Statistical Data Analysis

Cumulative Frequency Table

- Find the cumulative frequency by adding up the frequencies which are less than or equal to that value.

- Example question:

The lengths of 40 insects of a certain species were measured, to the nearest milimetre. The frequency distribution is given in the table below.

Length (x mm)	Frequency
$25 < x \leq 30$	1
$30 < x \leq 35$	3
$35 < x \leq 40$	6
$40 < x \leq 45$	12
$45 < x \leq 50$	10
$50 < x \leq 55$	6
$55 < x \leq 60$	2

Length (x mm)	Cumulative Frequency	
≤ 30	1	
≤ 35	4	(1+3)
≤ 40	10	(4+6)
≤ 45	22	(10+12)
≤ 50	32	(22+10)
≤ 55	38	(32+6)
≤ 60	40	(38+2)

Cumulative Frequency Curve

- How to plot a cumulative frequency curve:

1. Label the vertical axis "Cumulative Frequency"
2. Label the horizontal axis, i.e. "Number of..."
3. Plot the points on the graph paper
4. Join all the points with a smooth curve

Important!

- A reading of any number, x (person/animal/thing) indicates that x did something for less than/equal to y hours / no of people which produce less than/equal to y things etc

E.g. A reading of 14 indicates that 14 students surf the Internet for less than or equal to 5 hours.

- The answer can only be accurate up to half of a small square grid.

Discrete Data

- Median : volve of data in the middle position

Example:

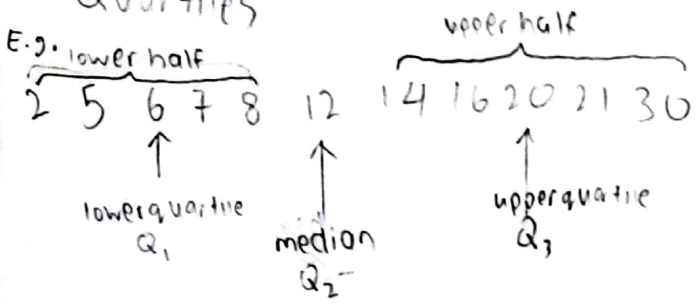
2 5 6 7 8 12 14 16 20 21 30

↑

median

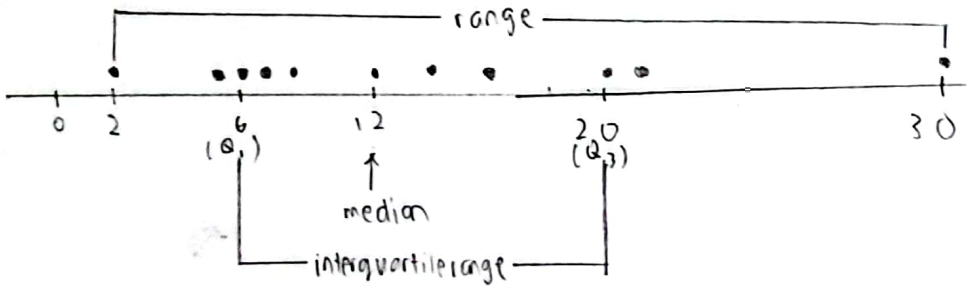
- Quartiles

E.g.



- Range : Lagest value − Smallest value
- Interquartile Range : $Q_3 - Q_1$

E.g.



- Continuous Data

- Data which can take on any value within a range of numbers

Example : Data set showing the height (cm) of 30 girls in a class can take on values such as 150.4, 169.34, 150, etc

- We can estimate the quartiles from the cumulative frequency curve

Addition Law of Probability and Mutually Exclusive Events

- If two events A and B cannot occur at the same time, they are mutually exclusive
- If two events A and C can occur at the same time, they are non-mutually events

- Addition Law of Probability:

Mutually Exclusive:

$P(A \text{ or } B)$ or $P(A \cup B) = P(A) + P(B)$

Non-mutually Exclusive:

$(A \text{ or } C)$ or $P(A \cup C) \neq P(A) + P(C)$

Multiplication Law of Probability and Independent Events

- Example Question:

There are 7 green marbles and 3 yellow marbles in a bag. The marbles are identical except for their colour. A marble is drawn at random from the bag and is replaced. A second marble is then drawn at random from the bag. Find the probability that

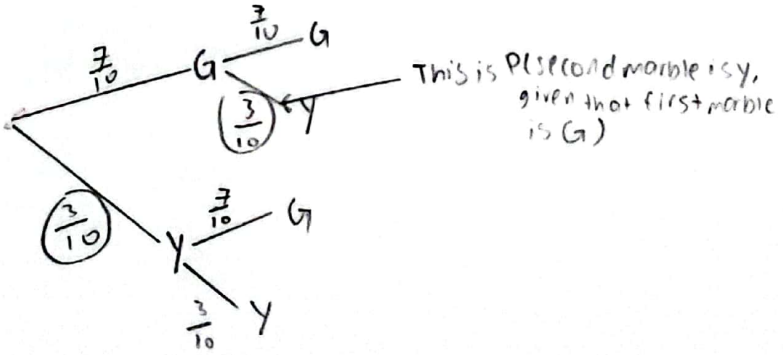
(i) the first marble drawn is yellow

(ii) the second marble drawn is yellow given that the first marble drawn is given

(iii) the first marble drawn is green and the second marble drawn is yellow

(iv) the second marble drawn is yellow

Tree Diagram:



i) P(first marble is Y) $= \frac{3}{10}$ [see ○ - $\frac{\text{Number of yellow marbles in bag for first draw}}{\text{Total number of marbles in bag for first draw}}$

ii) P(second marble is Y, given that first marble is G) $= \frac{3}{10}$

iii) P(first marble is G and second marble is Y) / P(GY) $= \frac{7}{10} \times \frac{3}{10}$

$= \frac{21}{100}$

iv) P(second marble is Y) = P(GY) + P(YY)

$= \left(\frac{7}{10} \times \frac{3}{10}\right) + \left(\frac{3}{10} \times \frac{3}{10}\right)$

$= \frac{21}{100} + \frac{9}{100}$

$= \frac{30}{100}$

$= \frac{3}{10}$

- Independent Events
- If the chance of one of two events occuring does not affect the chance of the other event occuring, both events are INDEPENDENT EVENTS
- If A and B are independent events, P(A and B) or $P(A \cap B) = P(A) \times P(B)$

Probability of Single Events

- $P(E) = \frac{\text{Number of favourable outcomes for event } E}{\text{Total number of possible outcomes}} = \frac{k}{m}$
- $0 \leqslant P(E) \leqslant 1$
- $P(E) = 0$ if and only if E is an impossible event, i.e. it will never occur
- $P(E) = 1$ if and only if E is a certain event, i.e. it will definitely occur
- $P(\text{not } E) = 1 - P(E)$
  $(E') \rightarrow$ E prime

- Sample Space & Events

Example 1: A card is chosen at random from a deck of 12 cards

$S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ All the numbers from 1 to 12 are all possible outcomes

$n(S) = 12$

Example 2: A card with a prime number is chosen

$A = \{2, 3, 5, 7, 11\}$

$n(A) = 5$

In general, in a sample space with a finite number of equally likely outcomes, the probability of an event occuring, i.e. $P(E)$ is

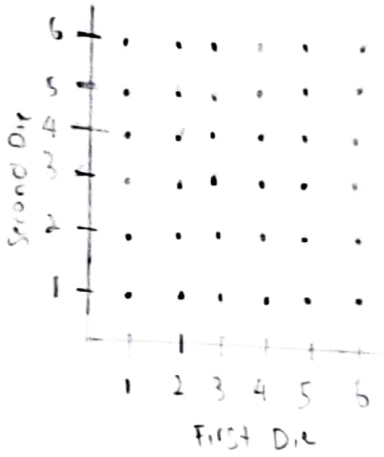
★ $P(E) = \frac{n(E)}{n(S)}$

Single Combined Events, Possibility Diagrams and Tree Diagrams

- Possibility Diagram

Example: Two fair dice are thrown.

1st Possibility Diagram



2nd Possibility Diagram

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

- Tree Diagram

Example : Three fair coins are tossed