Year 3 Express Additional Mathematics Paper 1 Solutions

2017 Final Examinations

1a. Given $f(x) = x^3 + ax^2 + 2x + 2$

$\therefore f(-1) = -1 + a - 2 + 2$

$-1 = -1 + a$

$\therefore a = 0$

1b. $x^3 + 2x + 2 = (x+1)(x^2 - x + 3) - 1$

$\therefore$ Quotient is $(x^2 - x + 3)$

2 Given $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$$\Rightarrow \frac{1}{R_2} = \frac{1}{\sqrt{3}-1} - \frac{3}{\sqrt{3}+1}$$

$$= \frac{\sqrt{3}+1-3\sqrt{3}+3}{3-1}$$

$$= \frac{4-2\sqrt{3}}{2}$$

$$= 2-\sqrt{3}$$

$$\therefore R_2 = \frac{1}{2-\sqrt{3}}$$

$$= \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$\therefore R_2 = 2+\sqrt{3}$$

3 **(i)** $\frac{3}{x} = 2x^{\frac{1}{4}}$

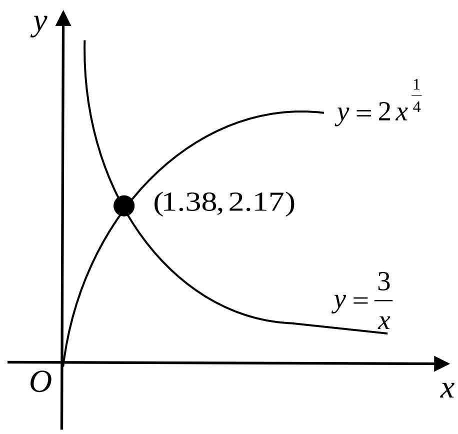
$x^{\frac{5}{4}} = \frac{3}{2}$

$\therefore x = \left(\frac{3}{2}\right)^{\frac{4}{5}} = 1.383...$

and $y = \frac{3}{1.383} = 2.17...$

Point of intersection: $(1.38, 2.17)$

(ii)



4i. Using $Y = mX + c$

$Y = 3X + c$

Passing through $(1,5)$:

$$5 = (3)(1) + c$$

$$\therefore c = 2$$

$$Y = 3X + 2$$

Given $x - q = \frac{px}{y}$

$$xy - qy = px$$

$$y - q\frac{y}{x} = p$$

$$\therefore \quad y = q\frac{y}{x} + p$$

$\Rightarrow$ Gradient: $m = 3 \qquad \therefore q = 3$

$\Rightarrow$ y-intercept: $c = 2 \quad \therefore p = 2$

4ii. From part (i): $y = 3\left(\frac{y}{x}\right) + 2$

or $Y = 3X + 2$

When $2x - y = 0 \quad \Rightarrow \quad \frac{y}{x} = 2$

Using $y = 3(2) + 2$

$$\therefore y = 8$$

5 **(a)** $(6-x)(1+x)\geq -8$

$6+6x-x-x^2+8\geq 0$

$-x^2+5x+14\geq 0$

$x^2-5x-14\leq 0$

$(x-7)(x+2)\leq 0$

$\therefore -2\leq x\leq 7$

(b) $mx^2-8=4x-5m$

$mx^2-4x+(5m-8)=0$

For line to meet curve:

$(-4)^2-4m(5m-8)\geq 0$

$16-20m^2+32m\geq 0$

$-20m^2+32m+16\geq 0$

$-5m^2+8m+4\geq 0$

$5m^2-8m-4\leq 0$

$(5m+2)(m-2)\leq 0$

$\therefore -\frac{2}{5}\leq m\leq 2$

6 **(a)** $e^{x} = 8 - 16e^{-x}$

$e^{2x} - 8e^{x} + 16 = 0$

$(e^{x} - 4)^{2} = 0$

$e^{x} = 4 \Rightarrow \quad x = \ln 4$

$\therefore x = 1.39$ (to 3 sig fig)

(b) <u>**Working with Base 3**</u>

$$2\log_9(2x^2) - \log_3(4-x) = \frac{1}{\log_9 3}$$

$$\log_3(2x^2) - \log_3(4-x) = 2$$

$$\log_3\left(\frac{2x^2}{4-x}\right) = 2$$

$$\therefore \frac{2x^2}{4-x} = 3^2 \Rightarrow \quad 2x^2 + 9x - 36 = 0$$

$$x = \frac{-9 \pm \sqrt{9^2 - 4(2)(-36)}}{2(2)}$$

$$\therefore x = 2.55 \text{ or } -7.05$$

<u>Alternatively:</u>

<u>**Working with Base 9**</u>

$$2\log_9(2x^2) - \log_9(4-x)^2 = 2$$

$$\left.\begin{aligned} &2\log_9(2x^2) - 2\log_9(4-x) = 2 \\ &\log_9(2x^2) - \log_9(4-x) = 1 \end{aligned}\right\} *$$

$$\log_9\left(\frac{2x^2}{4-x}\right) = 1 \qquad \therefore \frac{2x^2}{4-x} = 9$$

$$\Rightarrow \quad 2x^2 + 9x - 36 = 0$$

$$x = \frac{-9 \pm \sqrt{9^2 - 4(2)(-36)}}{2(2)}$$

$$\therefore x = 2.55 \text{ or } -7.05$$

7 **(i)** Given equation of C_1 : $x^2+y^2-2x-6y+6=0$

$a=\frac{-2}{-2}=1$; $b=\frac{-6}{-2}=3$ $\therefore$ centre of $C_1:(1,3)$

$r=\sqrt{1^2+3^2-6}=2$ $\therefore$ radius of $C_1=2$ units

Alternatively,

$x^2-2x+1+y^2-6y+3^2=-6+1+9$

$(x-1)^2+(y-3)^2=2^2$

$\therefore$ centre of $C_1:(1,3)$

$\therefore$ radius of $C_1=2$ units

(ii) Given $A(4,11)$ and $B(8,7)$ on C_2:

$m_{AB}=\frac{11-7}{4-8}=-1$ $\therefore m_{\perp}=1$

Midpoint of $AB=\left(\frac{4+8}{2},\frac{11+7}{2}\right)=(6,9)$

$\therefore$ Equation of perpendicular bisector of AB:

$y-9=1(x-6)$

$\Rightarrow y=x+3$ --------(1)

Centre of C_2 also lies on: $x=4$ --------(2)

Sub (2) into (1): $y=4+3=7$

$\therefore$ Centre of C_2 is $(4,7)$

$\therefore$ Radius of $C_2=\sqrt{(4-4)^2+(11-7)^2}=4$ units

$\therefore$ Equation of C_2: $(x-4)^2+(y-7)^2=4^2$

Alternatively:

Let centre of $C_2=(4,b)$

Using $AO=BO$:

$AO=BO$

$\sqrt{(4-4)^2+(11-b)^2}=\sqrt{(8-4)^2+(7-b)^2}$

Taking sq on both sides: $(11-b)^2=16+(7-b)^2$

Diff of 2 squares: $(11-b)^2-(7-b)^2=16$

$(11-b+7-b)(11-b-7+b)=16$

$(18-2b)(4)=16$ $\therefore b=7$

$\therefore$ Centre of $C_2=(4,7)$

$\therefore$ Radius of $C_2=\sqrt{(4-4)^2+(11-7)^2}=4$ units

$\therefore$ Equation of C_2: $(x-4)^2+(y-7)^2=4^2$

(iii) Distance between the centres of C_1 and C_2

$=\sqrt{(4-1)^2+(7-3)^2}=5$ units

Since $r_1+r_2=2+4>5$

$\therefore C_1$ and C_2 intersects.

8 Given the function $f(x) = 3\sin 2x - 2$,

(i) Period = 180°
Amplitude = 3

(ii) For $0° \le x \le 180° \quad \Rightarrow \quad 0° \le 2x \le 360°$

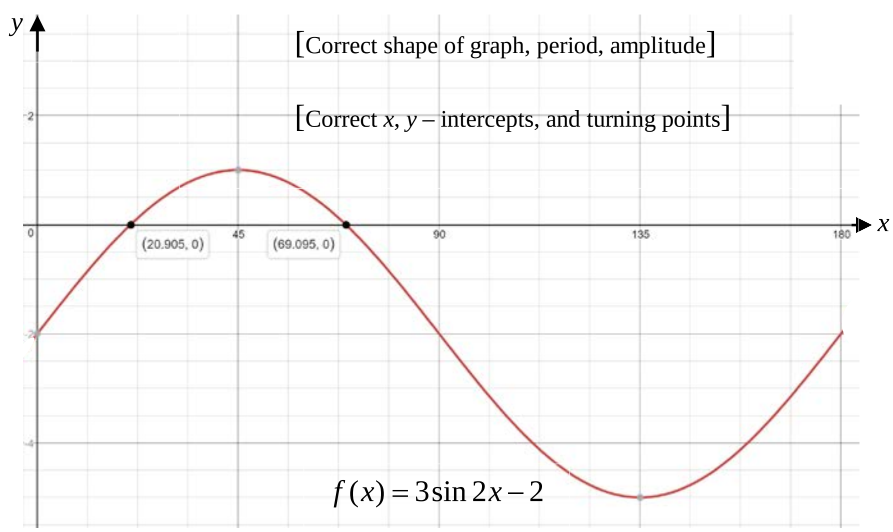
$3\sin 2x - 2 = 0 \;\Rightarrow \sin 2x = \dfrac{2}{3}$

$\alpha = \sin^{-1}\dfrac{2}{3} = 41.81^\circ$

$\therefore 2x = 41.81^\circ, \quad 138.19^\circ$

$\therefore x = 20.9^\circ, \quad 69.1^\circ$

(iii) sketch the graph of $f(x) = 3\sin 2x - 2$ for $0° \le x \le 180°$, showing the x – and y – intercepts clearly.



9 **(ai)** Given $f(x) = x^2(2x-9)+2 \Rightarrow f(x) = 2x^3 - 9x^2 + 2$.

let $x = \frac{1}{2}$: $f(\frac{1}{2}) = 2\left(\frac{1}{2}\right)^3 - 9\left(\frac{1}{2}\right)^2 + 2 = 0$* $\therefore (2x-1)$ is a factor

(aii) By synthetic division: $f(x) = \left(x - \frac{1}{2}\right)\left(2x^2 - 8x - 4\right)$

OR $= (2x-1)\left(x^2 - 4x - 2\right)$

$f(x) = 0 \Rightarrow x - \frac{1}{2} = 0$ or $2x^2 - 8x - 4 = 0$

$$\therefore x = \frac{1}{2} \quad \text{or} \quad x = \frac{8 \pm \sqrt{8^2 - 4(2)(-4)}}{2(2)} = 2 \pm \sqrt{6}$$

$$\therefore x = \frac{1}{2} \quad \text{or} \quad 4.45 \quad \text{or} \quad -0.449$$

(aiii) $2\sin^2\theta(\sin\theta - 4) + \cos^2\theta = -1$

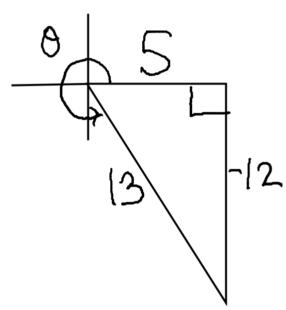
$2\sin^2\theta(\sin\theta - 4) + 1 - \sin^2\theta = -1$

$2\sin^3\theta - 8\sin^2\theta + 1 - \sin^2\theta + 1 = 0$

$\Rightarrow 2\sin^3\theta - 9\sin^2\theta + 2 = 0$

$\Rightarrow \sin\theta = \frac{1}{2}$,	$\sin\theta = 2 - \sqrt{6}$,	$\sin\theta = 2 + \sqrt{6}$ (NA)
$\alpha = \sin^{-1}\frac{1}{2} = 30^\circ$	$\alpha = \sin^{-1} 0.449$	
$\therefore \theta = 30^\circ, 150^\circ$	$= 26.67^\circ$	
	$\therefore \theta = 206.7^\circ, 333.3^\circ$	

(b) $\sec\theta = \frac{13}{5} \Rightarrow \cos\theta = \frac{5}{13}$



(i) $\sin\theta = -\frac{12}{13}$

(ii) $\tan\left(180^\circ - \theta\right) = -\tan\theta = \frac{12}{5}$