Forces

1 Introduction

This chapter is about recognising the different kinds of forces at work in different situations and knowing the characteristics of the forces. This is important whenever one is interested in explaining the motion of objects, which is the focus of the chapter on Dynamics. Understanding the forces at work is also important in calculating work done for a given situation.

2 Elastic Force

Most objects, e.g. a spring or a rubber block, when stretched or compressed, will exert a force opposite in direction to that applied on the object.

An elastic body is one which can return to its original size and shape after being deformed.

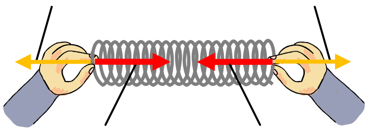


Fig. 2.1

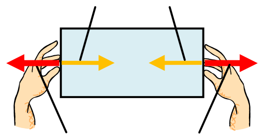


Fig. 2.2

The force exerted by the object on the external agent is an *elastic* force. In Fig. 2.1, the elastic force is also sometimes called *tensile* force or *tension*.

In response to an external applied force, an elastic body will exert an elastic force *on the external* body.

When the elastic force is proportional to the distance extended or compressed, the object is said to *obey Hooke's law*. Hooke's law states that

$$\vec{F} = -k\vec{x} \quad \text{or} \quad F = kx \text{ (magnitudes)}$$

where k is a positive constant and the negative sign indicates that the elastic force is opposite to the direction of displacement. Though called a 'law', Hooke's law is not universal or always obeyed. There are many objects or materials which do not obey Hooke's law at all or may obey it for a limited range of x only.

When the elastic force varies linearly with the extension or compression, the elastic body is said to obey Hooke's law.

k is known as the *constant of proportionality* or in the case of a spring, also called the *spring constant*. Note that k is constant only as long as the *dimensions* and *material* of the object are not changed. For a given x, the larger the value of k, the larger the elastic force and hence the larger the external force needed to cause the displacement x. Therefore k is also called the *stiffness*.

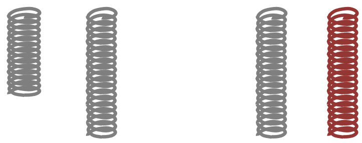


Fig. 2.3

Newton's Third Law

Note that in Fig. 2.1 the force exerted by *hand on spring* is equal in magnitude but opposite in direction to the force exerted by *spring on hand*. This is reflected by the arrows of equal length but opposite directions. In fact, Newton's third law states:

> When body A exerts a force on body B, body B must exert an equal but opposite force on body A.

Newton's third law implies that forces always occur in pairs

All forces exist in such equal and opposite pairs or action-reaction pairs.

Elastic Forces Inside a Spring

In Fig. 2.1, if we focus our attention on a middle segment of the spring as shown, would the elastic forces exerted by this segment on the neighbouring segments be the same or smaller than the elastic forces on the hands that pulled the spring?

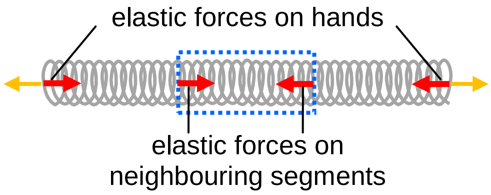


Fig. 2.4

Unlike Hooke's law, Newton's third law is universal. As shown in Fig. 2.5, while the middle segment **B** exerts elastic forces on the neighbouring segments **A** and **C**, the neighbouring segments also exert equal but opposite elastic forces on the middle segment.

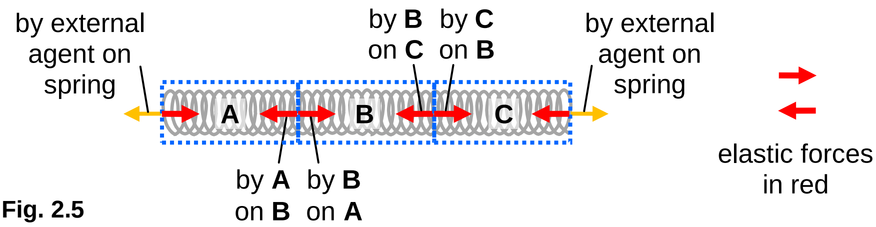


Fig. 2.5

Hence, the elastic forces(magnitude) exerted by the whole spring on the external agent and by any segment on neighbouring segments are the same. This situation is frequently described as 'the tension in a spring is the same at every point'. Strictly speaking, the tension is only the same everywhere in the spring if it is not accelerated or its mass is negligible.

So, assuming massless springs, when different springs of different spring constants are connected together and pulled at both ends as shown, the tensions(magnitude T) between the springs and inside them will be the same.

For a stretched or compressed elastic body at rest, elastic forces exist throughout the body and the magnitude is the same on every part of the body.

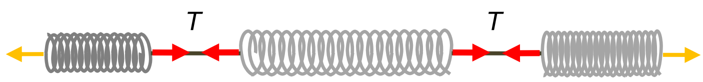


Fig. 2.6

However their extensions will be different unless they are totally identical springs. In the same way, ropes which are pulled will have the same tension throughout.

Potential Energy

In general, potential energy (PE) can be stored in a system with
(a) at least two bodies which
(b) exert forces - on each other - that depend only on their positions.

Potential energy (PE) can be stored in a system with at least 2 bodies which exert forces on each other in a way that depend only on their positions.

In the case of a stretched spring, energy is stored in the spring as can be seen when two masses are attached to the ends of the spring and released. The two masses will be accelerated towards one another gaining kinetic energy (KE) while the elastic PE (EPE) gets converted to KE. What if the stretched spring is released without any masses attached? In this case, the PE is converted to KE of the various spring segments and perhaps some sound energy when neighbouring coils knock into each other upon release.

Another type of PE you already know is the gravitational PE. When a ball is lifted higher from the ground and thus further from the Earth, PE is stored between Earth and the ball. When the ball is released, the mutual attraction between the Earth and ball will accelerate them towards one another. However due to the large mass of the Earth, it hardly moves while the ball moves much more easily and gets practically all the KE that comes from the stored PE.

F-e & F-L graphs

For different forces F_A applied to pull a spring, it is found that the spring will settle at different final lengths. If the spring obeys Hooke's law, then the graph of applied force F_A versus *extension* e will look as shown in Fig. 2.8a while the graph of F_A versus *length* of spring is as shown in Fig. 2.8b.

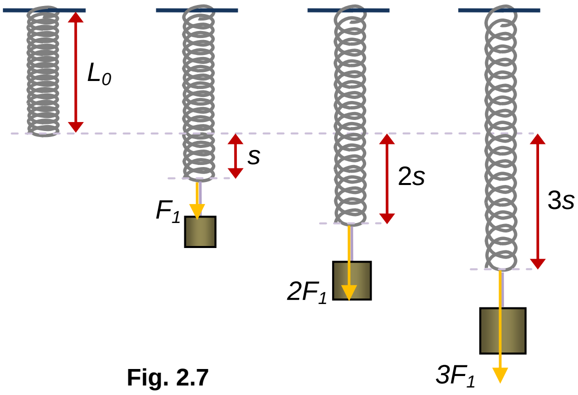


Fig. 2.7

Applied force vs extension

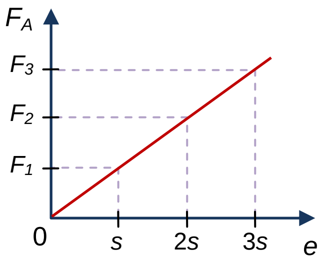


Fig. 2.8a

Applied force vs length

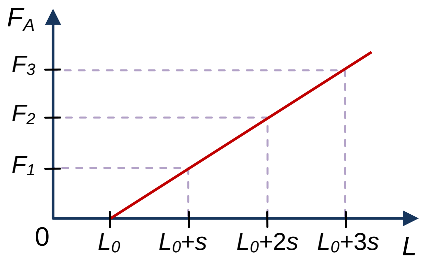


Fig. 2.8b

Force-extension or force-compression graphs are different from force-length graphs.

How Does EPE Get Stored?

Since energy cannot be created but only transferred from one body to another or converted from one form to another, where does the elastic PE come from? It will be learnt that *work done* by a force is equal to the energy transferred or converted from one form to another form. When a person exerts a force to extend a spring, the person uses chemical energy to do the work which gets stored inside the spring as elastic PE.

Elastic PE = work done on the elastic body.

Work done is calculated by the product of force and the displacement in the direction of the force. In the case of a spring being stretched, the force needed to stretch the first mm and the force needed to stretch the second mm are different. As the extension increases, the force needed also increases, so how do we find the work done?

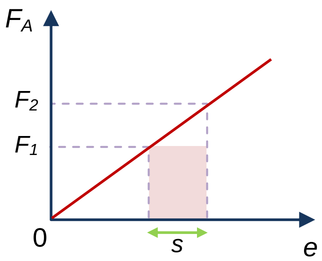


Fig. 2.9a

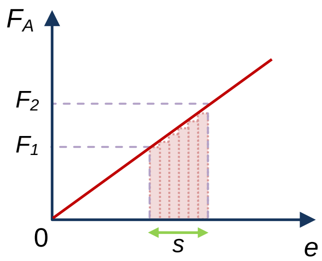


Fig. 2.9b

Consider the work done in increasing the length by s. The applied force varies from F_1 to F_2. $F_1 s$ is the area of the shaded rectangle as shown in Fig.2.9a. This area represents the work done by F_1 over a distance s. Since the force is actually not constant, we can always line up a series of very narrow rectangles within the distance s such that the total area of the rectangles would effectively be the total work done over s. Thus for any *F-e* plot; whether straight line or curve; the area under the plot represents the work done. This work done is equal to the *increase* in EPE over s.

Work done = area under F_A-e or F_A-L graph (regardless of whether straight line or not).

Calculation of EPE

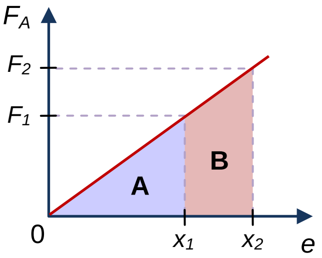


Fig. 2.10a

If the extension increases from x_1 to x_2:

$\boxed{\text{Initial EPE = area } \mathbf{A}}$ = ½ F_1x_1

By Hooke's law $F_1 = kx_1$

∴ Initial EPE = ½ kx_1^2

$\boxed{\textit{Increase} \text{ in EPE = area } \mathbf{B}}$

= ½ $(F_1+F_2)(x_2-x_1)$

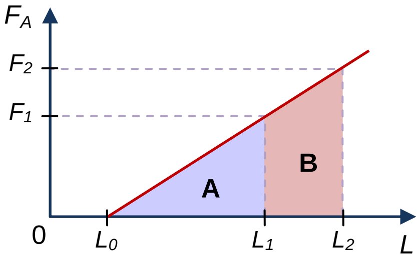


Fig. 2.10b

If the length of a spring increases from L_1 to L_2:

$\boxed{\text{Initial EPE = area } \mathbf{A}}$ = ½ $F_1(L_1-L_0)$

$\boxed{\textit{Increase} \text{ in EPE = area } \mathbf{B}}$

= ½ $(F_1+F_2)(L_2-L_1)$

Initial EPE is area **A** as shown.

Increase in EPE is area **B** as shown.

The mathematical expression for the EPE depends on the shape of the area - triangle or trapezium.

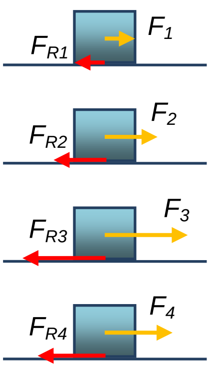
3 Friction, Viscous Force & Drag

Static and Sliding Friction

When two smooth surfaces in contact with one another is magnified using a microscope, we will see that the surfaces are actually not perfectly smooth at the microscopic level. Friction between the two surfaces can arise from

Fig. 3.1

1 interlocking jagged edges of the two surfaces &
2 intermolecular forces between molecules in one surface and the other.



When a small force F_1 is applied to pull a *heavy* box resting on a rough surface, the box does not move.

When applied force is increased to F_2, the box may still remain at rest.

Only when at a certain magnitude F_3, the box starts to move.

After that, the force F_4 needed to maintain a *constant* velocity is actually smaller than F_3.

Fig. 3.2

The reason for the above behaviour is due to a friction force (F_{R1} and F_{R2}) that is able to grow in magnitude to match the applied force but this friction force has a maximum value F_{R3}. When the box starts to move, the friction force F_{R4} at work becomes smaller than F_{R3}. In other words, the person will feel easier to maintain the motion (regardless of speed) of the box than to get it started to move.

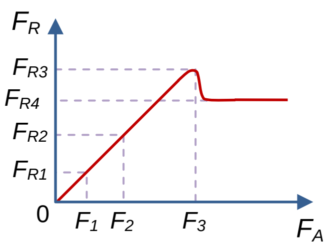


Fig. 3.3

The friction from F_{R1} to F_{R3} is called *static* friction while F_{R4} is called *sliding* or *kinetic* friction.

For two surfaces in contact:

The friction force that prevents sliding is called static friction.

The friction force that opposes the sliding motion is called sliding or kinetic friction.

Static friction can vary to match an applied force and it has a maximum value that is greater than sliding friction.

Friction in Action

Friction is often thought of as impeding motion, but it is frequently needed for motion. Let's look at a 'rear wheel drive' car that is initially at rest in Fig. 3.4. When the rear wheel is driven by the engine, 'F_R on road' is exerted by the rear wheel on the road. In accordance with Newton's third law, an equal but opposite reaction force 'F_R on car' acts on the wheel. The forward push of 'F_R on car' causes the car and front wheel to move forward, but the static friction on the front wheel 'F_F on car' prevents the wheel from sliding, causing it to rotate about its axle instead.

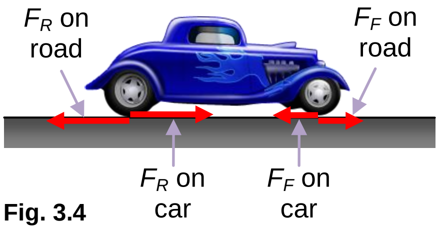


Fig. 3.4

When there is no skidding or sliding, the friction forces shown in Fig. 3.4 are all static friction.

The net forward force enables the car to accelerate.

If the car is rolling along the road and the wheels are disengaged from the engine, then the situation is as shown in Fig. 3.5.

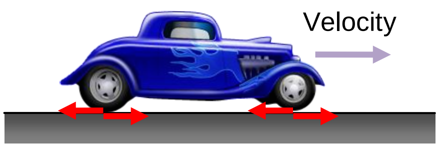


Fig. 3.5

Now, the static friction forces on all the wheels are backward, causing a resultant force on the car which decelerates it.

If the car is moving very fast when the brakes are used to jam all the wheels, thereby preventing them from rotating about their axles, the car will skid on the road surface as shown in Fig. 3.6.

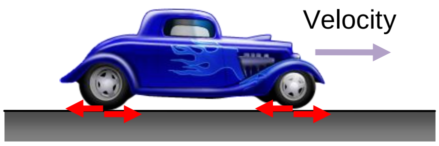


Fig. 3.6

Now, all the friction forces are sliding friction forces. The resultant backward force causes car to decelerate.

If the stationary car were on some icy surface with no friction and the rear wheels are driven, then the situation will be as shown in Fig. 3.7

Fig. 3.7

On a frictionless surface, the driven rear wheels slide on the ice with the car staying on the same spot.

Viscous Force & Drag

Viscous force is the resistive force between two fluid layers or between a fluid and a solid surface when they slide past each other. The higher the viscosity or relative speed of the layers, the greater the viscous force.

Viscous force is the resistive force between two fluid layers or between a fluid and a solid surface when they slide past each other.

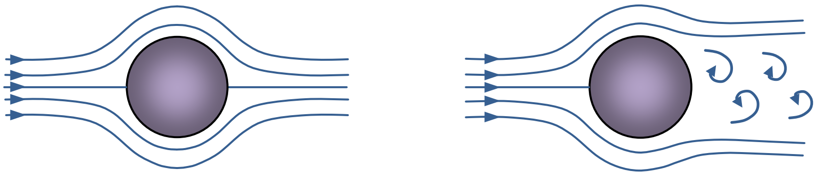
Cross section of a pipe showing different layers flowing at different velocities.

Viscous forces are exerted at:
X between neighbouring fluid layers &
Y between the fluid and pipe surface.

Fig. 3.8

Drag is a force exerted by a fluid(liquid or gas) that opposes the motion of an object through the fluid. In air, the drag is also called *air resistance*. A number of components contribute to the drag force, including viscous force and the force due to turbulent fluid flow.

Drag is a force exerted by a fluid that opposes the motion of an object through the fluid. It is dependent on viscosity, shape, size and speed.



non-turbulent when ball's speed is low. **Fig. 3.9** turbulent when ball's speed is high.

Drag is overall determined by factors such as fluid viscosity, shape and size of object and speed of object. In general at lower speeds, the drag force varies as v while at higher speeds it varies as v^2.

4 Contact Force

Normal Contact Force

Here, 'normal' means perpendicular. Normal contact force arises from contact between two solid surfaces and it acts perpendicular to the surface.

Normal contact force is always perpendicular to the surface of contact.

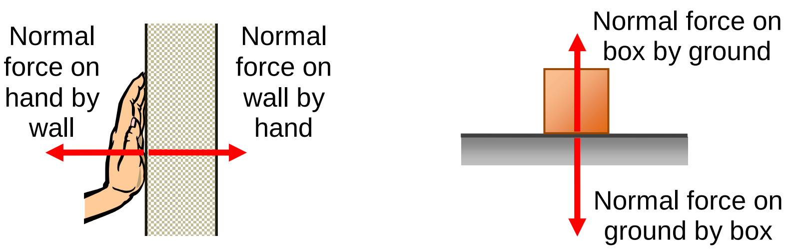


Fig. 4.1

Overall Contact Force

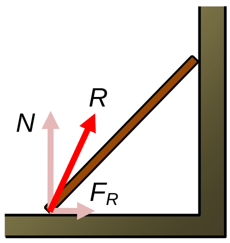


Fig. 4.2

For a ladder leaning against the wall, there will be friction forces and normal contact forces acting at its two ends. Friction F_R is also a contact force and it is sometimes combined with normal contact N force to give the overall contact force R as shown in Fig. 4.2. The direction of F_R is to the right because the tendency of the ladder is to slide to the left.

The *overall contact force* is the vector sum of the friction and normal contact force.

5 Lift

Fig. 5.1

When an aerofoil shape wing cuts through a gas or liquid, it generates a lift force that is perpendicular to the wing's velocity. The wing forces the fluid downwards with the help of the aerofoil shape and the angle of the wings. The fluid in turn exerts an upward force on the wings in accordance with Newton's third law. The lift need not be vertically upwards if the aircraft is climbing or even upside down.

Lift is a force generated by an aerofoil shape wing that is *perpendicular* to the velocity of the wing.

6 Upthrust

Pressure Difference due to Weight

Due to gravitational pull, a fluid column as shown in Fig. 6.1 exerts a force on the area A.

Pressure = Force/Area
= weight/Area
= volume $\times$ density $\times$ g / A
= $(Ah) \times \rho \times g / A$
= ρgh

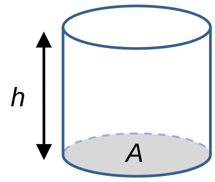


Fig. 6.1

Pressure *difference* due to the weight of a fluid column:
$\Delta P = \rho gh$

Upthrust Due to Pressure Difference

When an object such as a rectangular box is immersed in water of density ρ_w, the pressure on

top surface $P_T = P_{atm} + \rho_w g h_T$
bottom surface $P_B = P_{atm} + \rho_w g h_B$

Since $P_B > P_T$, net upward force on the box

$= (P_B - P_T)\,A$
$= \rho_w\, g(h_B - h_T)\,A$
$= \rho_w\, gV_{box}$

where V_{box} is the volume of the box.

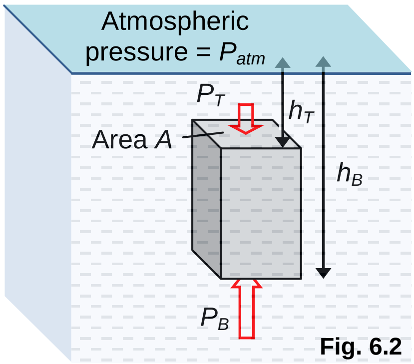


Fig. 6.2

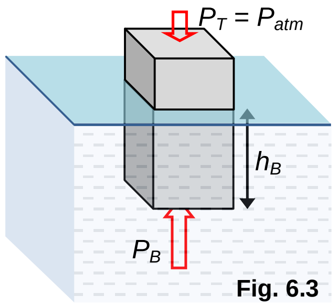


Fig. 6.3

If the box were floating as shown in Fig. 6.3, the pressure on

top surface $P_T = P_{atm}$
bottom surface $P_B = P_{atm} + \rho_w g h_B$

Net upward force $= \rho_w\, g h_B\, A$
$= \rho_w\, gV_{sub}$

where V_{sub} is the submerged volume of the box.

Origin of upthrust is the *difference in pressure* on the top and bottom surfaces of a fully or partially submerged body.

In both cases above, the upward force is called *upthrust* and is equal to the weight of water displaced by box. The pressures on the four vertical surfaces can be ignored as they lead to forces which cancel. It can also be shown that for any *irregular* body, the upthrust is still given by the weight of displaced fluid. This general result is called Archimedes' principle:

Upthrust is equal to the weight of the fluid displaced.

Archimedes' principle states that the upthrust on an immersed (partially or fully) body is equal to the weight of the fluid displaced.

7 Electric Force (Will be covered under the chapter Electric Field)

For now, the focus will be on some basics. An *electric field* is a human construct to demystify the fact that a charge can exert a force on another charge without touching.

An electric field is a region of space in which a charge experiences an electric force.

An electric field is a region of space in which a charge experiences an electric force.

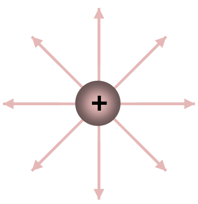


Fig. 7.1

Electric *field lines* are used to indicate the *direction* and *strength* of the field - the closer the lines, the stronger the field. Fig. 7.1 & 7.2 show the strength decreasing with distance from each charge.

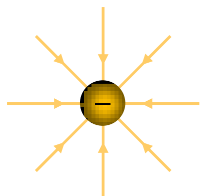


Fig. 7.2

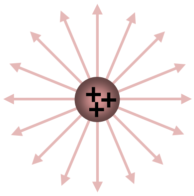


Fig. 7.3

In Fig. 7.3 a larger amount of charge produces a stronger field around it compared to that in Fig. 7.1

With the field concept, we then say that a charge placed in an electric field experiences a force because it interacts with the field. When two charges are placed near to one another, each charge interacts with the field of the other charge and thus they exert forces on each other. This pair of forces is a Newton's 3rd law pair.

Electric force arises from the interaction of a charge with the field it is in.

Also, in Fig. 7.4, the larger the amount of charge found in a field, the stronger the electric force F_E experienced by it.

For a charge q experiencing an electric field strength of E, the electric force $F_E = qE$.

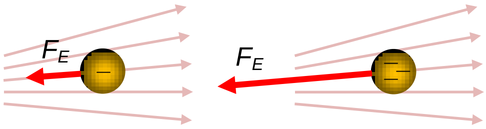


Fig. 7.4

Electric force $F_E = qE$.

8 Magnetic Force (Will be covered under the chapter Electromagnetism)

A magnetic field is a region of space in which a magnetic pole or current experiences a magnetic force.

Magnetic field can be produced by a magnet or current. A straight wire carrying current would produce a magnetic field as shown in Fig. 8.2. A coil of wires carrying current as shown in Fig. 8.3 would produce a magnetic field similar to that of a magnet as shown in Fig. 8.1.

A magnetic field is a region of space in which a magnetic pole or current experiences a magnetic force.

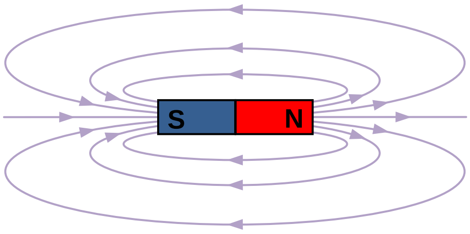


Fig. 8.1

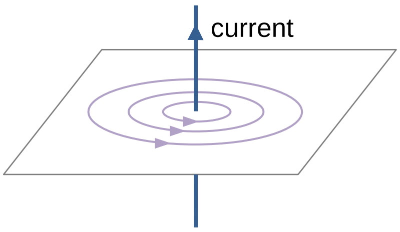


Fig. 8.2

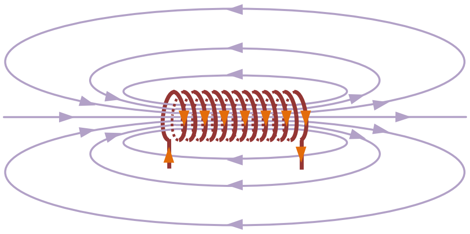


Fig. 8.3

9 Gravitational Force (Will be covered under the chapter Gravitation)

A gravitational field is a region of space in which a mass experiences a gravitational force.

A mass produces a field around it just like charges. However, there is only one type of mass and the field direction is always towards the mass.

When two masses are placed near to one another, each mass interacts with the field of the other mass and thus they exert gravitational forces on each other. This pair of forces is a Newton's 3rd law or action-reaction pair.

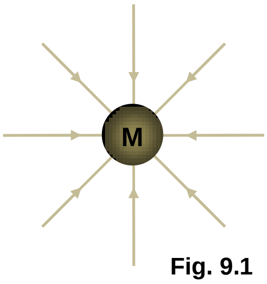


Fig. 9.1

A gravitational field is a region of space in which a mass experiences a gravitational force.

The stronger the field a given mass experiences, the stronger the gravitational pull on it. For a mass *m* experiencing a gravitational field strength of *g*, the gravitational force $F_G = mg$.

On the ground, the gravitational field strength *g* has a mean value of 9.81 N kg^{-1}. When you go to Mount Everest, about 8 km high, *g* will only be smaller by about 0.3%. However, for a GPS satellite 20 000 km high above ground, the *g* value is 0.573 N kg^{-1}.

Gravitational force arises from the interaction of a mass with the field it is in.

$F_G = mg$

10 Fundamental Forces

From sections 2 to 9, different kinds of forces were introduced. As time passed, physicists realised that many of those forces have the same origin. They identified four fundamental forces - electromagnetic force, gravitational force, nuclear weak force and nuclear strong force. By now, physicists have managed to unify electromagnetic and nuclear weak force into the electro-weak force. Of the four fundamental forces, the electromagnetic force is the origin of all the forces covered in sections 2 to 8.

11 Moment/Torque and Couple

Moment or Torque

Moment or *torque* is defined as the product of force and the perpendicular distance from a pivot.

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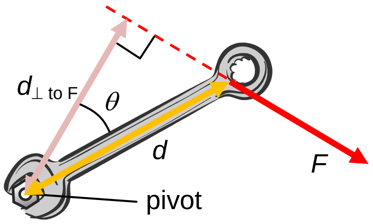


Fig. 11.1

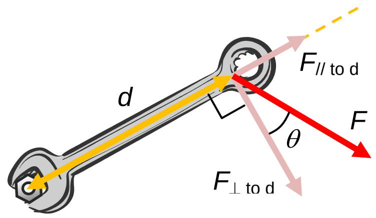


Fig. 11.2

Moment = F × perpendicular distance from pivot
$= Fd_{\perp \text{ to F}}$
$= F\,(d\cos\theta)$

$F_{\perp \text{ to d}} = F\cos\theta$ & $F_{// \text{ to d}} = F\sin\theta$

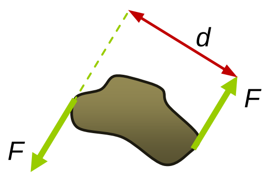
Moment $= F_{\perp \text{ to d}} \times d$
$= (F\cos\theta)\, d$

Fig. 11.2 shows an alternative way to calculate the moment. Here, the force is resolved into two components - $F_{// \text{ to d}}$ & $F_{\perp \text{ to d}}$. $F_{// \text{ to d}}$ does not contribute any moment since its line of action passes through the pivot and its perpendicular distance from the pivot is thus zero. $F_{\perp \text{ to d}}$ on the other hand has a perpendicular distance *d* from the pivot; hence the moment of *F* is due solely to the component $F_{\perp \text{ to d}}$.

A moment or torque tends to cause turning motion. If a *stationary* body is acted upon by two equal moments in opposite directions, the net moment is zero and the body will not turn. However, if there is a *net* moment the body will have an angular acceleration about the pivot.

Couple

A couple is defined as a pair of equal and opposite forces whose lines of action do not coincide. If there are no other forces except a couple applied to a body, there will be a turning moment of *Fd* regardless of the pivot point chosen for calculation, thus leading to angular acceleration or increasingly faster rotation. The couple gives rise to zero net force on the body; hence there is no linear acceleration.



Moment of the couple = *Fd*

Fig. 11.3

A couple is defined as a pair of equal and opposite forces whose lines of action do not coincide.

Moment of the couple = *Fd*

12 Equilibrium

As mentioned in the Introduction, being able to recognise the forces acting on a system is of utmost importance. After all, it is the forces which determine the type of motion or the lack of motion and they also determine the energy transfers via work done. Much more about the relationship between forces, motion and energy will be covered in the chapters Dynamics, Work Energy Power, Circular Motion and Oscillation.

It is very important to be able to recognise forces on a system and their characteristics.

In this section, we are interested in objects that are in equilibrium. That means no *linear* acceleration and *angular* acceleration for the object. Linear acceleration is just $\bar{a} = d\bar{v} / dt$, the same acceleration encountered in Kinematics. Angular acceleration is the rate change of *angular velocity*. Angular velocity is the rate of change of *angular displacement*. Angular displacement is the angle in radians or degrees swept out by a body that is in circular motion.

Just as linear acceleration is caused by a net force, angular acceleration is caused by a net moment or torque. Therefore, the conditions for zero linear and angular acceleration are zero net force and net moment respectively.

A system in equilibrium means it has no linear acceleration and angular acceleration.

A system is in equilibrium *if and only if* both its net force and net moment are zero.

Equilibrium	
Means	Conditions
1 No linear acceleration and 2 No angular acceleration	1 Net force = 0 along any direction and 2 Net moment = 0 about any pivot point

Equilibrium Conditions for Problem Solving

The two conditions/equations are useful when faced with a problem to find unknowns such as magnitude of a force, distance, angle or direction. Students who find it difficult to master the usage of the equations may refer to the following procedure which is adapted from a more general problem solving strategy:

1. Identify the system i.e. the object of focus or interest.
2. So that you can identify all the forces at work on the system.
3. Draw a diagram showing layout, known information, where the forces act and their directions.
4. Based on experience with the equations, decide which one you would use or perhaps you would use both. This decision can sometimes mean an easier path or much more tedious one to get the answer. Point is that you don't rush in but give some thought.
5. Execute the plan.

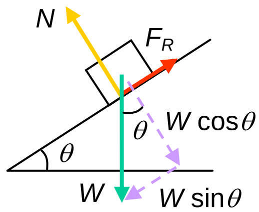
The examples below only show step 5.

Example - Using Net force = 0

Given a block of weight W resting on a slope with angle θ. Find the normal contact force N and the friction F_R.

Method I - Working with Components

(Step 1) Decide directions to resolve. Tip: resolve *one* force W ($\perp$ & // to slope) instead of *two* forces N & F_R horizontally and vertically.



(Step 2) F_{net} = 0 **along slope**

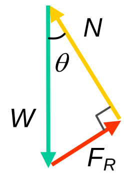
$\therefore F_R = W \sin\theta$ (Ans)

F_{net} = 0 **perpendicular to slope**

$\therefore N = W \cos\theta$ (Ans)

Method II - Working with Vector Diagrams

Net force = 0 ⇒ vector diagram will have all the force arrows joining head to tail giving a zero resultant.



(Step 1) Crucial to draw a correct vector diagram:
- arrows retain original directions
- join head to tail to form closed triangle/polygon

(Step 2) Use definitions of sin, cos, tan, Pythogoras, sine and cosine rules.
Using definitions of sin and cos here:
From diagram, $N = W \cos\theta$ (Ans)
$F_R = W \sin\theta$ (Ans)

Example - Using Net moment = 0

A rod of given weight W is held horizontal by a string and a force F at angle ϕ to rod. Find F.

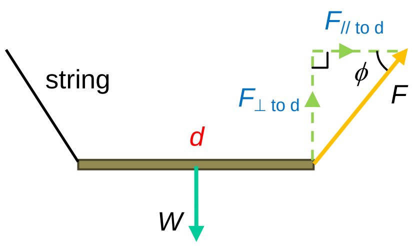
Method - CCW moment = CW moment

In using 'Counter clockwise moment = Clockwise moment', 2 skills are critical:

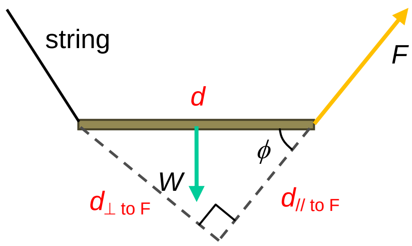
1. Choosing pivot point wisely.
2. Ability to find moment using either of 2 methods shown in Fig. 11.1 & 11.2.

(Step 1) Choosing the pivot to be the left end will ensure that unknown tension will not appear in the equation. See equations below:

(Step 2) Resolve the force.
Introduce unknown d but no problem as it will be cancelled.
CCW = CW moment
$(F \sin\phi)\, d = W (d/2)$
$F = W / (2 \sin\phi)$ (Ans)



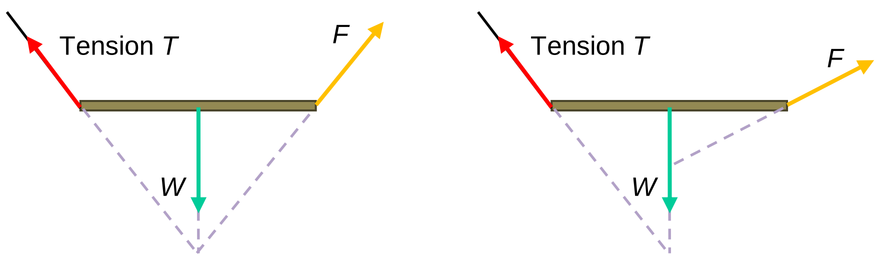
Alternatively,
(Step 2) Find and use $d_{\perp \text{ to F}}$.
CCW = CW moment
$F (d \sin\phi) = W (d/2)$
$F = W / (2 \sin\phi)$ (Ans)



Equilibrium of 3 Forces

In this special case where there are *only* 3 forces acting on a system; not more not less, the lines of action of the 3 forces must intersect at a point when the system is in equilibrium. Using the example of the rod above:

Special case: 3 forces in equilibrium must have their lines of action intersect at a point.



Correct directions **Fig. 12.1** Incorrect directions