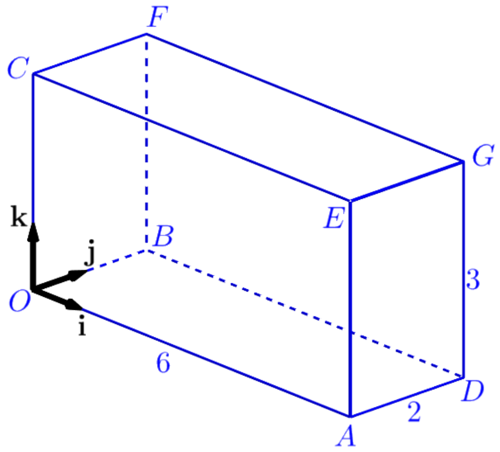
National Junior College
2025 – 2026 H2 Mathematics
Topic 1A Vectors I **Tutorial**

Check Your Understanding

1 The diagram below shows a rectangular cuboid *OADBFCEG* with dimensions $OA = 6$ units, $OB = 2$ units and $OC = 3$ units.



(a) Find the following vectors, giving your answers in column vector notation.

(i) $\overrightarrow{OE}$,

(ii) $\overrightarrow{OG}$,

(iii) $\overrightarrow{CB}$,

(iv) $\overrightarrow{CD}$.

State another two displacement vectors that are equal to $\overrightarrow{OE}$ and $\overrightarrow{CB}$ respectively.

(b) Find the magnitude of $\overrightarrow{CD}$. Hence, or otherwise, find two distinct vectors that are parallel to $\overrightarrow{CD}$ and has length 21 units each.

(c) The lines *OG* and *CD* intersect at point *P*. Find the position vector of *P*.

(d) The point *Q* is such that *E* lies on the line segment *BQ* and $BQ : EQ = 3 : 2$. Find the position vector of *Q*.

2 For each of the following pairs of vectors, find their scalar product.

(a) $\mathbf{i}+\mathbf{j}-3\mathbf{k}$ and $2\mathbf{i}+\mathbf{k}$,

(b) $\begin{pmatrix}1\\-4\\3\end{pmatrix}$ and $\begin{pmatrix}-1\\-3\\1\end{pmatrix}$.

3 Relative to an origin O, the position vectors of the points A and B are $\mathbf{i}-\mathbf{j}+2\mathbf{k}$ and $-2\mathbf{i}+\mathbf{j}$ respectively. Find

(a) $\angle AOB$,

(b) $\angle OBA$.

4 Relative to an origin O, A and B have position vectors $\mathbf{i}+c\mathbf{j}-2\mathbf{k}$ and $-c\mathbf{i}+2\mathbf{j}+\mathbf{k}$ respectively, where c is a real constant. Find the value of c

(a) if $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OB}$,

(b) if instead $\angle AOB = 120°$.

5 For each of the following pairs of vectors, find their vector product.

(a) $\mathbf{i}+\mathbf{j}-3\mathbf{k}$ and $2\mathbf{i}+\mathbf{k}$,

(b) $\begin{pmatrix}1\\-4\\3\end{pmatrix}$ and $\begin{pmatrix}-1\\-3\\1\end{pmatrix}$,

(c) $2\mathbf{i}-6\mathbf{k}$ and $\begin{pmatrix}-3\\0\\9\end{pmatrix}$.

6 Relative to an origin O, the position vectors of the vertices A, B and C of a triangle are $\mathbf{i}+\mathbf{j}-\mathbf{k}$, $2\mathbf{i}+3\mathbf{j}+\mathbf{k}$ and $5\mathbf{i}+4\mathbf{j}-\mathbf{k}$ respectively.

(i) Find the exact area of triangle ABC.

(ii) Find the length of projection of $\overrightarrow{AB}$ onto $\overrightarrow{AC}$.

(iii) Find the projection vector of $\overrightarrow{AB}$ onto $\overrightarrow{AC}$ in the form $\frac{1}{5}(a\mathbf{i}+b\mathbf{j}+c\mathbf{k})$ for some integers a, b and c to be determined.

(iv) Find the length of the vector component of $\overrightarrow{AB}$ perpendicular to $\overrightarrow{AC}$ in the form $\sqrt{n}$ for some integer n to be determined.

(v) Find the vector component of $\overrightarrow{AB}$ perpendicular to $\overrightarrow{AC}$ in the form $\frac{1}{5}(p\mathbf{i}+q\mathbf{j}+r\mathbf{k})$ for some integers p, q and r to be determined.

Practice Questions

1 **(a)** Relative to the origin O, the position vectors of two points A and B are $\mathbf{a}=\mathbf{i}+t\mathbf{k}$ and $\mathbf{b}=2\mathbf{i}-\mathbf{j}+\mathbf{k}$ respectively, where $t\in\mathbb{R}$. Given that the angle between $\mathbf{a}$ and $\mathbf{b}$ is $60°$, evaluate the possible exact values of t.

(b) Relative to the origin O, the position vectors of the points P and Q are $\mathbf{p}$ and $\mathbf{q}$ respectively. The point R lies on PQ produced such that $QR:PR=3:5$ and OR is perpendicular to OP. Show that $\mathbf{p}\cdot\mathbf{q}=\frac{3}{5}|\mathbf{p}|^2$.

[2015 / NJC / SH2 / T1LT / Q1]

2 **(i)** Find a unit vector $\mathbf{n}$ such that $\mathbf{n}\times(2\mathbf{i}-3\mathbf{j}+6\mathbf{k})=\mathbf{0}$.

(ii) Given that the vector $\mathbf{u}$ has a magnitude of 5 units and is parallel to the vector $2\mathbf{i}-3\mathbf{j}+6\mathbf{k}$, find the possible vectors of $\mathbf{u}$.

(iii) Find the cosine of the acute angle between $\mathbf{u}$ and the z-axis.

[MI Promo 9758/2018/PU2/01/Q6 modified]

3 **(a)** Given that $\mathbf{u}$ and $\mathbf{v}$ are non-zero vectors such that $\mathbf{u}\times\mathbf{v}=(\mathbf{u}-\mathbf{v})\times(\mathbf{u}+\mathbf{v})$, what can be deduced about $\mathbf{u}$ and $\mathbf{v}$?

(b) Referred to an origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, where $\mathbf{a}$ and $\mathbf{b}$ are not parallel to each other. Point C is the mid-point of AB. Point D lies on OA produced such that $OA:OD=1:p$, where $p>1$. Point E lies on BD, between B and D, such that $BE:ED=p:2$. Given that points O, C and E are collinear, find the value of p.

[EJC Promo 9758/2018/Q5]

4 Relative to the origin O, the position vectors of the points A, B and P are $-\mathbf{i}-3\mathbf{j}+2\mathbf{k}$, $5\mathbf{i}+2\mathbf{k}$ and $(1+2\lambda)\mathbf{i}+(\lambda-2)\mathbf{j}+2\mathbf{k}$ respectively, where $\lambda\in\mathbf{R}$, $\lambda\neq-1$.

(i) Show that A, B and P are collinear.

(ii) Find the value of λ such that P is on the line BA produced and area of triangle OAP is $162\sqrt{5}$ square units. Give a reason for your choice.

[2012 / NJC / Prelims / P1 / Q2]

5 Referred to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ such that $\mathbf{a}=\mathbf{i}-\mathbf{j}+\mathbf{k}$ and $\mathbf{b}=2\mathbf{i}+3\mathbf{k}$. The point C has position vector $\mathbf{c}$ given by $\mathbf{c}=\lambda\mathbf{a}+\mu\mathbf{b}$, where λ and μ are constants.

(i) Find the exact area of triangle OAB.

(ii) Given that $OABC$ forms a parallelogram, write down the values of λ and μ.

(iii) Given instead that $\mu=2$ and that $OC=\sqrt{59}$, find the possible coordinates of C, leaving your answers in exact form.

[2016 / MJC / Promo / Q4]

6 Referred to the origin O, the points A and B are such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$ where **a** and **b** are non-parallel vectors. It is also given that C, D and E are the midpoints of OA, OB and AB respectively, and M is the point of intersection of the lines AD and BC.

(i) Find $\overrightarrow{OM}$ in terms of **a** and **b**.

(ii) Show that M lies on the line OE.

(iii) Find the area of the quadrilateral $OCMD$, expressing your answer in the form $k|\mathbf{a}\times\mathbf{b}|$, where k is a constant to be determined.

7 **(i)** Given that $\mathbf{u}\times\mathbf{v} = \mathbf{0}$, what can be deduced about the vectors **u** and **v**?

Vectors **a**, **b** and **c** are such that $\mathbf{a} \neq \mathbf{0}$ and $\mathbf{a}\times 3\mathbf{b} = 2\mathbf{a}\times\mathbf{c}$.

(ii) Show that $3\mathbf{b} - 2\mathbf{c} = \lambda\mathbf{a}$, where λ is a constant.

(iii) It is now given that **a** and **c** are unit vectors, the modulus of **b** is 4 and that the angle between **b** and **c** is $60°$. Using a suitable scalar product, find exactly the two possible values of λ.

[2018 / A-Level / P1 / Q6(modified)]

8 Relative to the origin O, two points A and B have position vectors given by **a** and **b** respectively such that they are non-zero, non-parallel and $|\mathbf{a}| = |\mathbf{b}|$.

(i) Show that $(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}-\mathbf{b}) = 0$.

(ii) Give a geometrical interpretation of the result shown in part **(i)**.

(iii) Suppose $|\mathbf{a}| = |\mathbf{b}| = 1$, show that $|\mathbf{a}\cdot\mathbf{b}|^2 + |\mathbf{a}\times\mathbf{b}|^2 = 1$.

[2016 / NJC / SH2 / T2 CT / Q6]

9 Relative to the origin O, the position vectors of A and B are $\frac{11}{3}\mathbf{i} + 4\mathbf{j} + \frac{7}{3}\mathbf{k}$ and $2\mathbf{i} + 4\mathbf{k}$ respectively. It is given that R lies on BA produced such that $2BR = 3AR$.

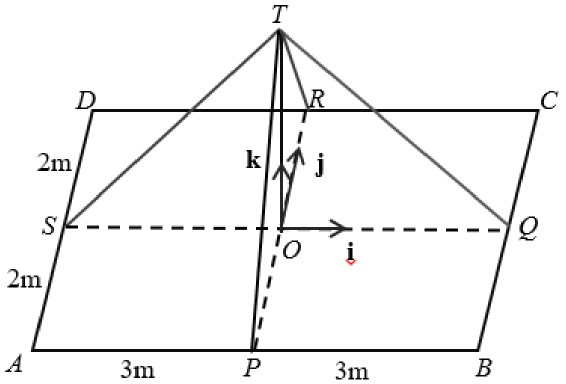
(i) Show that $\overrightarrow{OR}$ is given by $7\mathbf{i} + 12\mathbf{j} - \mathbf{k}$ and hence find the exact length of projection of $\overrightarrow{OR}$ on $\overrightarrow{OB}$.

(ii) Find the vector $\overrightarrow{NR}$, where N is the foot of perpendicular from R to OB.

(iii) Find the position vector of the point R', which is a reflection of the point R in the line passing through OB.

[2016 / SRJC / J1 CT / Q7]

10 During a camp, a group of scouts builds a structure using wooden poles as shown in the diagram. The structure has a rectangular base *ABCD* on the horizontal ground, with *AB* = 6 metres and *AD* = 4 metres. The top of the structure, *T*, is 2 metres vertically above *O*, the centre of *ABCD*. *T* is connected to *P*, *Q*, *R* and *S*, the mid-points of *AB*, *BC*, *CD* and *DA* respectively.



(i) If **i**, **j** and **k** are the unit vectors in the directions of *OQ*, *OR* and *OT* respectively, write down the position vectors of *Q*, *R* and *T*. Hence find angle *QTR* to the nearest degree.

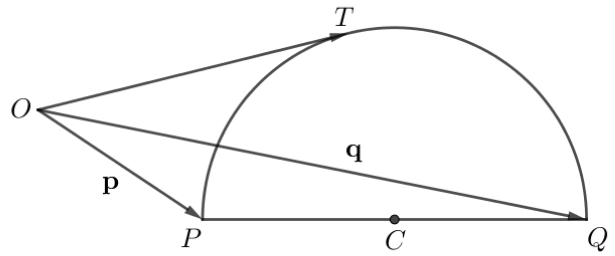
(ii) The scouts tie a rope from *P* to a point *E* on *ST* such that $4\overrightarrow{SE} = \overrightarrow{ST}$. Find the length of the rope between *E* and *P*, assuming that the rope is taut.

(iii) Another rope is tied from *P* to a point *F* on *TQ*. Find the position vector of *F* such that the length of the rope between *F* and *P* is minimised.

(iv) A flagpole of 5 metres is erected vertically at the point $G(\alpha, \beta, 0)$. Find the values of α and β if the top of the flagpole is collinear with *A* and *T*.

[2016 / PJC / J1 CT / Q10]

11



PQ is the diameter of a semicircle with centre *C* as shown above. *O* is the origin and $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$. The tangent to the semicircle drawn from *O* touches the semicircle at *T*.

(i) State, in terms of **p** and **q**, the position vector of *C*, and hence show that the radius of the semicircle is $\frac{1}{2}|\mathbf{p}-\mathbf{q}|$ units.

By considering the triangle *OTC*, show that $OT^2 = \mathbf{p}\cdot\mathbf{q}$.

(ii) Give a geometrical interpretation of $\dfrac{\left|\overrightarrow{OC}\cdot\overrightarrow{OT}\right|}{OT}$ and write down its value in terms of **p** and **q**.

(iii) Show that $\left|\overrightarrow{OC}\times\overrightarrow{OT}\right| = \dfrac{1}{m}|\mathbf{p}-\mathbf{q}|\sqrt{\mathbf{p}\cdot\mathbf{q}}$, where *m* is an integer to be determined.

[2014 / NJC / SH2 / Common Test / Q5 (adapted)]

Numerical Answers

Practice Questions

1 **(a)** $4 \pm \sqrt{21}$

2 **(i)** $\mathbf{n} = \pm \frac{1}{7}(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$ **(ii)** $\mathbf{u} = \pm \frac{5}{7}(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$

(iii) $\frac{6}{7}$ **(iv)** $p = \frac{3}{2}, \theta = \frac{\pi}{2}, q = \frac{3\sqrt{3}}{2}$

3 $p = \sqrt{2}$

4 **(iii)** -109

5 **(i)** $\frac{\sqrt{14}}{2}$ **(ii)** $\lambda = -1,\ \mu = 1$

(iii) $\left(\frac{13}{3}, -\frac{1}{3}, \frac{19}{3}\right),\ (-3, 7, -1)$

6 **(i)** $\frac{1}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$ **(iii)** $\frac{1}{6}|\mathbf{a} \times \mathbf{b}|$

7 $\lambda = \pm 2\sqrt{31}$

9 **(i)** $\sqrt{5}$ units **(ii)** $\overrightarrow{NR} = 6\mathbf{i} + 12\mathbf{j} - 3\mathbf{k}$

(iii) $\overrightarrow{NR'} = -5\mathbf{i} - 12\mathbf{j} + 5\mathbf{k}$

10 **(i)** $67°$ **(ii)** 3.05

(iii) $\overrightarrow{OF} = \frac{6}{13}\begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix}$ **(iv)** $\alpha = \frac{9}{2}, \beta = 3$