 12

0.2 mol of an ideal gas is trapped in a closed cylinder sealed by a piston at an initial temperature of 300 K and initial pressure *p*. The gas undergoes a cycle of changes *A*→*B*→*C*→*A* as shown in the figure below.



From *A* to *B*, the gas is compressed isothermally at a temperature of 300 K during which 800 J of work is done on the gas. From *B* to *C*, the gas is cooled at a constant volume of 1.0×10^{-3} m^3 till its pressure returns to *p*. From *C* to *A*, the gas expands at constant pressure *p*.

(a) Calculate the pressure *p*.

Solution:

$$pV = nRT$$

$$p = \frac{nRT}{V} = \frac{0.2 \times 8.31 \times 300}{5 \times 10^{-3}} = 99.7 \text{ kPa}$$

(b) Calculate the work done on the gas during the change from *C* to *A*.

Solution:

$$\text{Area under graph} = |W| = (5-1)\times 10^{-3} \times 99.72 \times 10^{3} = 399\text{ J}$$

As arrow points right, W is negative. Therefore

$$W = -399\text{ J}$$

(c) Calculate the net heat supplied in one complete cycle.

Solution:

$$W_{total} = W_{AB} + W_{BC} + W_{CA}$$

Process *B*→*C* is isochoric. Hence, $W_{BC} = 0$ J.

$$W_{total} = 800 + 0 - 399 = 401\text{ J}$$

Since process is cyclic,

$$\Delta U = 0\text{ J}$$

$$\Delta U = Q_{total} + W_{total}$$

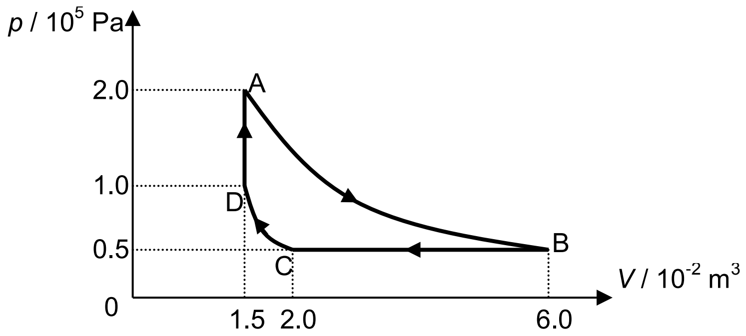
$$Q_{total} = 0 - 401 = -401\text{ J}$$

Example 13

A fixed mass of an ideal gas is subjected to various changes of pressure, volume and temperature through cycle *ABCDA*. The states *A*, *B*, *C*, and *D* of the gas are shown in the table below.

State	Pressure / 10^5 Pa	Volume / 10^{-2} m^3
A	2.0	1.5
B	0.5	6.0
C	0.5	2.0
D	1.0	1.5

The changes are shown in the P-V diagram below.



(a) The table below gives quantitative data for some of these changes. Complete the table.

State Change	Process	W / kJ	Q / kJ	ΔU / kJ
A to B	Isothermal	-4.16	4.16	0.00
B to C	Isobaric	2.00	-5.63	-3.63
C to D	Adiabatic	0.36	0.00	0.36
D to A	Isochoric	0	3.27	3.27

(b) This is a process in a cylinder of a diesel engine. If the cycle is repeated 10 times per second, calculate the power output of this engine.

Solution:

$$P = \frac{W}{t} = \frac{|-4.16 + 2 + 0.36| \times 10^3 \times 10}{1} = 18000 \text{ W}$$

(c) Calculate the efficiency ε of this engine where ε is defined as

$$\varepsilon = \frac{\text{work output}}{\text{heat input}}$$

Solution:

$$\varepsilon = \frac{\text{work output}}{\text{heat input}} = \frac{|-4.16 + 2 + 0.36| \times 10^3}{(4.16 + 3.27) \times 10^3} = 24.2\ \%$$

Kinetic Model of Matter

Self-Attempt Questions

1. Explain what is meant by internal energy of a system? Using the concept of internal energy, explain the following
   a) Compare the internal energy per unit mass of water and water vapour at the same temperature
   b) Why the temperature of a pure substance does not change when it experiences a change of state

2. (a) Explain, using a simple kinetic model for matter, why melting and boiling takes place without a change in temperature.

   (b) The ***specific latent heat of vaporisation*** of water is 2.26×10^6 J.kg^{-1} whereas the corresponding value of the ***specific latent heat of fusion*** is 3.34×10^5 J.kg^{-1}.

   Define the terms in **italics** and explain, using a *simple kinetic model for matter* why the first term is bigger than the second.

3. The assumptions of the simple kinetic theory of gases include:
   (i) gases are made up of many molecules moving randomly,
   (ii) the collisions of the molecules with the walls of the container are elastic (i.e. there is no loss in kinetic energy of the molecules in the collisions).
   What experimental evidence is there that justifies these assumptions? Describe the experiment in detail.

Discussion Question

4. One gram of water becomes 1671 cm^3 of steam when boiled at a constant pressure of 1.013×10^5 Pa. The specific latent heat of vaporisation at this pressure is 2.256×10^6 J.kg^{-1}. Calculate
   (a) the work done by the water when it vaporises;
   (b) its increase in internal energy. Using kinetic theory of gases, explain how a pressure is exerted by a gas.

[1.69×10^2 J, 2.09×10^3 J]

The Ideal Gas and the Ideal Gas Law

Self-Attempt Questions

5. What is the mass of 0.40 mol of Al_2O_3 ? Given that the mass numbers of Al and O are 27 and 16 respectively.

[40.8 g]

6. With reference to the assumptions made when describing an ideal gas, highlight the difference between a real and an ideal gas in terms of the forces of attraction or repulsion between molecules. How is the internal energy of an ideal gas different from a real gas?

7. A partially inflated balloon contains 500 m^3 of helium at 27 ˚C and 1 atm pressure. What is the volume of the helium at an altitude of 18000 ft where the pressure is 0.5 atm and the temperature is -3 ˚C.

[900 m^3]

8. A uniform capillary tube, closed at one end, contained air trapped by a thread of mercury 85 mm long. When the tube was held horizontal, the length of the air column was 50 mm; when it was held vertically with the closed end downwards, the length was 45 mm. Taking g to be 10 $m.s^{-2}$ and the density of mercury = 14 x 10^3 $kg.m^{-3}$ find the atmospheric pressure.

[1.1 x 10^5 Pa]

9. An air bubble of volume V_o is released by a fish at a depth h in a lake. The bubble rises to the surface. If the density of the water is ρ, the pressure at the surface is P_o, and the temperature of the lake is constant, find an expression for the volume of the bubble at the surface.

[$V = \left(1+\frac{\rho g h}{P_o}\right) V_o$]

<u>Discussion Questions</u>

10. Two identical gas cylinders each contain 20 kg of compressed air at 1000 kPa pressure and 275 K. One of the cylinders is fitted with a safety valve which releases air from the cylinder into the atmosphere if the pressure in the cylinder rises above 1100 kPa. The cylinders are then moved to a room where the temperature is 310 K. Calculate
    (a) the pressure in the cylinder which is not fitted with a safety valve,
    (b) the mass of gas lost from the cylinder fitted with the safety valve.

[1127kPa, 0.48 kg]

11. Two containers of volume V and 4V are connected by a capillary tube. Initially both containers are at 280 K and contain air at a pressure of 101 kPa. The larger container is now warmed to 350 K and the smaller container cooled to 210 K.
    (a) Why may we not apply pV/T = constant to either container separately?
    (b) Calculate the final pressure in both containers.

[111 kPa]

12. A car tire of volume 1.60×10^{-2} m^3 contains air at a temperature of 30.0 ˚C and a pressure of 2.60×10^5 Pa. A foot pump is used to pump air into the tire to increase the pressure to 3.10×10^5 Pa. Each stroke of the pump pushes 3.0×10^{-4} m^3 of air at 1.0×10^5 Pa into the tire. The temperature of the air stays constant at 30.0 ˚C throughout the whole process. You may assume that air behaves as an ideal gas.
    (a) Calculate the initial number of moles of air that the tire contains.
    (b) Calculate the number of strokes needed to raise the pressure of the car tire to 3.10×10^5 Pa.
    (c) Explain whether the answer calculate in the previous section is a minimum or a maximum number of strokes required.

[1.65 mol, 27, minimum]
PJ/09/P3/7b

13. In scuba diving, a greater water pressure acts on a diver at greater depths. The air pressure inside the body cavities (e.g. lungs) must be maintained at the same pressure as that of the surrounding water, otherwise they might collapse. A special valve automatically adjusts the pressure of the air breathed from a scuba tank to ensure that the air pressure equals the water pressure at all time.

    A 0.0150 m^3 scuba tank is filled with compressed air at an absolute pressure of 2.02×10^7 Pa.

    For the following questions, assume that air is consumed at a rate of 0.0300 m^3 per minute and that the temperature is the same at all depths.
    (a) Given that the density of sea water is 1025 kg m^{-3} and that the atmospheric pressure is 1.01×10^5 Pa, find the pressure at a depth of 10.0 m.
    (b) Hence calculate the volume of air available under a depth of 10.0 m.
    (c) Determine how long the diver can stay under a depth of 10.0 m.

[2.02×10^5 Pa, 1.50 m^3, 49.5 min]

Internal Energy of an Ideal Gas

Self-Attempt

14. Helium gas occupies a volume of 0.04 m^3 at a pressure of 2×10^5 Pa and temperature of 300 K. Calculate,
    (a) The mass of helium
    (b) The rms speed of the molecules
    (c) The rms speed at 432 K when the gas is heated at constant pressure to this temperature
    (d) The rms speed of hydrogen molecules at 432 K

    Take the relative molecular mass of hydrogen and helium to be 2 and 4 respectively.

[12.8 g, 1367 m.s^{-1}, 1641 m.s^{-1}, 2321 m.s^{-1}]

<u>Discussion Questions</u>

15. The atomic mass of Helium is 4. In a sample of Helium at 30 ˚C, find
    (a) Find the root mean square speed of a molecule of Helium.
    (b) The mean average translational kinetic energy of a Helium atom in this gas.
    (c) Given that the mass of gas involved is 50 g, find the total internal energy of the atoms in this gas sample. Assume the gas behaves as an ideal gas.

[1374 m.s^{-1}, 6.27×10^{-21} J, 4.72×10^{4} J]

16. The molecules of 2 ideal gases A and B have an average KE of 6.2×10^{-21} J at 27 ˚C. Molecular masses of gas A and B are 1.7×10^{-27} kg and 3.0×10^{-26} kg respectively.
    (a) Determine the root mean square speeds of the molecules of A and B.
    (b) The escape velocity from Earth is about 11 km.s^{-1} and from Jupiter is 60 km.s^{-1}, which of the gases, A or B is likely to be found in the Earth's atmosphere and which in Jupiter's? Why?

[2701 m.s^{-1}, 643 m.s^{-1}]

The First Law of Thermodynamics & Thermal Processes

<u>Self-Attempt Questions</u>

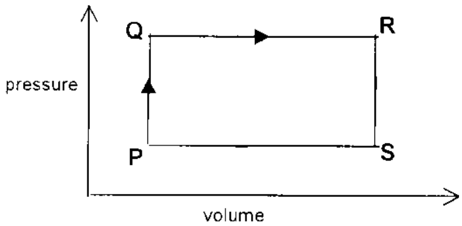
17. An ideal gas of volume 1.5×10^{-3} m^{3} and at pressure 1.0×10^{5} Pa is supplied with 70 J of energy. The volume increases to 1.7×10^{-3} m^{3}, the pressure remaining constant. Calculate the change in internal energy of the gas.

[+50 J]

18. The densities of water and steam at 100 ˚C and 1.01x10^{5} Pa are 1000 kg.m^{-3} and 0.590 kg.m^{-3} respectively. Calculate
    (a) the change in volume which occurs when 1.00 kg of water evaporates at 100 ˚C and 1.01x10^{5} Pa.
    (b) the work done against the atmosphere to produce the change in volume found in (a).
    (c) The specific latent heat of vaporization of water at 100 ˚C is 2.26x10^{6} J.kg^{-1}. Use the first law of thermodynamics to calculate the increase in internal energy of the molecules when 1.00 kg of water evaporates.
    (d) State with a reason, the increase in the potential energy of the molecules.

[1.69 m^{3}, 171 kJ, 2.09 MJ]

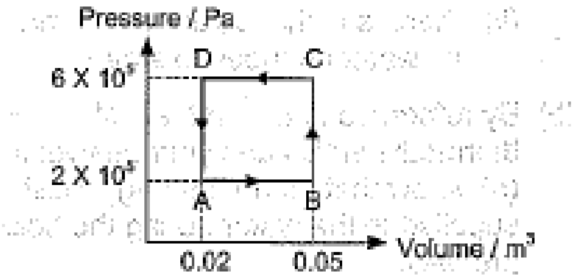
19. State the first law of thermodynamics.



When the gas is taken from state P to R by stages PQR, 8 J of heat is absorbed and 3 J of work is done by it. If the gas is taken from state P to R by stage PSR, it is found that 1 J of work is done by it. What is the heat exchange between the gas and the surroundings?

[+6 J]

20. Consider a system that is taken along the paths shown in the P-V diagram below. The internal energy of the system at A and at C is 30 kJ and 95 kJ respectively.



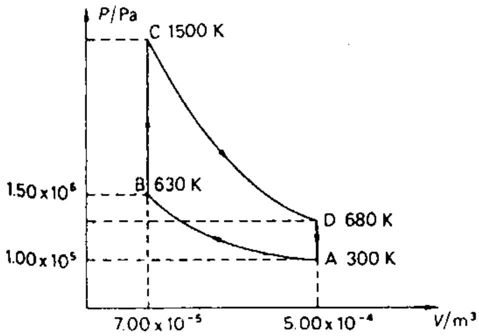
(a) Calculate the internal energy of the system at point B, if 21 kJ of heat is absorbed by the system in going from A to B.

(b) Find the amount of heat that enters the system along the path from B to C.

[45 J, 50 J]

Discussion Questions

21. The figure below shows some details concerning the 35ehavior of a fixed mass of a gas (assumed to be ideal) in a petrol engine.



(a) Use the equation of state for an ideal gas to find the number of moles in the fixed mass of gas.

(b) In the change from state B to state C, the temperature of the gas rises from 630 K to 1500 K. The molar heat capacity of the gas at constant volume is 21

J.mol^{-1}.K^{-1}. Calculate the thermal energy supplied in this change.

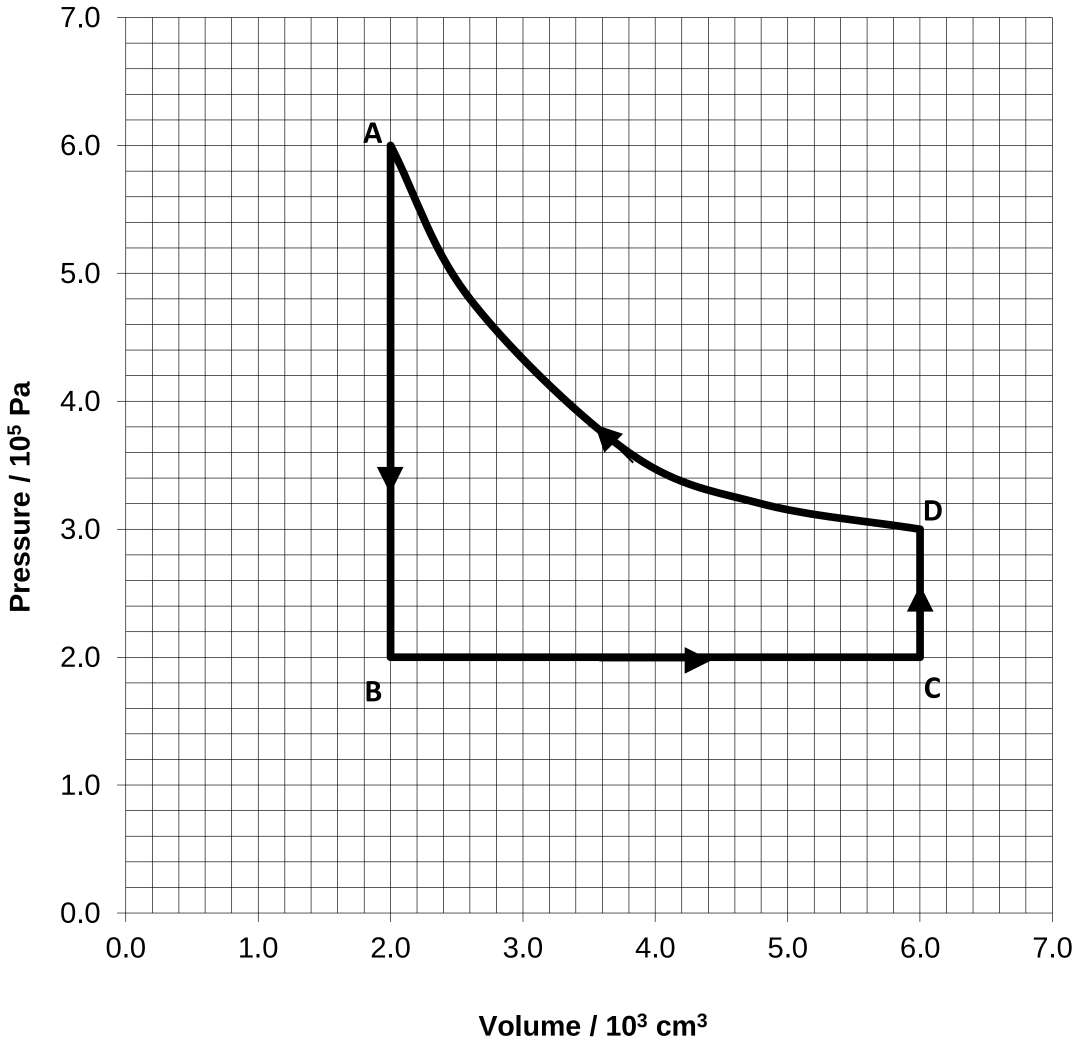
(c) How much work is done by the gas in the change from state B to C?

(d) In the change from state C to D the gas expands to its original volume. The temperature at state D is 680 K. Calculate the pressure at state D.

[0.0201 mol, 367 J, 0 J, 2.27×10^5 Pa]

** Pressure axis readings are 1.0×10^5 Pa and 1.5×10^5 Pa
** Volume axis readings are 7×10^{-5} m^3 and 5×10^{-4} m^3

22. A fixed mass of ideal gas undergoes a cycle of changes A→B→C→D→A as shown in the figure below.



(a) Use the figure to determine the work done on the gas during the change D→A.

(b) Fig. 7.2 shows a table with various parts of the cycle of change for the gas. Complete the table with the symbol + , – or 0 to indicate the work done on the gas corresponding to the various parts.

	Work done on the gas
A → B	
B → C	
C → D	
D → A	
A → B → C → D → A	

(c) Describe and explain the change in internal energy of the gas as it undergoes the change A→B and C→D.

(d) Explain why the total change in the internal energy of the gas during a complete cycle must be zero.

(e) The temperature of the gas at A is 600 K. Use the equation of state for an ideal gas to determine the temperature of the gas at B.

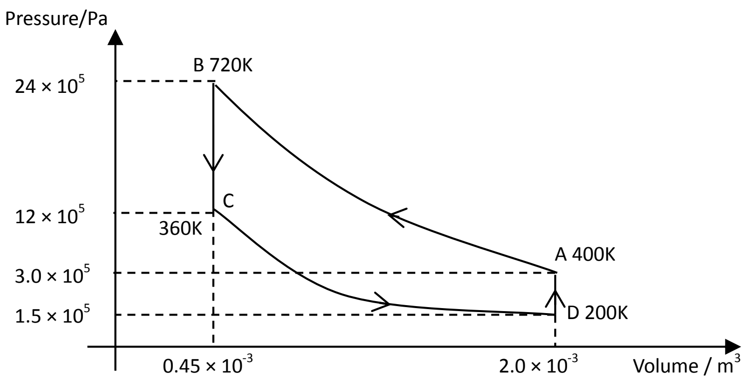
(f) Hence determine the heat loss when the gas undergoes the change A→B. The specific heat capacity of the gas is 940 $J.kg^{-1}.K^{-1}$ and the density of the gas at A is 1.6 $kg.m^{-3}$.

(g) A student states that the gas can undergo a change from A→C directly with no net heat transfer. Explain why his statement is not valid.

[1520±30 J, 200 K, $1.2×10^3$ J]

NY/09/P3/7b

23. A fixed mass of gas in a heat pump undergoes a cycle of changes of pressure, volume and temperature as illustrated in the graph below. The gas is assumed to be ideal.



The table below shows the increase in internal energy which takes place during each of the changes A to B, B to C and C to D. It also shows that in both sections A to B and C to D, no heat is supplied to the gas.

	Increase in internal energy /J	*Heat supplied to gas /J*	*Work done on gas /J*
A to B	1200	0	
B to C	–1350		
C to D	–600	0	
D to A			