SERANGOON JUNIOR COLLEGE

2015 JC2 PRELIMINARY EXAMINATION

MATHEMATICS

Higher 2 **9740/2**

Tuesday **25 Aug 2015**

Additional materials: Writing paper

List of Formulae (MF15)

TIME : 3 hours

READ THESE INSTRUCTIONS FIRST

Write your name and class on the cover page and on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets **[ ]** at the end of each question or part question.
At the end of the examination, fasten all your work securely together.

Total marks for this paper is 100 marks.

This question paper consists of 6 printed pages (inclusive of this page) and 2 blank pages.

Section A: Pure Mathematics [40 marks]

1 Functions f and g are defined as below.

$$\mathrm{f}: x \mapsto \frac{1}{x-1}, \qquad x \geq 2$$
$$\mathrm{g}: x \mapsto \ln(2-x) - x, \quad x < -2$$

(i) Sketch the graph of $y = \mathrm{g}(x)$ and state its exact range. [3]

(ii) Determine whether the composite functions fg and gf exist, justifying your answer. Find the range of the composite function if it exists. [4]

(iii) On the same diagram as part **(i)**, sketch the graph of $y = \mathrm{g}^{-1}(x)$, indicating on your sketch the line in which the graph of $y = \mathrm{g}(x)$ must be reflected in order to obtain the graph of $y = \mathrm{g}^{-1}(x)$. [2]

2 A tank contains water which is heated by an electric water heater working under the action of a thermostat. When the water heater is first switched on, the temperature of the water is $35^\circ\mathrm{C}$. The heater causes the temperature to increase at a rate $r^\circ\mathrm{C}$ per minute, where r is a constant, until the water temperature hits $75^\circ\mathrm{C}$. The heater then switches off.

(i) Write down, in terms of r, the time taken for the temperature to increase from $35^\circ\mathrm{C}$ to $75^\circ\mathrm{C}$. [1]

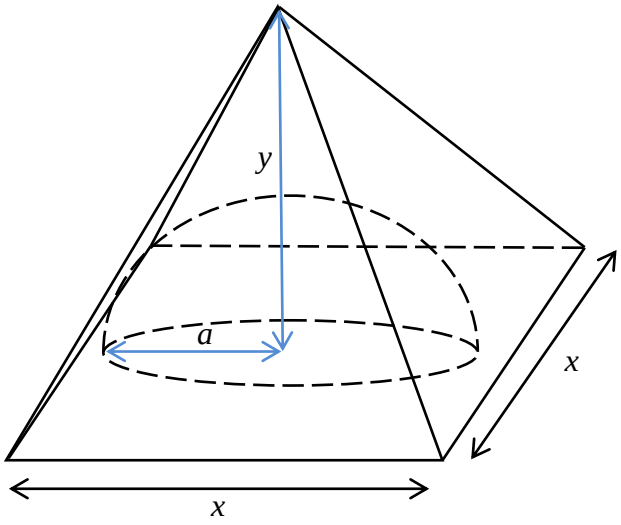
The temperature of the water then immediately starts to decrease. The temperature of the water at time t minutes after the heater is switched off is $\theta^\circ\mathrm{C}$. It is known that the temperature of the water decreases at a variable rate $k(\theta - 25)^\circ\mathrm{C}$ per minute, where k is a positive constant, until $\theta = 35$.

(ii) Write down a differential equation involving θ and t, to represent the situation as the temperature is decreasing. [1]

(iii) Given that when $\theta = 55$, the temperature is decreasing at a rate of $5^\circ\mathrm{C}$ per minute, find the total length of time for the temperature to increase from $35^\circ\mathrm{C}$ to $75^\circ\mathrm{C}$ and then decrease to $35^\circ\mathrm{C}$, leaving your answer in exact form, in terms of r. [7]

3 [It is given that the volume of a sphere of radius r is $\frac{4}{3}\pi r^3$ and that the volume of a right square pyramid with a square base of length x and height h is $\frac{1}{3}x^2h$.]

In the diagram below, a hemisphere of fixed radius a cm lies on the base of a right pyramid such that its curved surface is in contact with all four faces of the pyramid.



The pyramid has a square base of length x cm and height y cm. Given that the volume of region inside the pyramid that is not part of the hemisphere is denoted by V,

(i) show that $V = \frac{1}{3}\frac{ax^3}{\sqrt{x^2-4a^2}} - \frac{2}{3}\pi a^3$. [3]

(ii) use differentiation to find, in terms of a, the minimum value of V exactly, proving that it is a minimum. [7]

4 **(a)** The complex number z is given by $z = re^{i\theta}$, where $r > 0$ and $0 \leq \theta \leq \frac{1}{2}\pi$.

(i) Given that $w = (\sqrt{3} - i)z$, find $|w|$ in terms of r and arg w in terms of θ. [2]

(ii) For a fixed value of r, draw on the same Argand diagram the the locus of z and w as θ varies. [2]

(iii) If $r = 1.5$ units, calculate (in terms of π) the length of the locus of w for Im(w) ≥ 0 as θ varies. [2]

(b) Sketch on a single Argand diagram the set of points representing all complex numbers v satisfying both of the following inequalities:

$$|v - 5 - 8i| \leq 5 \quad \text{and} \quad |v - 12 - 8i| \geq |v - 12 - 10i|.$$

Hence find (in radians) the least value of arg $(v - 5 + 3i)$. [6]

Section B: Statistics [60 marks]

5 A company with eight hundred employees wishes to find out how much time its employees take to travel to work. It is given that the employees go to work either by car or by bus/train and that each of them takes the same form of transport to work every day.

The following table shows the number of employees going to work by car and the number of employees going to work by bus/train.

	Car	Bus/Train
Men	165	260
Women	82	293

The company wants to take a random sample of 180 employees.

(i) Explain what is meant, in this context, by the term "a random sample". [1]

(ii) Describe how a random stratified sample can be obtained. [2]

(iii) Give a reason why quota sampling is not as suitable in this context compared to stratified sampling. [1]

6 Salt is packed in bags to be sold. The manufacturer claims each bag contains at least 300 g of salt. To test this claim, a random sample of 15 bags of salt is examined and the mass, x g, of the contents of each bag is determined. It is found that the sample has a mean of 299.1 g and variance of 3.864 g^2.

(i) Test at the 10% significance level whether the manufacturer's claim is valid. [5]

(ii) State an assumption necessary for the test in **(i)** to be valid. [1]

7 A bag contains w white balls and b black balls. One ball is selected at random from the bag, its colour noted and it is then returned to the bag along with n additional balls of the same colour. A second ball is then randomly selected from the bag.

(i) Construct a probability tree showing this information. [2]

(ii) Show that the probability that the second ball selected is black is independent of n. [2]

It is now known that the second ball drawn is black. Show that the probability that the first ball drawn is white is $\dfrac{w}{w+b+n}$. [2]

8 The table below shows Singapore's GDP per capita over the years from 1965.

Year, x	GDP per capita (in thousands), y
1965	1.580
1970	2.832
1975	6.607
1980	10.714
1985	14.921
1990	23.139
1995	35.346
2000	41.018
2005	49.715
2010	63.498

Source: Department of Statistics Singapore

(a) Using the data available,

(i) draw the scatter diagram, labelling the axes clearly, [2]

(ii) find the least square regression line of y on x, [1]

(iii) estimate the GDP per capita in the year 2015, correct to the nearest whole number. Comment on the reliability of the value obtained. [2]

(b) It is suggested that the data from 1965 to 2010 can be modelled by $y = a + b(x - 1965)^2$ instead of a linear model. Find the value of the product moment correlation coefficient for each of the proposed models and determine which is the better model. [2]

9 In a factory manufacturing calculators, it is found that 1.5% of the calculators produced are defective.

In a random sample of 90 calculators, find the probability that

(i) there are exactly 2 defective calculators, [1]

(ii) the 90^{th} calculator is the second defective calculator given that there are exactly 2 defective calculators in that sample. [3]

The calculators are packed in boxes of 90.

(iii) Using a suitable approximation, find the probability that not more than 1 box of calculators, out of 60 boxes, contain more than 2 defective calculators. [5]

10 Seven men and seven women, including Sally and Andy, participated in a speed-dating session at a community centre.

All participants are to sit in a way such that no two persons of the same gender sit next to each other.

How many ways can the participants be arranged if

(i) they are seated at a round table of 14 seats, [2]

(ii) they are seated at 2 similar round tables of 6 seats each without Sally and Andy, [3]

(iii) they are seated on both sides of a rectangular table with 7 seats on each side, such that Sally and Andy sit next to each other on the same side? [3]

11 **(a)** A farm in the west of Singapore grows turnips for sale to the local market.

(i) Five turnips are randomly chosen. Find the probability that exactly one turnip weighs less than the lower quartile weight and exactly two turnips weigh more than the median weight. [2]

(ii) The mass of a randomly chosen turnip has mean 40 g and standard deviation of 3 g. If the probability that the mean mass of a large sample of n turnips is greater than 39.6 g exceeds 0.95, find the least value of n. [3]

(b) A random variable X has the distribution $X \sim \mathrm{N}(40, 3^2)$. The random variable Y is related to X by the formula $Y = aX - \frac{1}{b}$, where a and b are constants and $a > 0$. Given that $\mathrm{P}(Y < 85) = \mathrm{P}(Y > 155) = 0.075$, find the values of $\mathrm{E}(Y)$ and $\mathrm{Var}(Y)$, and hence find the values of a and b. [5]

12 **(a)** A teacher discovered that the probability that a randomly chosen student is late two days in a month is three times the probability that the student is late four days in a month. If the number of days a student is late in a month follows a Poisson distribution, find the non-zero variance of the Poisson distribution. [3]

(b) Records indicate that a certain hospital delivers an average of 3650 babies each year. Each day, there are 3 shifts of equal duration in the hospital. It is assumed that the number of deliveries in a day can be modelled by a Poisson distribution. Taking a year to consist of 365 days,

(i) show that the mean number of deliveries per shift is $\frac{10}{3}$ and find the most likely number of deliveries per shift, [3]

(ii) find the expected number of shifts with at least 5 deliveries in a week, [3]

(iii) explain why a Poisson distribution may not be a suitable model for the number of deliveries in a day for a hospital with a significant number of planned deliveries (e.g. cesarean section). [1]

End of Paper

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