RAFFLES INSTITUTION
H2 Mathematics (9758)
2016 Year 5

Tutorial 4A : Complex Numbers I

Section A (Basic Questions)

Do these questions without a GC first, then verify your answers using a GC wherever possible.

1 Express the following complex numbers in the form $x+\mathrm{i}y$, where $x,\ y \in \mathbb{R}$:

(a) $(4-\mathrm{i})-(3+3\mathrm{i})$ **(b)** $(2+\mathrm{i})(3-4\mathrm{i})$ **(c)** $(1+\mathrm{i})^3$

(d) $\dfrac{3+\mathrm{i}}{4-3\mathrm{i}}$ **(e)** $\dfrac{-8+5\mathrm{i}}{-2-4\mathrm{i}}-\dfrac{3+8\mathrm{i}}{1+2\mathrm{i}}$

[**a)** $1-4\mathrm{i}$ **b)** $10-5\mathrm{i}$ **c)** $-2+2\mathrm{i}$ **d)** $\dfrac{9}{25}+\dfrac{13}{25}\mathrm{i}$ **e)** $-4-\dfrac{5}{2}\mathrm{i}$]

2 Find $x,\ y \in \mathbb{R}$ such that

(a) $2x+3\mathrm{i}y=-x-6\mathrm{i}$ **(b)** $(x+\mathrm{i}y)(2-\mathrm{i})=8+\mathrm{i}$ **(c)** $(x+2\mathrm{i})(2+3\mathrm{i})=\mathrm{i}y$

[**a)** $x=0,\ y=-2$ **b)** $x=3,\ y=2$ **c)** $x=3,\ y=13$]

3 Solve the equations

(a) $z^2=3-4\mathrm{i}$, **(b)** $z^2+2z+10=0$, **(c)** $z^3-2z-4=0$.

[**a)** $\pm(2-\mathrm{i})$ **b)** $-1\pm 3\mathrm{i}$ **c)** $2,\ -1\pm\mathrm{i}$]

4 Find $a,\ b\in\mathbb{R}$ if

(a) $3+\mathrm{i}$ is a root of the equation $z^2+az+b=0$,

(b) $a+\mathrm{i}a$ is a root of the equation $z^2+4z+b=0$.

[**a)** $a=-6,\ b=10$ **b)** $a=-2,\ b=8$ *or* $a=0,\ b=0$]

5 For each of the following complex number, represent it on an Argand diagram.
Find also its modulus and argument.

(a) $\sqrt{3}+\mathrm{i}$ **(b)** $-1+\mathrm{i}\sqrt{3}$ **(c)** $\mathrm{i}^2(1+\mathrm{i})$

(d) $-\mathrm{i}(1+\mathrm{i})$ **(e)** $\sin\theta+\mathrm{i}\cos\theta$, where $0<\theta<\dfrac{\pi}{2}$

[**a)** $2,\ \dfrac{\pi}{6}$ **b)** $2,\ \dfrac{2\pi}{3}$ **c)** $\sqrt{2},\ -\dfrac{3\pi}{4}$ **d)** $\sqrt{2},\ -\dfrac{\pi}{4}$ **e)** $1,\ \dfrac{\pi}{2}-\theta$]

Section B (Standard Questions)

6 Given that $\arg(a+\mathrm{i}b)=\theta$, where $a>0$, $b>0$, find, in terms of θ and π, the values of

(a) $\arg(-a+\mathrm{i}b)$, **(b)** $\arg(-a-\mathrm{i}b)$, **(c)** $\arg(b+\mathrm{i}a)$.

[**i)** $\pi-\theta$ **ii)** $\theta-\pi$ **iii)** $\dfrac{\pi}{2}-\theta$]

7 It is given that $w=\dfrac{z-1}{z^*+1}$, where $z=a+\mathrm{i}b$, $a,\ b\in\mathbb{R}$.
By expressing w in the form $u+\mathrm{i}v$, $u,\ v\in\mathbb{R}$, find the conditions under which
(a) w is real, **(b)** w is purely imaginary.

[**a)** $ab=0$ **b)** $a^2-b^2=1$]

compare coefficients

8(a) Solve for $\lambda,\ \mu\in\mathbb{R}$ if $(4-\mathrm{i})^2+(8\lambda+\mathrm{i})(3\mu-\mathrm{i})+8\mathrm{i}=43$.

(b) Solve for $z=a+\mathrm{i}b$, $a,\ b\in\mathbb{R}$ if $(z+\mathrm{i})^*=2\mathrm{i}z+\mathrm{i}$.

[**a)** $\lambda=\dfrac{3\sqrt{3}}{8}$, $\mu=\sqrt{3}$ or $\lambda=-\dfrac{3\sqrt{3}}{8}$, $\mu=-\sqrt{3}$ **b)** $-\dfrac{4}{3}+\dfrac{2}{3}\mathrm{i}$]

9 Solve the following simultaneous equations:
(a) $w+z=6+2\mathrm{i}$, $w-3z=\dfrac{20}{2-\mathrm{i}}$; **(b)** $z=w+3\mathrm{i}+2$, $z^2-\mathrm{i}w+5-2\mathrm{i}=0$.

[**a)** $w=\dfrac{13}{2}+\dfrac{5}{2}\mathrm{i},\ z=-\dfrac{1}{2}-\dfrac{1}{2}\mathrm{i}$ **b)** $w=-2-\mathrm{i},\ z=2\mathrm{i}$ or $w=-2-4\mathrm{i},\ z=-\mathrm{i}$]

10 **[9740/2007/01/Q3]**
The complex number w is such that $ww^*+2w=3+4\mathrm{i}$, where w^* is the complex conjugate of w. Find w in the form $a+\mathrm{i}b$, where a and b are real.

[$w=-1+2\mathrm{i}$]

11 It is given that $-1+2\mathrm{i}$ satisfies the equation $2z^3+3z^2+az+b=0$, where $a,b\in\mathbb{R}$.
Find a and show that $b=-5$. Hence obtain the exact values of all the roots of the equation.

[$a=8$; $\dfrac{1}{2}$, $-1\pm 2\mathrm{i}$]

12 **[9740/2010/02/Q1]**
(i) Solve the equation $x^2-6x+34=0$.
(ii) One root of the equation $x^4+4x^3+x^2+ax+b=0$, where a and b are real, is $x=-2+\mathrm{i}$. Find the values of a and b, and the other roots.

[**(i)** $3\pm 5\mathrm{i}$ **(ii)** $a=-16, b=-20$ roots are: $-2\pm\mathrm{i}, -2, 2$]

13 **[9740/2013/01/Q4]**
The complex number w is given by $1+2\mathrm{i}$.
(i) Find w^3 in the form $x+\mathrm{i}y$, showing your working.
(ii) Given that w is a root of the equation $az^3+5z^2+17z+b=0$, find the values of the real numbers a and b.
(iii) Using these values of a and b, find all the roots of this equation in exact form.

[**(i)** $w^3=-11-2\mathrm{i}$ **(ii)** $a=27,\ b=295$ **(iii)** $z=-\dfrac{59}{27}, 1\pm 2\mathrm{i}$]