RAFFLES INSTITUTION

2014 YEAR 6 PRELIMINARY EXAMINATION

MATHEMATICS PAPER 2 **9740/2**

Higher 2 **23 September 2014**

Total Marks: 100 **3 hours**

Additional materials: Answer Paper
Graph Paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your name and CT group on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages.

[Turn over

Section A : Pure Mathematics [40 marks]

1 **(i)** Differentiate $\dfrac{1}{\sqrt{1-4x^2}}$ with respect to x. **[2]**

(ii) Find $\displaystyle\int \frac{x\sin^{-1}(2x)}{\sqrt{(1-4x^2)^3}}\,\mathrm{d}x$. **[3]**

2 It is given that a is a constant, where $a>1$. Find, in terms of a, the solution to each of the following inequalities.

(i) $\dfrac{2a}{x^2} > \dfrac{1+6a^2-3ax}{x}$, **[4]**

(ii) $\dfrac{2a}{x} > \dfrac{1+6a^2-3a\sqrt{x}}{\sqrt{x}}$. **[2]**

3 The function f is defined by

$$\mathrm{f}: x \mapsto x^2-4x+3 \quad \text{for} \quad x \in \mathbb{R},\ x \le 3.$$

(i) Sketch the graph of $y=\mathrm{f}(x)$. **[1]**

(ii) If the domain of f is further restricted to $x \le k$, state with a reason the greatest value of k for which the function f^{-1} exists. **[2]**

In the rest of this question, the domain of f is $x \in \mathbb{R},\ x \le 3$, as originally defined.

The function g is defined by

$$\mathrm{g}: x \mapsto \ln(2x+a) \quad \text{for} \quad x \in \mathbb{R},\ x > -\frac{a}{2},\ a \in \mathbb{R}.$$

(iii) Find the smallest possible value of a for which the function gf exists. **[2]**

(iv) With this value of a, find gf. **[2]**

(v) Find the range of gf. **[2]**

4 **Do not use a calculator in answering this question.**

The complex number z satisfies both the relations $\left|z+2\sqrt{3}-\mathrm{i}\right| \leq 4$ and $\frac{5}{6}\pi \leq \arg\left(z+\mathrm{i}\right) \leq \pi$.

(i) On an Argand diagram, shade the region in which the point representing z can lie. **[4]**

(ii) Find the least possible value of $|z|$. **[2]**

(iii) State the cartesian form of the complex number z when $|z+\mathrm{i}|$ is greatest. **[1]**

(iv) Find the range of values of $\arg\left(z+4\sqrt{3}+\mathrm{i}\right)$. **[2]**

5 **(a)** A geometric progression has first term 9, and the tenth term is three times the fourth term. Find the 22nd term in the form $c\sqrt{3}$, where c is a non-zero constant to be determined. **[3]**

If all the terms of the geometric progression are positive and the k^{th} term is more than 10^{40}, find the least value of k. **[2]**

(b) Runners *A* and *B* compete in a race by running laps around the school.

Runner *A* runs the first lap in 5 minutes 15 seconds but finds that his speed drops steadily so that each lap takes him *d* minutes more than the preceding one.

Runner *B* runs the first lap in 6 minutes and increases his speed steadily so that each successive lap takes him 0.99 of the time taken for the preceding one.

(i) Runner A takes 6 minutes 39 seconds to run the 8^{th} lap. Find the value of *d*. **[2]**

(ii) Find expressions for the time taken for each runner to cover the first *n* laps. **[2]**

(iii) What is the least number of laps they complete if one of the runners took at least 38 minutes more than the other? **[2]**

Section B : Statistics [60 marks]

6 A company engages an opinion polling organization to undertake an in-depth survey of the views of its staff towards the proposed changes in its staff welfare benefits. The company has 50 "management", 150 "professional" and 300 "administrative" staff. The budget for the survey allows for a sample of 50 staff to be selected for the in-depth survey.

(i) Explain how systematic sampling could be carried out to choose the 50 staff. Explain why this may not provide a representative sample of the staff. **[3]**

(ii) Name a more appropriate sampling method to provide a representative sample of the staff. **[1]**

[Turn over

7 A random sample of 50 Year 6 female students is taken and their 2.4 km run times are tabulated as follows:

2.4 km run time (min)	10.0	11.0	12.5	13.0	13.5	14.0	14.5	15.0	16.0	17.5
Number of students	2	3	6	8	8	8	7	4	2	2

(i) Calculate the unbiased estimates of the population mean and population variance of the 2.4 km run times, in minutes, of Year 6 female students. **[2]**

A test at the 5% significance level is carried out to determine whether the mean 2.4 km run times of Year 6 female students has improved from their Year 5 2.4 km mean run times of μ_0 minutes.

(ii) State the appropriate null and alternative hypotheses for the test. **[1]**

(iii) Find the set of values of μ_0 for which the null hypothesis would be rejected. **[3]**

8 To take a test, a group of 10 students are to be seated in two rows; the front and the back row, where each row has 5 seats. Find the number of possible distinct seating arrangements if

(i) there is no restriction, **[1]**

(ii) 2 particular students are not to be seated next to each other on the same row, **[3]**

(iii) 2 particular students are absent on that day and no student seated in the front row is taller than any student seated in the back row. (You may assume that no 2 students have the exact same height.) **[4]**

9 p% of the students in a school take Mathematics A and the remaining $(100-p)$% take Mathematics B. 30% of the students who had taken Mathematics A failed while 40% of the students who had taken Mathematics B failed.

Two students from the school are selected at random.

(i) Show that the probability that exactly one of the two students failed Mathematics is $0.0002(2400-2p-0.01p^2)$. **[5]**

(ii) The probability that both students take Mathematics A, given that exactly one of the two students failed Mathematics is $\frac{7}{48}$. Find p. **[3]**

10 Air bubbles occur in glass panels during its production. The number of large and small air bubbles in a glass panel are modelled as having independent Poisson distributions with means 0.2 and 1.8 respectively.

A glass panel is randomly chosen.

(i) Find the probability that the glass panel contains at most 2 air bubbles. **[2]**

(ii) Find the probability that the glass panel contains at least 1 large air bubble, given that the glass panel contains at most 2 air bubbles. **[3]**

12 glass panels are randomly chosen.

(iii) Find the probability that at least 7 glass panels contain at most 2 air bubbles. **[2]**

(iv) Using a suitable approximation, find the probability that the total number of air bubbles in the 12 glass panels is less than 30. **[3]**

11 A hair stylist tabulated the number of bottles of shampoo sold in his salon in each month. The discounts that was given, x %, and the number of bottles of shampoo sold, y, in each month are as follows.

x	5	10	15	20	25	30	40
y	25	35	55	84	118	151	300

(i) Draw a scatter diagram for these values, labelling the axes. **[1]**

(ii) Calculate the equation of the regression line of y on x. **[1]**
Hence calculate the corresponding estimated value of y when $x = 5$. **[1]**

(iii) Comment on the suitability of the linear model for this data set. **[2]**

It is suggested that x and y are related by the equation $y = a + bx^2$.

(iv) Calculate the product moment correlation coefficient between y and x^2 and comment on its value. **[2]**

(v) Estimate the values of a and b. **[2]**

The cost price of each bottle of shampoo is \$10, and the selling price before discount is \$20.

Using the suggested model above, estimate the discount that will maximize the profit for the hair stylist in a month, giving your answer to the nearest whole number. **[3]**

[Turn over

12 The masses, in grams (g), of chicken and duck eggs are modelled as having independent normal distributions with means and standard deviations as shown in the table below.

	Mean (g)	Standard deviation (g)
Chicken Eggs	50	2
Duck Eggs	65	3

Chicken eggs are packaged in cartons of 6 and duck eggs are packaged in cartons of 4. It is given that the mass of the cartons are negligible.

(i) Find the probability that the total mass of two randomly chosen cartons of chicken eggs exceeds twice the mass of a randomly chosen carton of duck eggs by at least 100 g. **[3]**

Chicken eggs that have a mass within 2 standard deviations of 50 g are classified as "good".

(ii) Using a suitable approximation, find the probability that out of ten randomly chosen cartons of chicken eggs, at least 55 chicken eggs in total are "good". **[5]**

A random sample of n cartons of duck eggs are chosen and their mean mass M in grams(g) per carton is recorded.

(iii) Given that $n = 4$, find $P(M > 265)$. **[2]**

(iv) Find the approximate value of $P(M > 265)$ as n becomes large. **[2]**

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