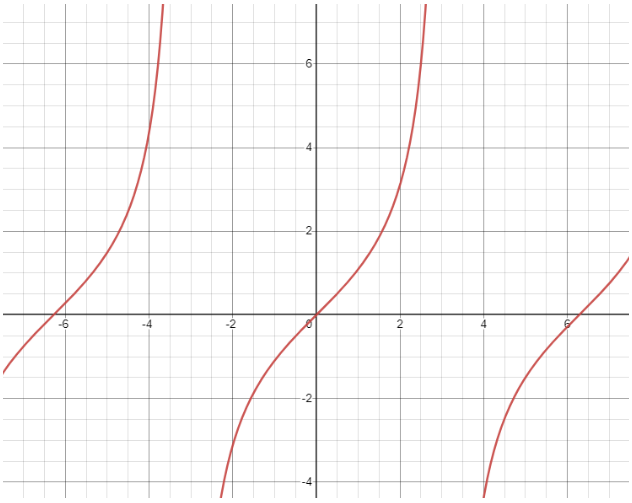
YISHUN TOWN SECONDARY SCHOOL

Mathematics Department

MARKING SCHEME

Examination : 4NA Preliminary Examination **Date**: ___ Aug 2023

Subject : Additional Maths **Paper No.** : 2

Q^{n}	Key Steps / Solution	Scheme
1a	$\int (x^2 - 1)^2 \text{ d}x$ $= \int x^4 - 2x^2 + 1 \text{ d}x$ $= \frac{x^5}{5} - \frac{2}{3}x^3 + x + c$	For expansion
1b	$\int \frac{x^3 - x^2}{2x} \text{ d}x$ $= \frac{1}{2}\left(\int x^2 - x \text{ d}x\right)$ $= \frac{1}{2}\left(\frac{x^3}{3} - \frac{1}{2}x^2\right) + c$ $= \frac{x^3}{6} - \frac{x^2}{4} + c$	For simplifying fraction
2a	2π or 360^o	
2b		1 cycle in 2π critical points ($\frac{\pi}{2}$,-2), ($\frac{3\pi}{2}$,2), ($-\frac{\pi}{2}$,2), ($-\frac{3\pi}{2}$,-2) asymptotes at $\theta = \pi$, $\theta = -\pi$
3a	$\text{f}(x) = 3x^2 - 9x + 7$ $= 3(x^2 - 3x) + 7$ $= 3\left[\left(x - \frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2\right] + 7$ $= 3\left(x - \frac{3}{2}\right)^2 - \frac{27}{4} + 7$ $= 3\left(x - \frac{3}{2}\right)^2 + \frac{1}{4}$	factorizing out 3 completing the square multiplying 3 back
3b	Since $3\left(x - \frac{3}{2}\right)^2 \geq 0$, $3\left(x - \frac{3}{2}\right)^2 + \frac{1}{4} \geq \frac{1}{4} (> 0)$.	

4	$\frac{\mathrm{d}q}{\mathrm{d}t} = -\frac{5}{\sqrt{(2t+1)^3}}$ $q = -5\int (2t+1)^{-\frac{3}{2}} dt$ $= (-5)\frac{(2t+1)^{-\frac{1}{2}}}{-\frac{1}{2}(2)} + c$ $= \frac{5}{\sqrt{2t+1}} + c$ $q = \frac{5}{\sqrt{2t+1}} + c$ $5 = \frac{5}{\sqrt{2(0)+1}} + c$ $c = 0$ $q = \frac{5}{\sqrt{2t+1}}$ $1 = \frac{5}{\sqrt{2t+1}}$ $\sqrt{2t+1} = 5$ $2t+1 = 25$ $t = 12$	
5a	$y = \frac{6x^2}{2-5x}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{12x(2-5x) - 6x^2(-5)}{(2-5x)^2}$ $= \frac{24x - 60x^2 + 30x^2}{(2-5x)^2}$ $= \frac{24x - 30x^2}{(2-5x)^2}$ $= \frac{6x(4-5x)}{(2-5x)^2}$ $a = 6$ and $b = 5$	Quotient rule Expansion and simplification Factorisation

5b	For decreasing function, $\frac{dy}{dx} < 0$ $\frac{6x(4-5x)}{(2-5x)^2} < 0$ $(2-5x)^2 > 0$ $6x(4-5x) < 0$ $x < 0 \;\; or \;\; x > \frac{4}{5}$	Identifying $\frac{dy}{dx} < 0$ With indication of $(2-5x)^2 > 0$ leading to $6x(4-5x) < 0$
6a	$\frac{dy}{dx} = 3x^2 - 2$ At $x = 1, \frac{dy}{dx} = 1, y = 4$ Gradient of normal at $x = 1$ $= -1$ Equation normal at $x = 1$ $4 = -1(1) + c$ $c = 5$ $y = -x + 5$ Area $= \frac{1}{2}(5)(5)$ $= 12.5$ units^2	gradient function for value of y gradient of normal equation of normal
6b	$3x^2 - 2 = 1$ $3x^2 = 3$ $x = \pm 1$ When $x = -1$, $y = 6$ $Q(-1, 6)$	
7a	$\cos 3\theta = \cos(2\theta + \theta)$ $= \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$ $= (2\cos^2\theta - 1)\cos\theta - (2\sin\theta\cos\theta)\sin\theta$ $= 2\cos^3\theta - \cos\theta - 2\sin^2\theta\cos\theta$ $= 2\cos^3\theta - \cos\theta - 2(1-\cos^2\theta)\cos\theta$ $= 2\cos^3\theta - \cos\theta - 2\cos\theta + 2\cos^3\theta$ $= 4\cos^3\theta - 3\cos\theta$	for addition formula for double angle for changing to cosine

7b	$4\cos^3\theta - 3\cos\theta = \frac{1}{2}$ $\cos 3\theta = \frac{1}{2}$ Reference angle = 60^o $3\theta = 60^o, 300^o, 420^o, -60^o, -300^o, -420^o$ $\theta = 20^o, 100^o, 140^o, -20^o, -100^o, -140^o$	$\cos 3\theta$ Ref angle 1^{st} 3 angles next 3 angles
8a	$\pi r^2 h = 108\pi$ $h = \frac{108}{r^2}$	
8b	$A = \pi r(2r) + 2\pi rh + \pi r^2$ $= 2\pi r^2 + 2\pi r\left(\frac{108}{r^2}\right) + \pi r^2$ $= 3\pi r^2 + \left(\frac{216\pi}{r}\right)$ $= 3\pi\left(r^2 + \frac{72}{r}\right)$	surface area substituting *h* and simplifying to answer
8c	At minimum, $\frac{\mathrm{d}A}{\mathrm{d}r} = 0$ $3\pi\left(2r - \frac{72}{r^2}\right) = 0$ $2r^3 = 72$ $r = 3.30$ $\frac{\mathrm{d}^2 A}{\mathrm{d}r^2} = 3\pi\left(2 + \frac{144}{r^3}\right)$ > 0 $\therefore A$ is a minimum at r = 3.30	differentiation and = 0 solving 2^{nd} derivative conclusion
8d	$h = \frac{108}{\left(\sqrt[3]{36}\right)^2}$ $= 9.9058$ Height of container = $9.9058 + \sqrt{(2(3.3019)^2 - (3.3019)^2}$ $= 15.6$ Since 15.6 < 35 cm, the container can be displayed in the shop.	

9a	$10x - x^2 = 16$ $x^2 - 10x + 16 = 0$ $(x-2)(x-8) = 0$ $x = 2, 8$ $A(2,16)$	
9b	$\int_0^2 10x - x^2 dx$ $= \left[5x^2 - \frac{1}{3}x^3\right]_0^2$ $= 5(2)^2 - \frac{1}{3}(2)^3$ $= 17\frac{1}{3}$ units^2 Shaded area $=17\frac{1}{3} + (6)(16) - \frac{1}{2}(8)(16)$ $= 49\frac{1}{3}$units^2	correct limits integration substitution correct method to find area 2 triangles
10a	$y = x\left(\frac{1}{2}x - k\right)^3$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \left(\frac{1}{2}x - k\right)^3 + 3x\left(\frac{1}{2}x - k\right)^2\left(\frac{1}{2}\right)$ $= \left(\frac{1}{2}x - k\right)^2\left(\frac{1}{2}x - k + \frac{3}{2}x\right)$ $= \left(\frac{1}{2}x - k\right)^2(2x - k)$ When $x = 2, \frac{\mathrm{d}y}{\mathrm{d}x} = 0$ $\left(\frac{1}{2}(2) - k\right)^2(2(2) - k) = 0$ $(1-k)^2(4-k) = 0$ $k = 1,\ 4$ (rej)	Product Rule Substitution and equating to 0 Solving and leading to shown + rejection

<table>
<tr>
<td>10b</td>
<td>
$\frac{dy}{dx} = \left(\frac{1}{2}x - 1\right)^2 (2x - 1)$
When $\frac{dy}{dx} = 0$,
$x = 2,\ \frac{1}{2}$
<table>
<tr><td></td><td>2^-</td><td>2</td><td>2^+</td></tr>
<tr><td>$\frac{dy}{dx}$</td><td>>0</td><td>0</td><td>>0</td></tr>
</table>
By 1st derivative test, $x = 2$ is a point of inflexion.
<table>
<tr><td></td><td>$\frac{1}{2}^-$</td><td>$\frac{1}{2}$</td><td>$\frac{1}{2}^+$</td></tr>
<tr><td>$\frac{dy}{dx}$</td><td><0</td><td>0</td><td>>0</td></tr>
</table>
By 1st derivative test, the curve has a minimum point at $x = \frac{1}{2}$.
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<td>
$x = ½$
1st derivative test
1st derivative test
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