Solutions to WEP LN Examples

Example 1

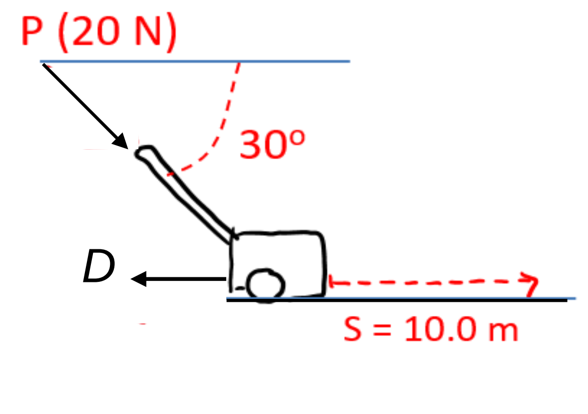
A lawn mower is pushed by a force *P* of 20.0 N at 30° to the horizontal, against a horizontal resistive force *D* of 8.0 N. It moves a horizontal distance of 10.0 m. Calculate

(i) the work done by the force *P* on the lawn mower.

(ii) the work done by the resistive force *D.*

(iii) the work done by the normal contact force from the ground and the weight of the lawn mower.

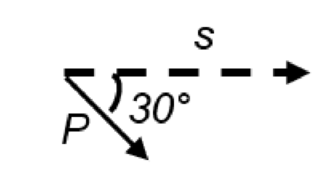
(iv) the change in kinetic energy of the mower.



(i) $W = Fs\cos\theta$

$= (20.0)(10.0\cos 30^{\circ})$

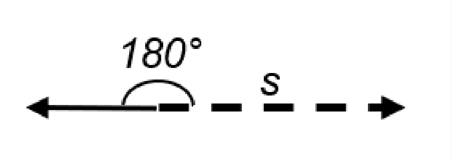
$= 173$ J



(ii) $W = Fs\cos\theta$

$= (8.0)(10.0\cos 180^{\circ})$

$= -80.0$ J

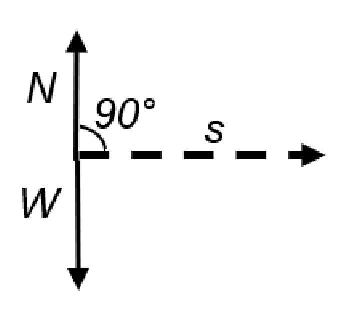


(iii) $W = Fs\cos\theta$

$= (N)(10.0\cos 90^{\circ})$

$= 0$ J

Since weight is also normal to displacement, Work done by weight = 0 J



(iv) *Net WD = Total work done by all forces*

$= 173 + (-80.0) + (0) + (0)$

$= 93$ J

Increase in KE = Net WD = 93 J

Example 2

A mass of weight 5.0 N is lifted by a vertical force of 7.0 N through a vertical distance of 2.0 m.

(a) Calculate the work done by the lifting force.

(b) Calculate the work done by the gravitational force.

(c) Calculate the change in gravitational potential energy.

(d) Calculate the total work done on the mass.

(e) Assuming it was initially at rest. calculate the final speed of the mass.

(a) Work done by vertical liftng force = $F\,s\cos\theta = (7.0)(2.0)(1) = +14$ J

(b) Work done by gravitational force = $mg\,s\cos\theta = (5.0)(2.0)(-1) = -10$ J

(c) ΔG.P.E. = $mg\Delta h = 5.0 \times 2.0 = +10$ J (Gain in GPE)

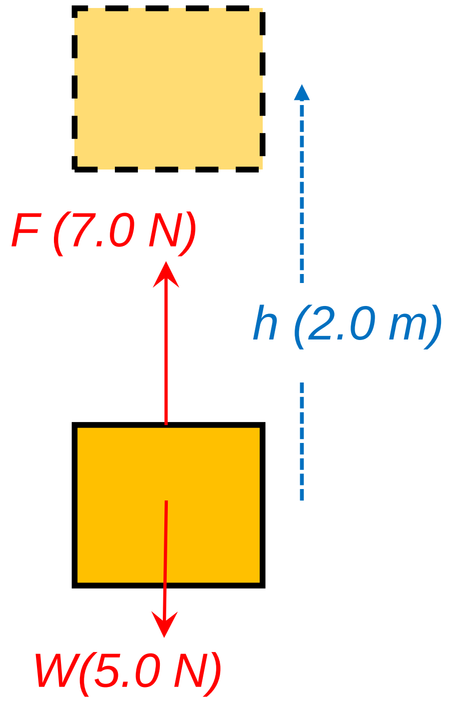
(d) Total work done on mass = Sum of work done by lifting force & weight
= 14 + (-10) = 4 J

(e) KE gained by mass = total work done on mass = 4 J

$$\frac{1}{2}m(v^2-0^2) = 4$$

$$\frac{1}{2}\left(\frac{5.0}{9.81}\right)v^2 = 4$$

$$v = 4 \text{ m s}^{-1}$$



Example 3 (J99/I/23 mod)

A sample is placed in a tensile testing machine. It is extended by known amounts and the tension is measured. (i) What is the work done in stretching the sample to the first point where it no longer obeys Hooke's Law? (ii) What is the work done on the sample when it is given a total extension of 9.0 mm?

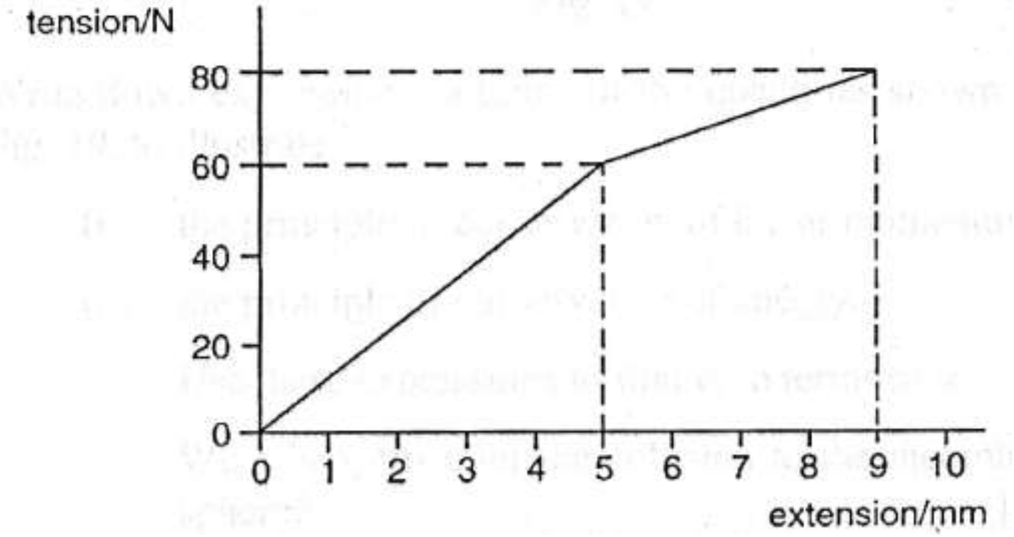
W = Area under the force-displacement graph

(i) $W = \frac{1}{2}Fx = \frac{1}{2}(60)(0.005) = 0.15\text{ J}$

(ii) $W = 0.15 + \left(\frac{60+80}{2}\right)(0.009 - 0.005)$

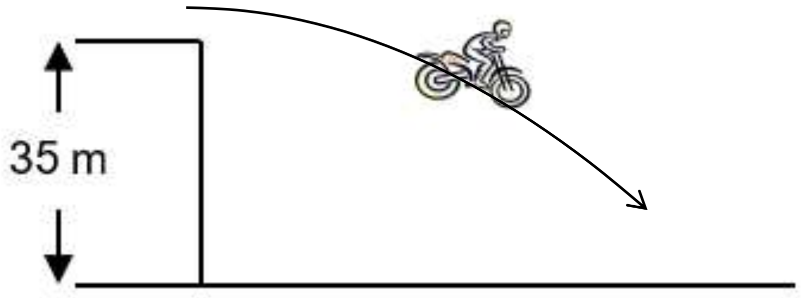
$= 0.15 + 0.28$

$= 0.43\text{ J}$



Example 4

(a) A motorcyclist drives horizontally off a cliff at a speed of 38.0 m s^{-1}. Ignoring air resistance, find the speed of the motorcycle just before it reaches the ground.



Applying Principle of Conservation of Energy on the motorcyclist,

$$KE_i + GPE_i = KE_f + GPE_f$$

$$GPE_i – GPE_f = KE_f - KE_i \qquad \text{(Loss in GPE = Gain in KE)}$$

$$mg\Delta h = \tfrac{1}{2} m(v^2 - u^2)$$

$$v = \sqrt{2g\Delta h + u^2}$$

$$v = \sqrt{2(9.81)(35) + 38.0^2}$$

$$= 46.2 \text{ m s}^{-1}$$

Example 4

b) Would the answer change if the motorcyclist leaps across at the same 35 m height with an initial upward angle while maintaining the same initial speed of 38.0 m s^{-1}?

As air resistance is negligible, as long as the motorcycle has fallen by a vertical height of 35 m (regardless of the actual path), the loss in gravitational PE and therefore the gain in KE remains the same.

Example 5

A car of mass 800 kg moving at 30 km h^{-1} along a horizontal road is brought to rest by a constant retarding force of 5000 N. Calculate the distance the car moves before coming to rest.

By Principle of Conservation of Energy,

KE_i + Net WD by ext forces = KE_f

Net WD by ext forces = $KE_f - KE_i$ (This is known as Work-Energy theorem)

$$-fs = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$(-5000)s = \frac{1}{2}(800)(0)^2 - \frac{1}{2}(800)(8.33)^2$$

$$s = 5.6 \text{ m}$$

Example 6

A block of mass 4.0 kg is released at point P on a curved frictionless track as shown. The block slides down a vertical distance of 80 cm and enters a sand pit. Determine the distance the block can move along the sand pit before coming to rest if the friction acting on the block is 85 N.

Initial state: Block of mass at top of slope
Final state: Block of mass has come to rest at sand pit

By Conservation of Energy

$KE_i + GPE_i + \text{WD by ext forces} = KE_f + GPE_f$

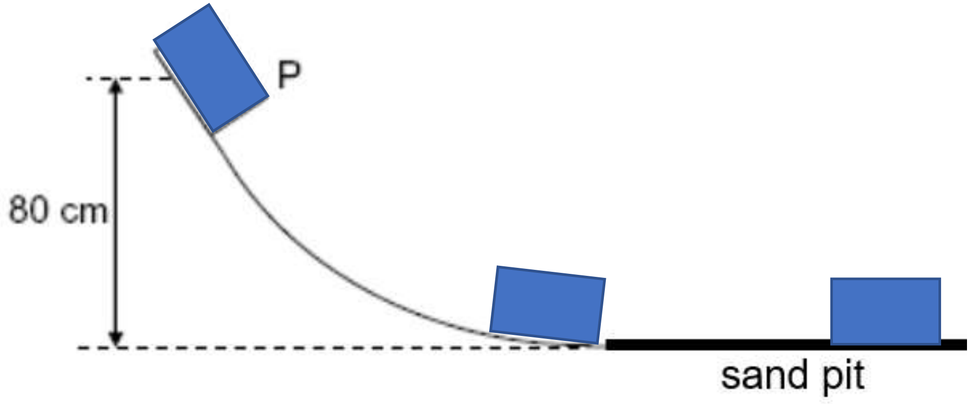
$GPE_i - GPE_f = -\text{WD by ext forces} \quad (KE_i = KE_f = 0)$

$mg(\Delta h) = -(-f\,s)$

where f is the frictional force &
s is the displacement along sand pit

$(4.0)(9.81)(0.80) = (85)\,s$

$s = 0.37$ m



See the elegance of using the energy approach?

We do not need to consider what happens during the process. We just need to consider the initial and final energy states to solve the problem.

Example 6

A block of mass 4.0 kg is released at point P on a curved frictionless track as shown. The block slides down a vertical distance of 80 cm and enters a sand pit. Determine the distance the block can move along the sand pit before coming to rest if the friction acting on the block is 85 N.

Initial state: Block of mass at top of slope
Final state: Block of mass has come to rest at sand pit

By Conservation of Energy
$KE_i + GPE_i$ + WD by ext forces = $KE_f + GPE_f$
$GPE_i - GPE_f$ = -WD by ext forces   ($KE_i = KE_f = 0$)
$mg(\Delta h) = -(-f\,s)$

where f is the frictional force &
s is the displacement along sand pit

$(4.0)(9.81)(0.80) = (85)\,s$
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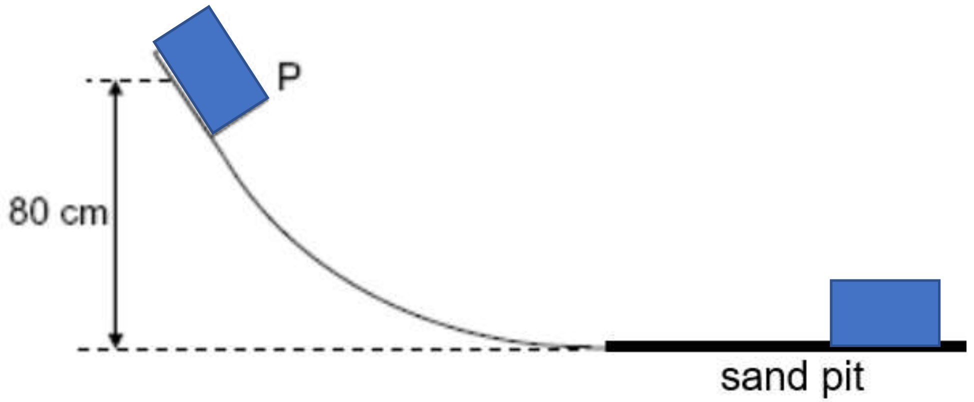


Alternative Approach
WD **against** friction = Increase in thermal energy

By Principle of Conservation of Energy,

Loss in GPE = Gain in thermal energy = WD against friction
$mg(\Delta h) = f\,s$

$(4.0)(9.81)(0.80) = (85)\,s$
$s = 0.37$ m

Note the subtle difference between Work Done **by** friction and Work Done **against** friction.

Example 7

A 1.10 x 10^3 kg car, starting from rest, accelerates for 5.00 s. The magnitude of the acceleration is 4.60 m s^{-2}. Determine the average power generated by the net force that accelerates the vehicle.

$$\langle P \rangle = \frac{W}{t} = \frac{Fs}{t} = \frac{(ma)s}{t}$$

$$= \frac{(ma)\left(ut + \frac{1}{2}at^2\right)}{t}, \qquad \text{where } u = 0$$

$$= \frac{1}{2}ma^2t = \frac{1}{2}(1.10 \times 10^3)(4.60)^2(5.00)$$
$$= 5.82 \times 10^4 \text{ W}$$

Example 8

A 1000 kg car experiences a total resistive force of 2000 N moving with a constant velocity of 20.0 m s^{-1} along a straight horizontal road.

What is the power developed by the engine of the car when it travels at 20.0 m s^{-1}

(a) horizontally, and

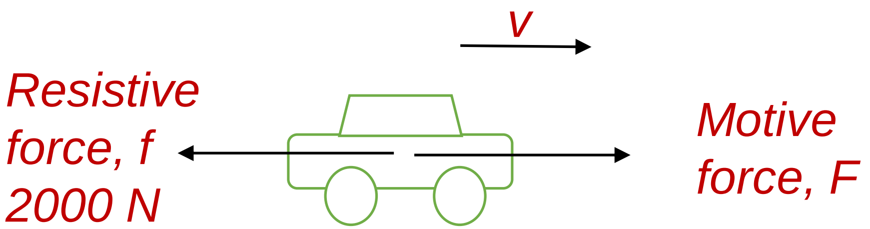
(b) up a slope with gradient of 5°?

(a) Horizontally, velocity is constant => net force is zero.

$$F - f = ma = 0$$

Thus motive force F = resistive force f = 2000 N

$$P = Fv = (2000)(20.0) = 40\,000 \text{ W}$$



(b) Along the slope, net force is zero

$$F' - f - mg\sin\theta = ma = 0$$

$$F' = f + mg\sin\theta$$

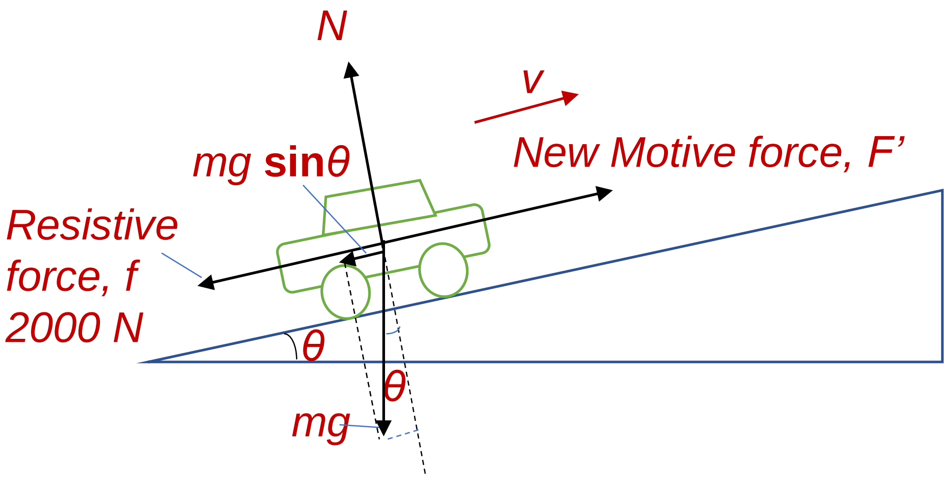
$$F' = (2000) + (1000)(9.81)\sin(5^\circ)$$

$$F' = 2855 \text{ N}$$

$P = F'v$

$= (2855)(20.0)$

$= 57100$ W



Example 9

A small motor is used to raise a weight of 2.0 N at constant speed through a vertical height of 80 cm in 4.0 s. The efficiency of the motor is 20 %. What is the electrical power supplied to the motor?

As the weight is moving at constant speed,
net force on weight = 0.

Motive force provided by the motor = Weight = 2 N

Power output from motor = Fv = $F\left(\frac{s}{t}\right) = 2\left(\frac{0.80}{4.0}\right)$ =0.40 W

Efficiency of motor = $\frac{Power\ Output}{Power\ Input} \times 100\%$

$0.20 = \frac{0.40}{Power\ Input}$

Power Input to motor = 2.0 W