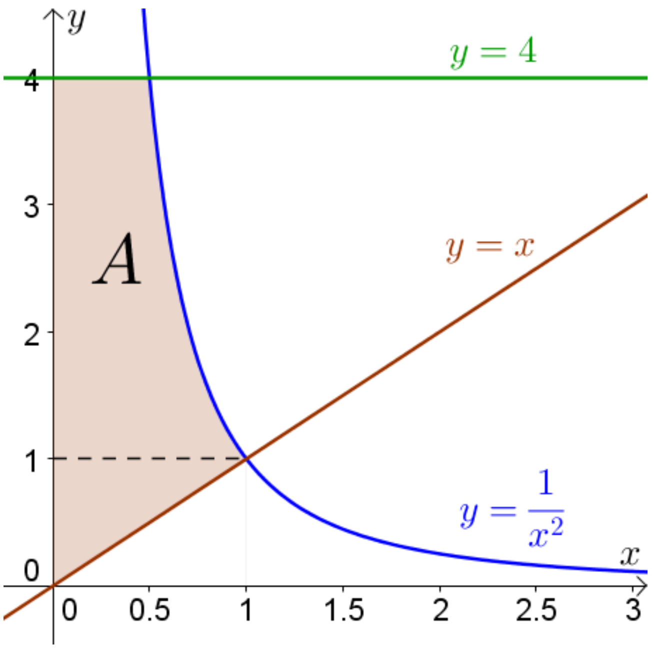
National Junior College
2016 – 2017 H2 Mathematics
Applications of Integration (Area and Volume) **Assignment 2 Solutions**

1(i)



$$y=\frac{1}{x^2} \Rightarrow x^2=\frac{1}{y} \Rightarrow x=\pm\frac{1}{\sqrt{y}}$$

$$\Rightarrow x=\frac{1}{\sqrt{y}} \quad (\because x\ge 0).$$

Integrating with respect to y, area of region A

$$=\int_1^4\left(\frac{1}{\sqrt{y}}\right)\mathrm{d}y+\frac{1}{2}(1)(1)$$

$$=\left[2\sqrt{y}\right]_{y=1}^{y=4}+\frac{1}{2}$$

$$=\frac{5}{2}.$$

[M1] Formed an area integral.

[A1] Correct expression for area of A.

[A1] $\frac{5}{2}$.

1(ii)

Volume generated

$$=\frac{1}{3}\pi(1)^2(1)+\pi\int_1^4\frac{1}{y}\,\mathrm{d}y$$

$$=\frac{\pi}{3}+\pi\left[\ln y\right]_1^4$$

$$=\frac{\pi}{3}+\pi\left[\ln 4-\ln 1\right]$$

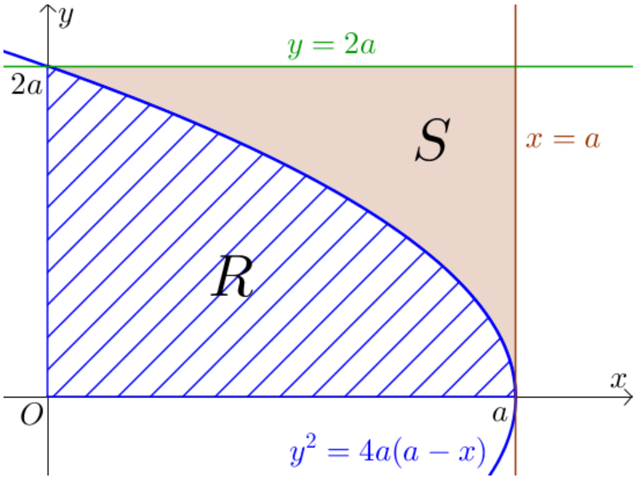
$$=\pi\left(\frac{1}{3}+\ln 4\right).$$

[M1] Formed a volume integral.

[A1] Correct expression for volume of solid.

[A1] $\left[\ln y\right]_1^4$

[A1] $\pi\left(\frac{1}{3}+\ln 4\right)$.

2(i)	 When $x = 0$, $y^2 = 4a^2 \Rightarrow y = \pm 2a$. When $y = 0$, $4a(a-x) = 0 \Rightarrow x = a$. Area of $R = \int_0^{2a} a - \frac{y^2}{4a}\,\mathrm{d}y$ $= \left[ay - \frac{y^3}{12a}\right]_0^{2a}$ $= a(2a) - \frac{(2a)^3}{12a}$ $= \frac{4}{3}a^2$.	[M1] Formed area integral. [A1] $\left[ay - \frac{y^3}{12a}\right]_0^{2a}$. [A1] $\frac{4}{3}a^2$.
2(ii)	$V_x = \pi\int_0^a 4a(a-x)\,\mathrm{d}x$ $= \pi\left[4a^2x - 2ax^2\right]_0^a$ $= \pi\left(4a^3 - 2a^3\right)$ $= 2\pi a^3$.	[M1] Formed volume integral w.r.t. x. [A1] $\left[4a^2x - 2ax^2\right]_0^a$. [A1] $2\pi a^3$.
2(iii)	$V_y = \pi\int_0^{2a}\left(a - \frac{y^2}{4a}\right)^2 \mathrm{d}y = \pi\int_0^{2a} a^2 - \frac{y^2}{2} + \frac{y^4}{16a^2}\,\mathrm{d}y$ $= \pi\left[a^2y - \frac{y^3}{6} + \frac{y^5}{80a^2}\right]_0^{2a}$ $= \pi\left[a^2(2a) - \frac{(2a)^3}{6} + \frac{(2a)^5}{80a^2}\right]$ $= \frac{16}{15}\pi a^3$ $= \frac{8}{15}\left(2\pi a^3\right) = \frac{8}{15}V_x$.	[M1] Formed volume integral w.r.t. y. [A1] $\left[a^2y - \frac{y^3}{6} + \frac{y^5}{80a^2}\right]_0^{2a}$ [A1] $V_y = \frac{8}{15}V_x$ **(a.g.)**.

2 (iv)

Volume of solid formed when S is rotated completely about the y-axis

$$=\pi(a)^2(2a)-V_y$$

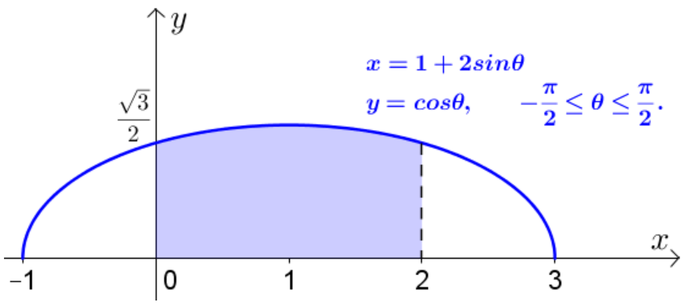
$$=2\pi a^3-\frac{16}{15}\pi a^3$$

$$=\frac{14}{15}\pi a^3.$$

[M1] Expressing volume as a difference.

[A1] $\frac{14}{15}\pi a^3$.

3(i)



When $y=0$, $\cos\theta=0\Rightarrow\theta=\pm\frac{\pi}{2}$. Hence,

$$x=1+2\sin\left(-\frac{\pi}{2}\right)=-1 \text{ or } x=1+2\sin\left(\frac{\pi}{2}\right)=3.$$

When $x=0$, $1+2\sin\theta=0\Rightarrow\sin\theta=-\frac{1}{2}\Rightarrow\theta=-\frac{\pi}{6}$.

Hence $y=\cos\left(-\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$.

[B1] Shape of curve

[B2,1,0] Axial intercepts

3(ii)

When $x=2$, $2=1+2\sin\theta\Rightarrow\theta=\frac{\pi}{6}$.

Area of region

$$=\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\cos\theta(2\cos\theta)\,\mathrm{d}\theta$$

$$=2\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\cos^2\theta\,\mathrm{d}\theta$$

$$=\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\cos2\theta+1\,\mathrm{d}\theta$$

$$=\left[\frac{1}{2}\sin2\theta+\theta\right]_{-\frac{\pi}{6}}^{\frac{\pi}{6}}$$

$$=\left[\frac{1}{2}\sin2\left(\frac{\pi}{6}\right)+\left(\frac{\pi}{6}\right)\right]-\left[\frac{1}{2}\sin2\left(-\frac{\pi}{6}\right)+\left(-\frac{\pi}{6}\right)\right]$$

$$=\frac{\sqrt{3}}{2}+\frac{\pi}{3}.$$

[M1] Method of substitution.

[A1] $2\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}}\cos^2\theta\,\mathrm{d}\theta$.

[A1] $\left[\frac{1}{2}\sin2\theta+\theta\right]_{-\frac{\pi}{6}}^{\frac{\pi}{6}}$.

[A1] $\frac{\sqrt{3}}{2}+\frac{\pi}{3}$.