O	JURONG SECONDARY SCHOOL 2023 GRADUATION EXAMINATION SECONDARY 4 EXPRESS/ SECONDARY 5 NORMAL ACADEMIC

CANDIDATE NAME	

CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS **4049/02**

PAPER 2 **2023**
Candidates answer on the Question Paper. **2 hours 15 minutes**
Additional Materials : Writing Paper (1 sheet)

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 90.

For Examiner's Use
90

This document consists of **17** printed pages including this page.

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n\,,$$

where n is a positive integer, and $\binom{n}{r} = \dfrac{n!}{r!(n-r)!} = \dfrac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 At the beginning of a virus outbreak, the number of cases of infected people increased with time. After t days, the number of recorded cases was N. It was observed that N can be modelled by the equation $N = 1200e^{kt}$.

(a) Write down the initial number of cases recorded. [1]

Solution	Marks
1200	B1

The number of cases recorded after 6 days rose to 4800.

(b) Estimate the number of cases recorded after 10 days. [4]

Solution	Marks
$4800 = 1200e^{6k}$	M1 – substitute values correctly
$e^{6k} = \frac{4800}{1200}$	
$6k = \ln\frac{4800}{1200}$	
$k = 0.231049$	M1 – value of k – accept rounded off values.
$N = 1200e^{10(0.231049)}$	M1
$N = 12095.2$	
≈ 12100	A1

A pandemic is declared if the number reaches 20 000 cases.

(c) Assuming the trend continues, estimate after how many days will it take for a pandemic to be declared. [2]

Solution	Marks
$20000 = 1200e^{0.231049t}$	M1 – substitute values correctly
$t = 12.1766$	
≈ 13 days	A1 – Do not accept 12 days

2 The expression $x^3 + px^2 + qx + r$ is divisible by both x and $x-2$ and it leaves a remainder of 8 when divided by $x+2$.

(a) Find the values of p, q and r. [4]

Solution	Marks
$f(0)=0$ $r=0$ $f(2)=0$ $8+4p+2q=0......(1)$ $f(-2)=8$ $-8+4p-2q=8$ $-16+4p-2q=0......(2)$ $(1)+(2)$ $8p=8$ $p=1$ $q=-6$	[B1] [M1] – Forming correct equations (either one) [A1] [A1]

(b) Hence, find the remainder when it is divided by x^2+2x-3. [2]

Solution	Marks
Long division $$\begin{array}{r\|l} & x-1 \\ \hline x^2+2x-3 & x^3+x^2-6x \\ & -(x^3+2x^2-3x) \\ \hline & -x^2-3x \\ & -(-x^2-2x+3) \\ \hline & -x-3 \end{array}$$ Remainder = $-x-3$	[M1 – Long division] A1 – Writing out remainder

Alternative Method $x^2+2x-3=(x+3)(x-1)$ $x^3+x^2-6x=Q(x)(x^2+2x-3)+R$ $x^3+x^2-6x=Q(x)(x^2+2x-3)+(ax+b)$ When $x=1$, $1+1-6=a+b$ $a=-4-b......(1)$ When $x=-3$, $-27+9-6(-3)=-3a+b$ $0=-3a+b......(2)$ Sub (1) into (2), $0=-3(-4-b)+b$ $0=12+3b+b$ $b=-3$ $a=-1$ Remainder = $-x-3$	[M1] – Forming of equations [A1]

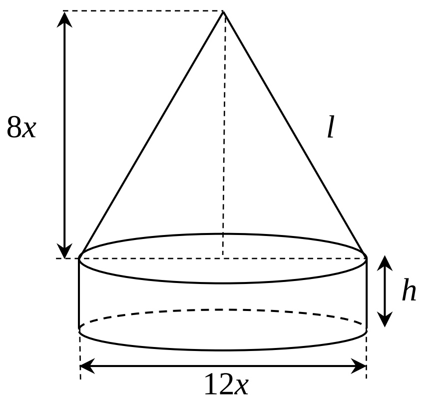
3 **(a)** Show that $2\cos\theta + \cot\theta - 1 = 2\cos\theta\cot\theta$ can be written as $(2\cos\theta - 1)(\sin\theta - \cos\theta) = 0$. [3]

Solution	Marks
$2\cos\theta + \cot\theta - 1 = 2\cos\theta\cot\theta$	
$2\cos\theta + \cot\theta - 1 - 2\cos\theta\cot\theta = 0$	
$2\cos\theta - 1 + \cot\theta(1 - 2\cos\theta) = 0$	M1 – factorisation
$(2\cos\theta - 1) - \cot\theta(2\cos\theta - 1) = 0$	
$(2\cos\theta - 1)(1 - \cot\theta) = 0$	M1 – conversion of $\cot\theta$
$(2\cos\theta - 1)\left(1 - \frac{\cos\theta}{\sin\theta}\right) = 0$	A1
$(2\cos\theta - 1)(\sin\theta - \cos\theta) = 0$	
Alternatively	
$2\cos\theta + \frac{\cos\theta}{\sin\theta} - 2\cos\theta \times \frac{\cos\theta}{\sin\theta} - 1 = 0$	M1 – conversion of cot
$2\sin\theta\cos\theta + \cos\theta - 2\cos^2\theta - \sin\theta = 0$	M1 – simplification
$2\cos\theta(\sin\theta - \cos\theta) - (\sin\theta - \cos\theta) = 0$	A1 – factorise by grouping
$(\sin\theta - \cos\theta)(2\cos\theta - 1) = 0$	

(b) Hence, solve the equation $2\cos 2x + \cot 2x - 1 = 2\cos 2x\cot 2x$ for $0° < x < 180°$. [4]

Solution	Marks
$\cot 2x + 2\cos 2x - 1 = 2\cos 2x\cot 2x$	
$(2\cos 2x - 1)(\sin 2x - \cos 2x) = 0$	M1 – With $2x$
$\cos 2x = \frac{1}{2}$ *or* $\sin 2x = \cos 2x$	M1, M1 – Correct Equations
$\tan 2x = 1$	
$\text{Basic}\angle = 60°$ $\quad$ $\text{Basic}\angle = 45°$	
$2x = 60°, 300°$ $\quad$ $2x = 45°, 225°$	
$x = 30°, 150°$ $\quad$ $x = 22.5°, 112.5°$	
$x = 30°, 22.5°, 112.5°, 150°$	A1

4 The diagram below shows a mould made of a cylinder and a right circular cone. The diameter of the cylinder is $12x$ cm and its height is h cm. The vertical height of the cone is $8x$ cm.



(a) Find an expression, in terms of x, for the slant height l of the cone. [1]

Solution	Marks
$l=\sqrt{(8x)^2+(6x)^2}$ $=10x$	B1

(b) Given that the entire mould is covered with a plastic sheet whose area is 240π cm^2, express h in terms of x. [2]

Solution	Marks
$\pi rl+2\pi rh+\pi r^2=240\pi$ $\pi(6x)(10x)+2\pi(6x)h+\pi(6x)^2=240\pi$ $60x^2+12xh+36x^2=240$ $12xh=240-96x^2$ $h=\dfrac{240-96x^2}{12x}$ $h=\dfrac{20-8x^2}{x}$	 M1 A1

(c) Show that the volume, V cm^3, of the mould is given by $V = 720\pi x - 192\pi x^3$. [3]

Solution	Marks
Volume $= \frac{1}{3}\pi r^2 h_{cone} + \pi r^2 h$ $= \frac{1}{3}\pi(6x)^2(8x) + \pi(6x)^2(\frac{20-8x^2}{x})$ $= 96\pi x^2 + 720\pi x - 288\pi x^3$ $= 720\pi x - 192\pi x^3$ (shown)	M2 – Cone, cylinder A1

(d) Hence find the value of x for which the volume has a stationary value and determine whether this value for the volume is a maximum or minimum. [4]

Solution	Marks
Volume $V = 720\pi x - 192\pi x^3$ $\frac{dV}{dh} = 720\pi - 576\pi x^2$ $\frac{dV}{dh} = 0$ $720\pi - 576\pi x^2 = 0$ $x^2 = \frac{720\pi}{576\pi}$ $x = 1.11803, \quad -1.11803$(rej) $x = 1.12$ $\frac{d^2V}{dh^2} = -1152\pi x$ $\frac{d^2V}{dh^2} < 0$ since x is positive. Hence, maximum volume.	 M1 A1 M1 A1

5 **(a)** Without using a calculator, show that $\cos 15° = \dfrac{\sqrt{2}+\sqrt{6}}{4}$. [3]

Solution	Marks
$\cos 15° = \cos(60° - 45°)$ $= \cos 60 \cos 45 + \sin 60 \sin 45$ $= \frac{1}{2}(\frac{\sqrt{2}}{2}) + \frac{\sqrt{3}}{2}(\frac{\sqrt{2}}{2})$ $= \frac{\sqrt{2}+\sqrt{6}}{4}$ (shown)	M1 – use of correct formulae with appropriate values M1 – either term A1

(b) Hence, state the value of $\cos(-15°)$. [1]

Solution	Marks
$\cos(-15°) = \frac{\sqrt{2}+\sqrt{6}}{4}$	[B1]

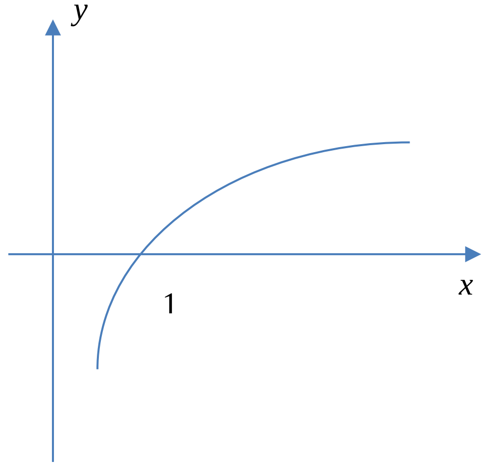
(c) Using your answer from **part (i)**, find the exact value of $\sec(15°)$ [3]

Solution	Marks
$\sec 15° = \frac{1}{\frac{1}{4}(\sqrt{2}+\sqrt{6})}$ $= \frac{4}{(\sqrt{2}+\sqrt{6})} \times \frac{(\sqrt{2}-\sqrt{6})}{(\sqrt{2}-\sqrt{6})}$ $= \frac{4\sqrt{2}-4\sqrt{6}}{2-6}$ $= \frac{4\sqrt{2}-4\sqrt{6}}{-4}$ $= \sqrt{6}-\sqrt{2}$	 [M1] – Rationalising [M1] – Either denominator or numerator. [A1]

6 **(a)** Solve $6^x = \frac{10}{3} - 6^{-x}$. [4]

Solution		Marks
$6^x = \frac{10}{3} - 6^{-x}$		
$6^x = \frac{10}{3} - \frac{1}{6^x}$		[M1]
Let $u = 6^x$		
$u = \frac{10}{3} - \frac{1}{u}$		
$3^2 u - 10u + 3 = 0$		[M1]
$(3u-1)(u-3) = 0$		
$u = \frac{1}{3}$	*or* $u = 3$	[M1]
$6^x = \frac{1}{3}$	$6^x = 3$	
$x = \frac{\lg\frac{1}{3}}{\lg 6} = -0.613$	$x = \frac{\lg 3}{\lg 6} = 0.613$	[A1]

(b) Sketch the graph of $\ln x$, showing any points of intersection with the axes. [2]

Solution	Marks
	[G1 – Shape] [G1 – Intersection]

(c) To solve $e^{1-2x} = x^3$, a straight line can be drawn on the same axes as the graph in **part (b)**.

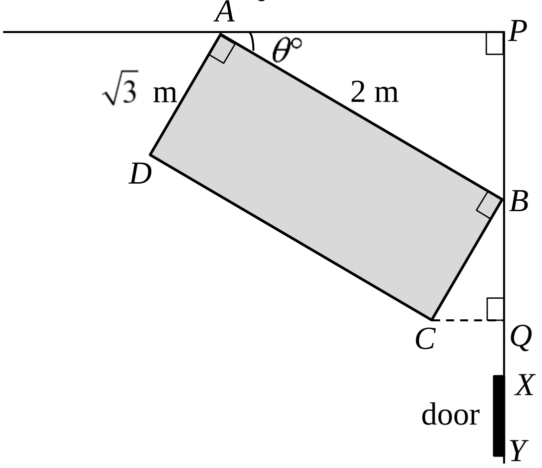
(i) Determine the equation of the straight line to be drawn. [2]

Solution	Marks
$e^{1-2x} = x^3$ $\ln e^{1-2x} = \ln x^3$ $1-2x = 3\ln x$ $\frac{1-2x}{3} = \ln x$ $y = \frac{1-2x}{3}$	 [M1] [A1]

(ii) Hence, state the number of solutions for $e^{1-2x} = x^3$. [1]

Solution	Marks
Number of solutions = 1	[B1]

7 The diagram below shows a rectangular table, *ABCD* placed at the corner of a classroom. It is given that the table has length *AB* = 2 m and width *AD* = $\sqrt{3}$ m. It is also given that $\angle APB = \angle BQC = 90°$ and $\angle PAB = \theta°$.



(a) Show that the length of *PQ*, *L* can be expressed as $L = 2\sin\theta + \sqrt{3}\cos\theta$. [2]

Solution	Marks
$\frac{PB}{2} = \sin\theta, \frac{BQ}{\sqrt{3}} = \cos\theta$	M1
$L = PB + BQ = 2\sin\theta + \sqrt{3}\cos\theta$	A1

(b) Express *L* in the form $R\sin(\theta + \alpha)$ where $0° < \alpha < 90°$ and *R* > 0. [3]

Solution	Marks
$R = \sqrt{2^2 + 3}$	M1
$\alpha = \tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$	M1
$L = \sqrt{7}\sin\left(\theta + 40.9°\right)$	A1

(c) Find the value of θ for *L* = 2.3 m. [2]

Solution	Marks
$\sin\left(\theta + 40.894°\right) = 0.869318$	
$b.a. = 60.3795°$	M1
$\theta = 19.5°$ (1 d.p.)	A1

(d) Find the maximum value of *L* and the corresponding value of θ. [2]

Solution	Marks
$\max L = \sqrt{7}$ m	B1
corr value of $\theta = 49.1°$	B1

8 A curve has the equation $y = (3 - x)\sqrt{2x+5}$.

(a) Show that $\dfrac{dy}{dx} = \dfrac{ax+b}{\sqrt{2x+5}}$, where a and b are constants to be determined. [3]

Solution	Marks
$\dfrac{dy}{dx} = (3-x)\times\dfrac{1}{2}(2x+5)^{-\frac{1}{2}}\times 2 + (2x+5)^{\frac{1}{2}}\times(-1)$ $= \dfrac{3-x-(2x+5)}{\sqrt{2x+5}}$ $= \dfrac{-3x-2}{\sqrt{2x+5}}$	M1 product rule M1 combine into a single fraction A1

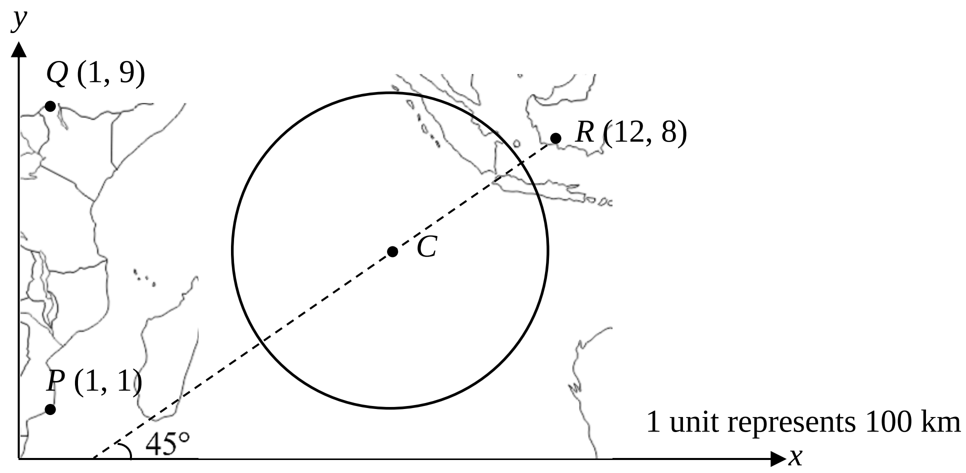
(b) A point (x, y) moves along the curve. Find the value of x when the y-coordinate is increasing at the same rate as the x-coordinate. [3]

Solution	Marks
$\dfrac{dy}{dx} = 1$ $-3x - 2 = \sqrt{2x+5}$ $9x^2 + 12x + 4 = 2x + 5$ $9x^2 + 10x - 1 = 0$ $x = \dfrac{-10 \pm \sqrt{10^2 - 4(9)(-1)}}{2(9)}$ $= 0.0923$ (rej) or -1.20 (3 s.f.) x cannot be 0.0923 as $-3x-2 \geq 0$	M1 M1 square both sides A1 B1

(c) Using your answer in **(a)**, evaluate $\displaystyle\int_{-2}^{2} \frac{-3x}{\sqrt{2x+5}}\, dx$. [4]

Solution	Marks
$\displaystyle\int_{-2}^{2} \frac{-3x}{\sqrt{2x+5}}\, dx = \left[(3-x)\sqrt{2x+5}\right]_{-2}^{2} + \int_{-2}^{2} \frac{2}{\sqrt{2x+5}}\, dx$ $= (3-5) + \left[\dfrac{2\sqrt{2x+5}}{2\times\frac{1}{2}}\right]_{-2}^{2}$ $= 2$	M1 M1 correct integral M1 substitution A1

9 The map below shows part of the Indian Ocean.
Geological stations $P(1, 1)$, $Q(1, 9)$ and $R(12, 8)$ detected an earthquake and a geologist is attempting to locate the epicentre, C of the earthquake.



Instruments at P and Q detected the earthquake at exactly the same time, indicating that the epicentre, C is equidistant from P and Q. Instrument at R detected it in the direction indicated by the line l, which makes an angle of $45°$ with the positive x-axis.

(a) Show that the line l can be represented by the equation $y = x - 4$. [2]

Solution	Marks
Gradient of $l = 1$ $y - 8 = x - 12$ $y = x - 4$ (shown)	M1 correct gradient A1 with substitution

(b) Find the coordinates of C. [2]

Solution	Marks
y − coordinate = $\frac{1+9}{2} = 5$ $5 = x - 4$ $x = 9, C = (9, 5)$	M1 M1 A1

(c) It is given that the earthquake detected can be felt at places as far as 450 km from the epicentre.
Find the equation of the circle that represents the places affected. [2]

Solution	Marks
radius $= \frac{450}{100} = 4.5$ units $(x-9)^2 + (y-5)^2 = \frac{81}{4}$	M1 A1

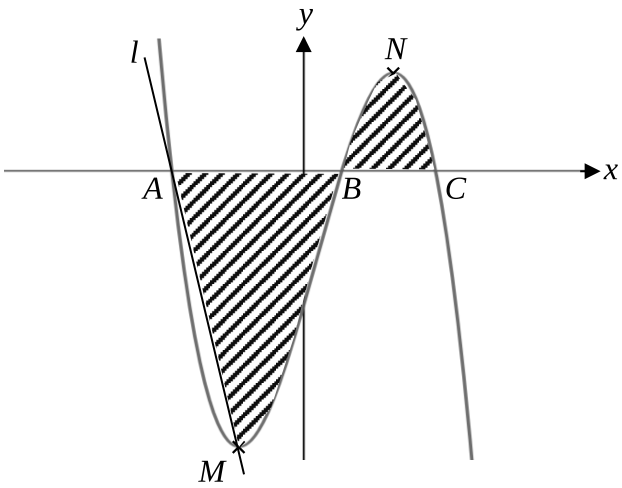
(d) Hence, or otherwise, determine if geological station R is inside the circle. [2]

Solution	Marks
$CR=\sqrt{(12-9)^2+(8-5)^2}$ $=4.24<4.5$ R lies inside the circle	M1 A1

(e) Explain why it is not possible to draw such a circle that passes through all three geological stations P, Q and R.
Support your answer with mathematical calculations. [3]

Solution	Marks
Gradient of $PR=\dfrac{8-1}{12-1}=\dfrac{7}{11}$ Gradient of $RQ=\dfrac{8-9}{12-1}=-\dfrac{1}{11}$ Since gradient of $PR\times$ gradient of $RQ\neq-1, \angle QRP\neq 90°$ PQ is a vertical line and neither PR nor QR are horizontal. PQR is not a right-angled triangle, by right angle in a semicircle, P, Q and R do not lie on the same circle.	M1 find gradients (or using Pythagoras' Theorem) M1 conclusion A1 with circle property stated

10 The diagram below shows part of the curve $y = (2x - 1)(3 - x^2)$. The curve has a minimum point at M and a maximum point at N. The curve intersects the x-axis at A, B and C. The line l pass through A and M.



(a) Find the coordinates of A, B and C. [3]

Solution	Marks
$(2x-1)(3-x^2)=0$ $A=\left(-\sqrt{3},0\right)$ $B=(\frac{1}{2},0)$ $C=(\sqrt{3},0)$	M1 A1 three correct coordinates (accept 3 sf) A1 correct reference to the diagram -1 if not in coordinate form

(b) Find the coordinates of M. (You are not required to prove that it is the minimum point.) [3]

Solution	Marks
$y=-2x^3+x^2+6x-3$ $\frac{dy}{dx}=-6x^2+2x+6$ $-6x^2+2x+6=0$ $x=\frac{-2\pm\sqrt{4-4(-6)(6)}}{-12}$ $=-0.847$ or 1.18 coordinates of $M=(-0.847, 6.15)$	M1 correct derivative M1 '= 0' A1

(c) Hence, find the area of the shaded region. [6]

Solution	Marks
Area of the shaded region = $-\frac{1}{2}\times 6.1493\times 0.88505$ $-\int_{-0.847}^{0.5}\left(-2x^3+x^2+6x-3\right)\,\mathrm{d}x$ $+\int_{0.5}^{\sqrt{3}}\left(-2x^3+x^2+6x-3\right)\,\mathrm{d}x$ $=2.72122-\left[-\frac{1}{2}x^4+\frac{1}{3}x^3+3x^2-3x\right]_{-0.847}^{0.5}+\left[-\frac{1}{2}x^4+\frac{1}{3}x^3+3x^2-3x\right]_{0.5}^{\sqrt{3}}$ $=2.72122-\left(-\frac{71}{96}-4.23334\right)+\left(1.03589+\frac{71}{96}\right)$ $=9.47\text{ units}^2\,(3\text{s.f.})$	M1,M1,M1 each part M1 correct integral M1 correct substitution A1