	Class	Register Number
Name ______________________		

4052/01 **23/S4PR/EM/1**

MATHEMATICS PAPER 1

Wednesday **23 August 2023** **2 hours 15 minutes**

VICTORIA SCHOOL

PRELIMINARY EXAMINATION
SECONDARY FOUR

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question

If working is needed for any question, it must be shown with the answer.
The omission of essential working will result in loss of marks.
The total number of marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Gives answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

This paper consists of **18** printed pages, including the cover page.

[Turn over

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 Solve $7 - \frac{3x-4}{5} = 0$.

Answer $x =$ ..... [2]

---

2 **(a)** Express 24 and 2160 as a product of their prime factors. Leave your answers in index notation.

Answer $24 =$.....

$2160 =$..... [2]

(b) Find the smallest integer x such that the lowest common multiple of $24x$ and 2160 is 4320.

Answer $x =$ ..... [1]

---

3 **(a)** Calculate $150 - \dfrac{1.5^3 \times \left(-\dfrac{7}{8}\right)}{\sqrt[3]{6.01} + 6}$.

Write your answer correct to 4 significant figures.

Answer ..... [1]

(b) Write your answer to **part (a)** in standard form.

Answer ..... [1]

---

4 Expand and simplify $(3x+2y)(3x-5y)-(x-y)^2$.

Answer ..... [2]

---

5 y is inversely proportional to x^2. It is given that $y = 7$ for a particular value of x.
Find the value of y when this value of x is increased by 250%.

Answer ..... [2]

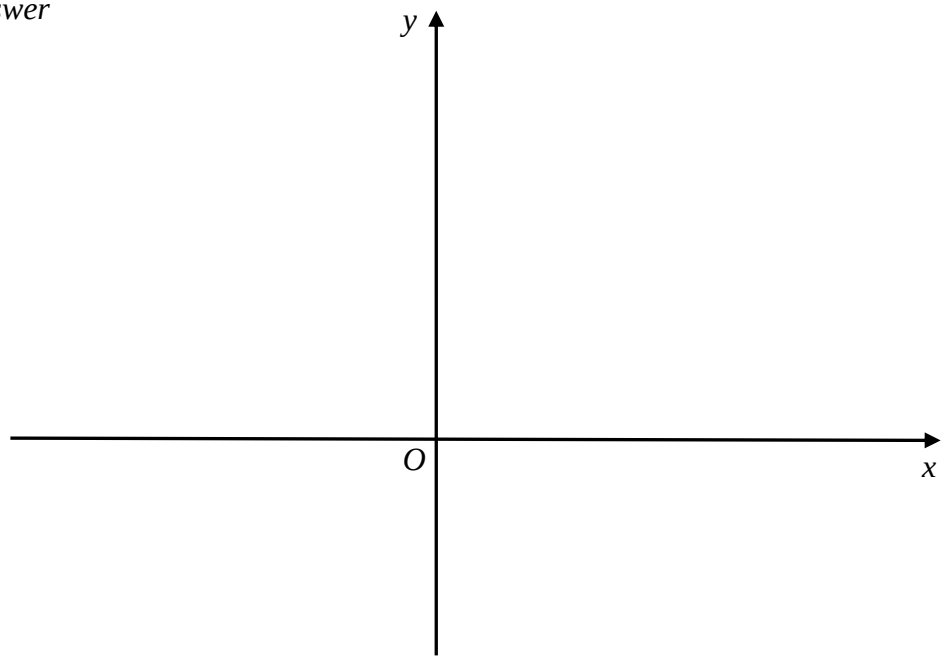
---

6 **(a)** Express $y = 12 + 4x - x^2$ in the form of $y = p - (x - q)^2$.

Answer ..... [2]

(b) Sketch the graph of $y = 12 + 4x - x^2$.
Indicate clearly the coordinates of the points where the graph crosses the axes (where applicable) and the maximum point on the curve.

Answer



[3]

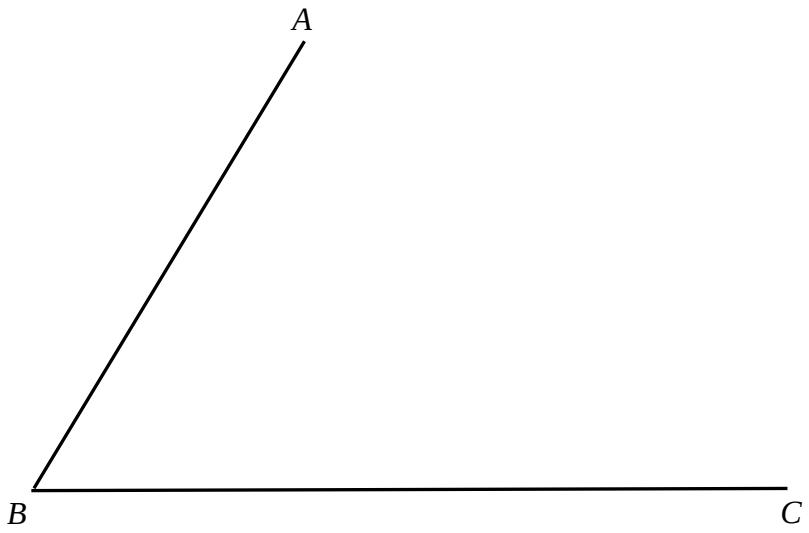
(c) Explain why the equation $12 + 4x - x^2 = 16.5$ does not have any solutions.

.....

.....

..... [1]

7 The diagram shows the positions of three points *A*, *B* and *C*.



(a) Construct the perpendicular bisector of *AB*. [1]

(b) Construct the bisector of angle *ABC*. [1]

(c) Point *P* is equidistant from *A* and *B* **and** equidistant from *AB* and *BC*.
Mark the point *P* on the diagram and measure the length *BP*.

Answer $BP =$ .....cm [1]

(d) Construct the parallelogram *ABCD* by completing the diagram. [1]

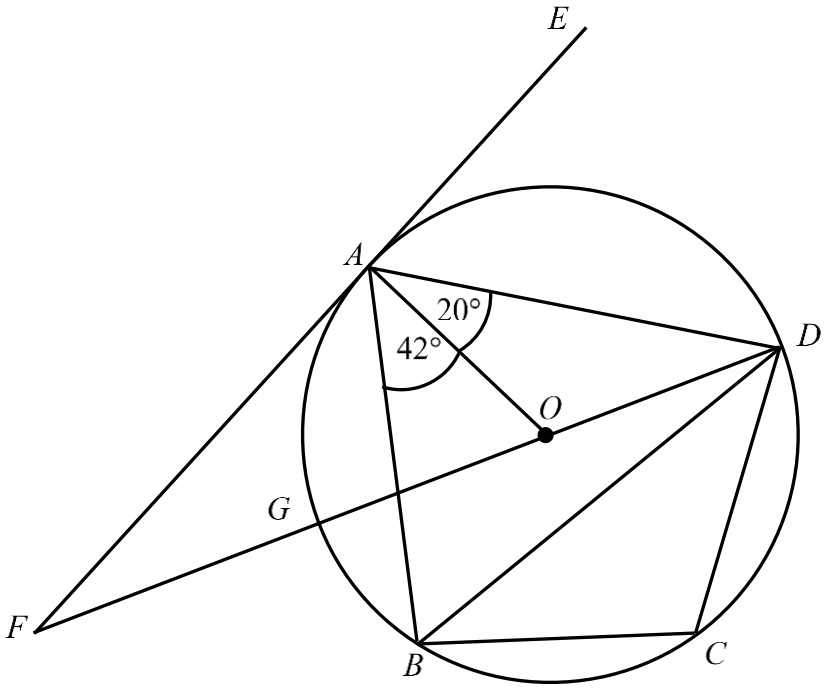
---

8 At an event, $\frac{1}{3}$ of the participants are children. 37.5% of the remaining participants are adults wearing spectacles and the remaining 50 participants are adults not wearing spectacles. Find the total number of participants at the event.

Answer ..... [2]

---

9 In the diagram, *A*, *B*, *C* and *D* are points on a circle, centre *O*. *EAF* is a tangent to the circle at *A*. *DOGF* is a straight line. Angle $OAD = 20°$ and angle $OAB = 42°$.



(a) Find angle *ABG*.
Give reasons for each step of your working.

Answer Angle *ABG* = ... ° [2]

(b) Show that angle $BCD = 118°$.
Give reasons for each step of your working.

.....

.....

.....[2]

10 Two bags *A* and *B* contain some blue and yellow marbles. The ratio of the number of blue marbles in bag *A* to bag *B* is 5 : 3. The ratio of the number of yellow marbles in bag *A* to bag *B* is 3 : 7. If both bags contain the same number of marbles, find the ratio of the number of blue marbles in bag *A* to the number of yellow marbles in bag *A*.

Answer ..... [3]

---

11 A group of 80 students took part in a challenge.
The table below shows the distribution of the time taken to complete the challenge.

Time (t minutes)	$30 < t \leq 40$	$40 < t \leq 50$	$50 < t \leq 60$	$60 < t \leq 70$	$70 < t \leq 80$
Number of students	12	32	23	9	4

(a) Calculate the estimate of the mean time taken.

Answer ..... mins [1]

(b) Calculate the estimate of the standard deviation of the time taken.

Answer ..... mins [1]

(c) The timer used to keep time was found to be inaccurate. The correct timing for each student was 2 minutes more than the recorded. Explain how the mean and the standard deviation of the time taken will be affected by this error.

.....

.....

..... [2]

---

12 Simplify $\frac{10x^2+17xy+3y^2}{4x^2-9y^2}$.

Answer ..... [2]

---

13 Solve the inequality $2-\frac{2x-1}{3} \leq \frac{x-2}{2} < \frac{3x+7}{4}$.

Answer ..... [3]

---

14 Simplify $\frac{\left(3x^{-1}y^2\right)^3}{5x^2y^4} \div \frac{2x^3y}{x^6y^{-2}}$, leaving your answer in positive indices.

Answer ..... [3]

---

15 $A(3,1)$, $B(3,5)$ and $C(-1,-3)$ are the vertices of a triangle.

Find

(a) the length of *AC*,

Answer ..... units [1]

(b) the equation of the line passing through *A* and parallel to *BC*,

Answer $y =$ ..... [2]

(c) the perpendicular distance from *B* to *AC*.

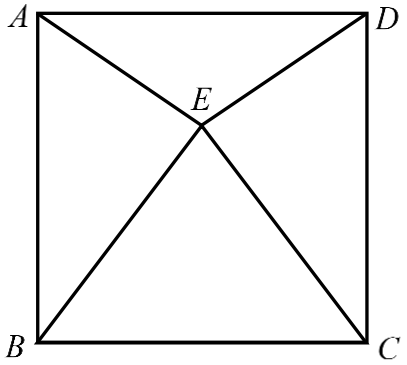
Answer ..... units [2]

---

16 A polygon has *n* sides. Two of its exterior angles are $55°$ and $65°$. The remaining $(n-2)$ interior angles are each $156°$. Calculate the value of *n*.

Answer $n =$ ..... [2]

17



In the above figure, *ABCD* is a square. Angle *EAD* = angle *EDA* = $x°$.
Show that triangle *AEB* and triangle *DEC* are congruent.
Give a reason for each statement you make.

Answer

[3]

---

18 Factorise completely,

(a) $12y - 10x - 40xy + 3$,

Answer ..... [2]

(b) $w^3 + 2w^2 - 16w - 32$.

Answer ..... [2]

---

19 A bakery shop sells three types of pastry packages, namely Regular, Supreme and Deluxe. The items in each of the pastry package are given in the table below.

	Blueberry tarts	Kiwi tarts	Apple tarts	Lemon tarts
Regular	10	6	8	5
Supreme	12	8	6	10
Deluxe	14	10	8	12

The cost price of a piece of Blueberry tart, Kiwi tart, Apple tart and Lemon tart is \$1.50, \$2.50, \$1.00 and \$2.00 respectively.

(a) Write down a 4×1 matrix **A** to represent the cost price of the pastries.

Answer **A** = [1]

(b) Write down a matrix **B** such that the product **BA** will give the cost price of each type of pastry package.

Answer **B** = [1]

(c) Evaluate **C = BA**.

Answer **C** = [1]

(d) Hadi buys 7 Regular, 3 Supreme and 2 Deluxe pastry packages.
Represent his purchase in a row matrix **D**.

Answer **D** = [1]

(e) Evaluate **E = DC**.

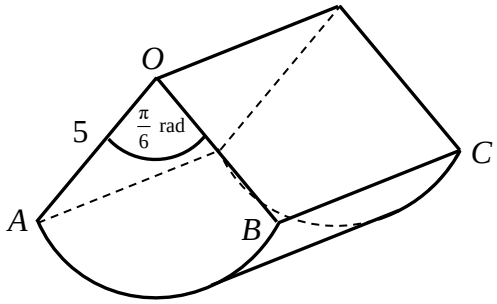
Answer **E** = [1]

(f) State what the element of matrix **E** represents.

.....

..... [1]

20



The diagram shows the cross-section of a prism which is a sector of a circle with centre O and radius 5 cm. Angle $AOB = \frac{\pi}{6}$ radians. It is given that the volume of the prism is 25π cm^3.

Find

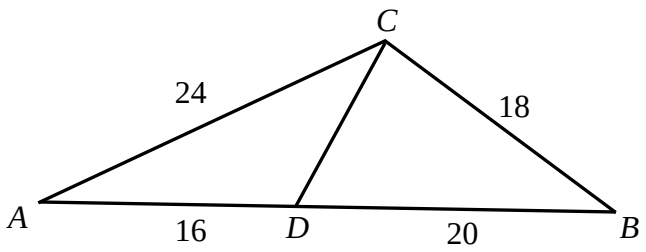
(a) the length BC,

Answer $BC =$ ..... cm [2]

(b) the total surface area of the prism.

Answer ..... cm^2 [3]

21



In the diagram, $AD = 16\,\text{cm}$, $AC = 24\,\text{cm}$, $BC = 18\,\text{cm}$ and $BD = 20\,\text{cm}$.

(a) Name a pair of similar triangles and show that they are similar.

.....

.....

.....

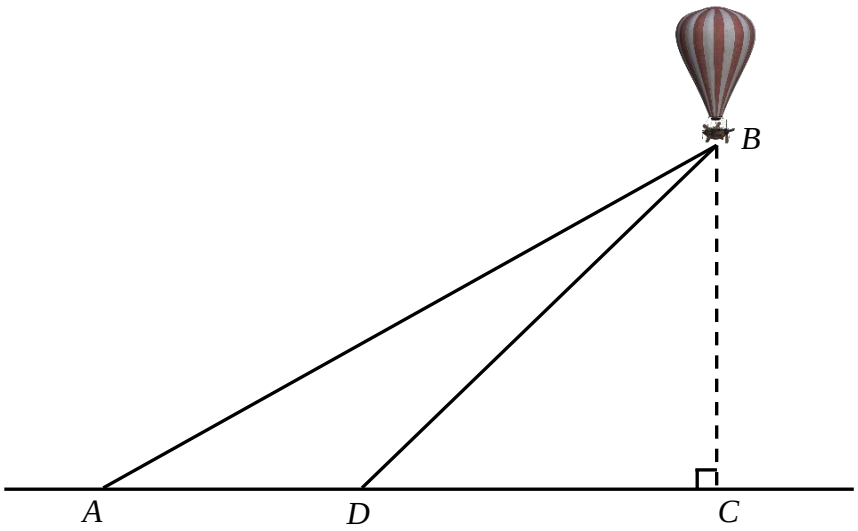
.....

..... [3]

(b) Find the length of CD.

Answer $CD =$ ..... cm [2]

22



In the diagram, *A* and *D* are two points on the ground, 350 m apart.
B is a hot-air balloon above a point *C* on the ground.
The angle of elevation of *B* from *A* is 32°.
The angle of depression of *D* from *B* is 55°.

Calculate the height of the hot-air balloon above the ground.

Answer ..... m [5]

23 The scale of a map of a housing estate is $1:50\,000$.

(a) A road on the map is 9 cm long. Calculate the actual length of the road, giving your answer in metres.

Answer ..... m [1]

(b) The housing estate occupies an area of 200 cm^2 on the map. On another map of scale $1:100\,000$, what is the area that will be occupied by the same housing estate? Give your answer in cm^2.

Answer ..... cm^2 [2]

24 **(a)** $\mathscr{E} = \{x : x \text{ is an integer between 20 and 30}\}$

$A = \{x : x \text{ is a multiple of 8}\}$

$B = \{x : x \text{ divided by 6 leaves a remainder of 3}\}$

(i) List the elements of A and B.

Answer $A =$ .....

$B =$ ..... [2]

(ii) Find $\text{n}(A \cup B)'$.

Answer ..... [1]

(b) The sets C, D and E satisfy the following three conditions:
$D \subset C$, $D \cap E = \varnothing$ and $C \cup E = C$.

Represent these sets on a Venn diagram.

Answer

[2]

25 In a sequence, the difference between the consecutive terms is constant.
The first five terms of the sequence are

$$-3 \qquad f \qquad g \qquad h \qquad 17.$$

(a) Find the values of f, g and h.

Answer $f =$ .....

$g =$ .....

$h =$ ..... [3]

(b) Write down an expression for the nth term of the sequence.

Answer ..... [1]

(c) Determine if 217 is a term of this sequence.

.....

.....

.....

.....

..... [1]

End of Paper

Class Register Number

Name Solutions

4052/01 **23/S4PR/EM/1**

MATHEMATICS PAPER 1

Wednesday **23 August 2023** **2 hours 15 minutes**

VICTORIA SCHOOL

PRELIMINARY EXAMINATION
SECONDARY FOUR

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
The number of marks is given in brackets [ ] at the end of each question or part question

If working is needed for any question, it must be shown with the answer.
The omission of essential working will result in loss of marks.
The total number of marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Gives answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

1 Solve $7-\frac{3x-4}{5}=0.$

$7-\frac{3x-4}{5}=0$

$35-(3x-4)=0$

$39-3x=0$

$\therefore\ x=13$

Answer $x=$ .....13..... [2]

---

2 **(a)** Express 24 and 2160 as a product of their prime factors. Leave your answers in index notation.

$24=2^3\times3$

$2160=2^4\times3^3\times5$

Answer $24=$ .....$2^3\times3$.....

$2160=$ .....$2^4\times3^3\times5$..... [2]

(b) Find the smallest integer x such that the lowest common multiple of $24x$ and 2160 is 4320.

$24x = 2^3\times3\times x$

$2160 = 2^4\times3^3\times5$

LCM $4320 = 2^5\times3^3\times5$

$\therefore$ the smallest integer $x=2^2=4.$

Answer $x=$ .....4..... [1]

---

3 **(a)** Calculate $150 - \dfrac{1.5^3 \times \left(-\dfrac{7}{8}\right)}{\sqrt[3]{6.01} + 6}$.

Write your answer correct to 4 significant figures.

Answer 150.4 [1]

(b) Write your answer to **part (a)** in standard form.

Answer 1.504×10^2 [1]

---

4 Expand and simplify $(3x+2y)(3x-5y)-(x-y)^2$.

$$
\begin{aligned}
&(3x+2y)(3x-5y)-(x-y)^2 \\
&= 9x^2 - 15xy + 6xy - 10y^2 - (x^2 - 2xy + y^2) \\
&= 9x^2 - 9xy - 10y^2 - x^2 + 2xy - y^2 \\
&= 8x^2 - 7xy - 11y^2
\end{aligned}
$$

Answer $8x^2 - 7xy - 11y^2$ [2]

---

5 y is inversely proportional to x^2. It is given that $y = 7$ for a particular value of x.
Find the value of y when this value of x is increased by 250%.

$y = \dfrac{k}{x^2}$, where k is a constant.

$\therefore k = yx^2$

$y_1 x_1^{\,2} = y_2 x_2^{\,2}$

$7x^2 = y_2 (3.5x)^2$

$= y_2 12.25x^2$

$\therefore y_2 = 7 \div 12.25$

$= \dfrac{4}{7}$

Answer $\dfrac{4}{7}$ [2]

---

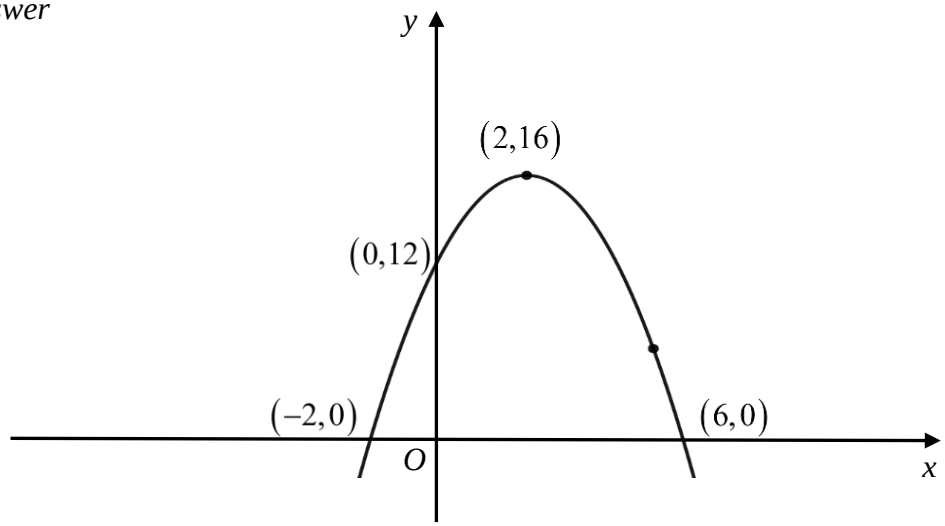
6 **(a)** Express $y = 12 + 4x - x^2$ in the form of $y = p - (x - q)^2$.

$$
\begin{aligned}
y &= 12 + 4x - x^2 \\
&= -x^2 + 4x + 12 \\
&= -\left[x^2 - 4x \right] + 12 \\
&= -\left[x^2 - 4x + (-2)^2 - (-2)^2 \right] + 12 \\
&= -\left[(x-2)^2 - 4 \right] + 12 \\
&= -(x-2)^2 + 16 \\
&= 16 - (x-2)^2
\end{aligned}
$$

Answer $y = 16 - (x-2)^2$ [2]

(b) Sketch the graph of $y = 12 + 4x - x^2$.
Indicate clearly the coordinates of the points where the graph crosses the axes (where applicable) and the maximum point on the curve.

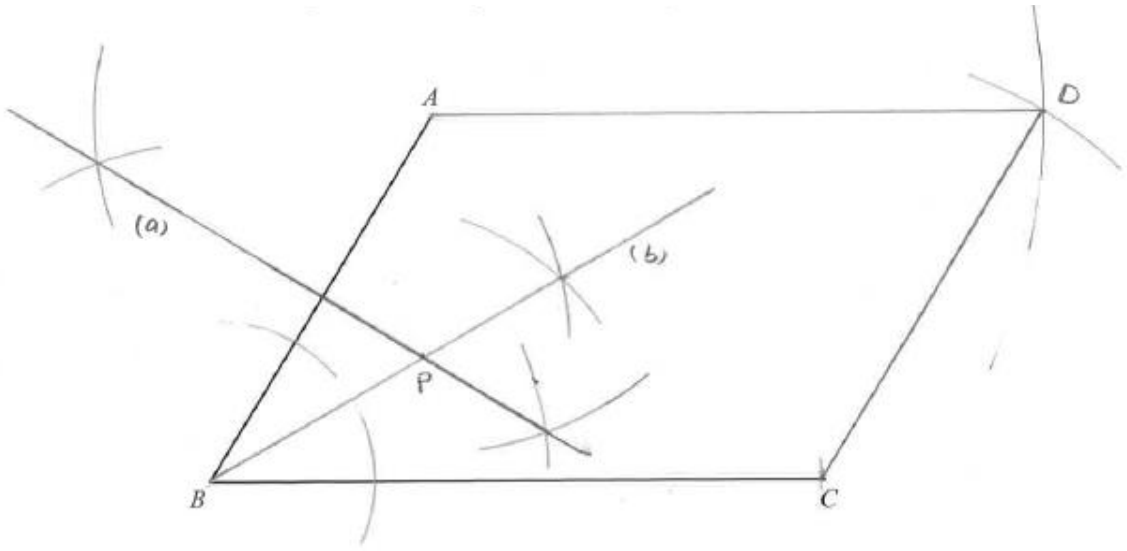
Answer



(c) Explain why the equation $12 + 4x - x^2 = 16.5$ does not have any solutions.

The graphs of $y = 12 + 4x - x^2$ and $y = 16.5$ do not intersect since the curve has a maximum point at $y = 16$. Hence the equation $12 + 4x - x^2 = 16.5$ does not have any solutions. [1]

7 The diagram shows the positions of three points *A*, *B* and *C*.



(a) Construct the perpendicular bisector of *AB*. [1]

(b) Construct the bisector of angle *ABC*. [1]

(c) Point *P* is equidistant from *A* and *B* **and** equidistant from *AB* and *BC*.
Mark the point *P* on the diagram and measure the length *BP*.

Answer $BP =$ 4 cm [1]

(d) Construct the parallelogram *ABCD* by completing the diagram. [1]

---

8 At an event, $\frac{1}{3}$ of the participants are children. 37.5% of the remaining participants are adults wearing spectacles and the remaining 50 participants are adults not wearing spectacles. Find the total number of participants at the event.

Let the total number of participants be *x*.

$$\therefore \frac{37.5}{100} \times \frac{2}{3}x + 50 = \frac{2}{3}x$$

$$\frac{1}{4}x + 50 = \frac{2}{3}x$$

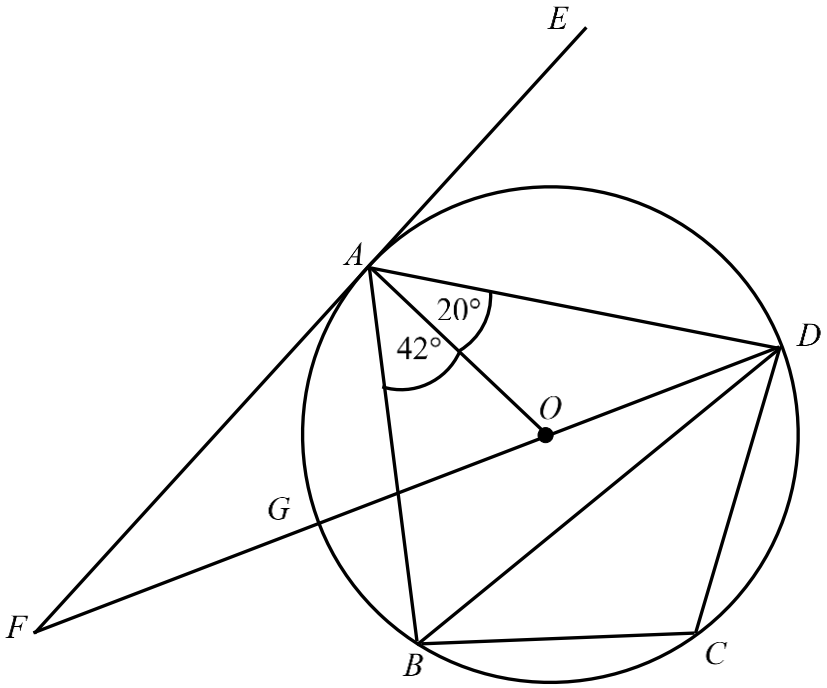
$$\frac{5}{12}x = 50$$

$$x = 120$$

Answer 120 [2]

---

9 In the diagram, *A*, *B*, *C* and *D* are points on a circle, centre *O*. *EAF* is a tangent to the circle at *A*. *DOGF* is a straight line. Angle $OAD = 20°$ and angle $OAB = 42°$.



(a) Find angle *ABG*.
Give reasons for each step of your working.

Since $OA = OD$ (radii of circle),
ΔOAD is isosceles.
$\angle ODA = \angle OAD$ (base $\angle$s of isosceles Δ)
$= 20°$
$\angle ABG = \angle ODA$ ($\angle$s in the same segment)
$= 20°$

Answer Angle *ABG* = 20 ° [2]

(b) Show that angle $BCD = 118°$.
Give reasons for each step of your working.

$\angle BAD = 42° + 20° = 62°$

$\angle BCD = 180° - 62°$ (opposite $\angle$s of a cyclic quad.)

$= 118°$ (shown)

[2]

10 Two bags *A* and *B* contain some blue and yellow marbles. The ratio of the number of blue marbles in bag *A* to bag *B* is 5 : 3. The ratio of the number of yellow marbles in bag *A* to bag *B* is 3 : 7. If both bags contain the same number of marbles, find the ratio of the number of blue marbles in bag *A* to the number of yellow marbles in bag *A*.

Let the number of blue marbles and number of yellow marbles in bag *A* be x and y respectively.

$$\therefore x + y = \frac{3}{5}x + \frac{7}{3}y$$

$$\frac{2}{5}x = \frac{4}{3}y$$

$$\therefore \frac{x}{y} = \frac{10}{3}$$

Hence, $x : y = 10 : 3$

Answer 10 : 3 [3]

---

11 A group of 80 students took part in a challenge.
The table below shows the distribution of the time taken to complete the challenge.

Time (t minutes)	$30 < t \leq 40$	$40 < t \leq 50$	$50 < t \leq 60$	$60 < t \leq 70$	$70 < t \leq 80$
Number of students	12	32	23	9	4

(a) Calculate the estimate of the mean time taken.

$$\text{Mean time} = \frac{\sum fx}{\sum f}$$

$$= \frac{4010}{80} = 50.125 \text{ mins}$$

Answer 50.125 mins [1]

(b) Calculate the estimate of the standard deviation of the time taken.

$$\text{Standard deviation of the time taken} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

$$= \sqrt{\frac{209\,600}{80} - \left(\frac{4010}{80}\right)^2} \approx 10.367 \approx 10.4 \text{ mins}$$

Answer 10.4 mins [1]

(c) The timer used to keep time was found to be inaccurate. The correct timing for each student was 2 minutes more than the recorded. Explain how the mean and the standard deviation of the time taken will be affected by this error.

The mean time taken will increase by 2 mins to 52.125 mins.

The standard deviation of the time taken will remain the same.

[2]

---

12 Simplify $\dfrac{10x^2+17xy+3y^2}{4x^2-9y^2}$.

$$\frac{10x^2+17xy+3y^2}{4x^2-9y^2}$$
$$=\frac{(2x+3y)(5x+y)}{(2x+3y)(2x-3y)}$$
$$=\frac{5x+y}{2x-3y}$$

Answer $\dfrac{5x+y}{2x-3y}$ ..... [2]

---

13 Solve the inequality $2-\dfrac{2x-1}{3}\le\dfrac{x-2}{2}<\dfrac{3x+7}{4}$.

$$2-\frac{2x-1}{3}\le\frac{x-2}{2}<\frac{3x+7}{4}$$

$2-\dfrac{2x-1}{3}\le\dfrac{x-2}{2}$ and $\dfrac{x-2}{2}<\dfrac{3x+7}{4}$

$12-2(2x-1)\le 3(x-2)$ $\quad$ $2(x-2)<3x+7$

$12-4x+2\le 3x-6$ $\quad$ $2x-4<3x+7$

$-7x\le -20$ $\quad$ $-x<11$

$x\ge 2\dfrac{6}{7}$ $\quad$ $x>-11$

$\therefore\ x\ge 2\dfrac{6}{7}$

Answer $x\ge 2\dfrac{6}{7}$ ..... [3]

---

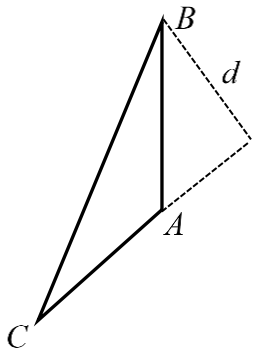
14 Simplify $\dfrac{\left(3x^{-1}y^2\right)^3}{5x^2y^4}\div\dfrac{2x^3y}{x^6y^{-2}}$, leaving your answer in positive indices.

$$\frac{\left(3x^{-1}y^2\right)^3}{5x^2y^4}\div\frac{2x^3y}{x^6y^{-2}}$$
$$=\frac{27x^{-3}y^6}{5x^2y^4}\times\frac{x^6y^{-2}}{2x^3y}$$
$$=\frac{27x^3y^4}{10x^5y^5}$$
$$=\frac{27}{10x^2y}$$

Answer $\dfrac{27}{10x^2y}$ ..... [3]

---

15 $A(3,1)$, $B(3,5)$ and $C(-1,-3)$ are the vertices of a triangle.



Find

(a) the length of *AC*,

$$\text{Length of } AC = \sqrt{(3+1)^2 + (1+3)^2}$$
$$= \sqrt{32}$$
$$\approx 5.66 \text{ units}$$

Answer 5.66 units [1]

(b) the equation of the line passing through *A* and parallel to *BC*,

$$\text{Gradient of } BC = \frac{-3-5}{-1-3} = 2$$
$\therefore$ eqn of line is,
$$y-1 = 2(x-3)$$
$$= 2x-6$$
$$y = 2x-5$$

Answer $y =$ $2x-5$ [2]

(c) the perpendicular distance from *B* to *AC*.

Let the perpendicular distance from *B* to *AC* be *d* units.
$$\frac{1}{2} \times AC \times d = \frac{1}{2} \times AB \times (3+1)$$
$$\sqrt{32} \times d = (5-1) \times (3+1)$$
$$d = \frac{16}{\sqrt{32}} \approx 2.83$$

Answer 2.83 units [2]

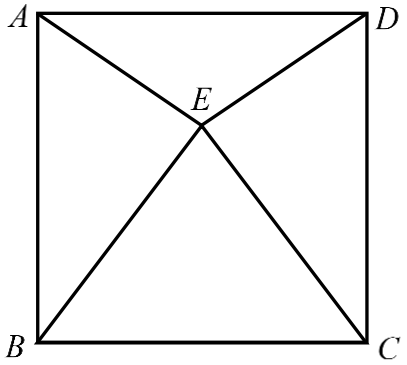
---

16 A polygon has *n* sides. Two of its exterior angles are $55°$ and $65°$. The remaining $(n-2)$ interior angles are each $156°$. Calculate the value of *n*.

Total sum of exterior angles $= 360°$
$$55° + 65° + (n-2)(180° - 156°) = 360°$$
$$120° + (n-2)(24°) = 360°$$
$$120 + 24n - 48 = 360$$
$$n = 12$$

Answer $n =$ 12 [2]

17



In the above figure, *ABCD* is a square. Angle *EAD* = angle *EDA* = $x°$.
Show that triangle *AEB* and triangle *DEC* are congruent.
Give a reason for each statement you make.

Answer

$\angle EAB = 90° - x°$ (complementary $\angle$s)

$\angle EDC = 90° - x°$ (complementary $\angle$s)

Hence, $\angle EAB = \angle EDC$

Since $\angle EAD = \angle EDA$ (given)

Using angle property of base angles of isosceles Δ, ΔEAD is isosceles.

Hence, $EA = ED$.

$AB = DC$ (sides of square are equal)

$\therefore \Delta AEB \equiv \Delta DEC$ (SAS)

[3]

---

18 Factorise completely,

(a) $12y - 10x - 40xy + 3$,

$$\begin{aligned} 12y - 10x - 40xy + 3 &= 12y + 3 - 10x - 40xy \\ &= 3(4y + 1) - 10x(1 + 4y) \\ &= (3 - 10x)(4y + 1) \end{aligned}$$

Answer $(3-10x)(4y+1)$ ..... [2]

(b) $w^3 + 2w^2 - 16w - 32$.

$$\begin{aligned} & w^3 + 2w^2 - 16w - 32 \\ &= w^2(w + 2) - 16(w + 2) \\ &= (w^2 - 16)(w + 2) \\ &= (w + 4)(w - 4)(w + 2) \end{aligned}$$

Answer $(w+4)(w-4)(w+2)$ ..... [2]

---

19 A bakery shop sells three types of pastry packages, namely Regular, Supreme and Deluxe. The items in each of the pastry package are given in the table below.

	Blueberry tarts	Kiwi tarts	Apple tarts	Lemon tarts
Regular	10	6	8	5
Supreme	12	8	6	10
Deluxe	14	10	8	12

The cost price of a piece of Blueberry tart, Kiwi tart, Apple tart and Lemon tart is \$1.50, \$2.50, \$1.00 and \$2.00 respectively.

(a) Write down a 4×1 matrix $\mathbf{A}$ to represent the cost price of the pastries.

Answer $\mathbf{A} = \begin{pmatrix} 1.50 \\ 2.50 \\ 1 \\ 2 \end{pmatrix}$ [1]

(b) Write down a matrix $\mathbf{B}$ such that the product $\mathbf{BA}$ will give the cost price of each type of pastry package.

Answer $\mathbf{B} = \begin{pmatrix} 10 & 6 & 8 & 5 \\ 12 & 8 & 6 & 10 \\ 14 & 10 & 8 & 12 \end{pmatrix}$ [1]

(c) Evaluate $\mathbf{C = BA}$.

$$\mathbf{C} = \mathbf{BA} = \begin{pmatrix} 10 & 6 & 8 & 5 \\ 12 & 8 & 6 & 10 \\ 14 & 10 & 8 & 12 \end{pmatrix} \begin{pmatrix} 1.50 \\ 2.50 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 48 \\ 64 \\ 78 \end{pmatrix}$$

Answer $\mathbf{C} = \begin{pmatrix} 48 \\ 64 \\ 78 \end{pmatrix}$ [1]

(d) Hadi buys 7 Regular, 3 Supreme and 2 Deluxe pastry packages. Represent his purchase in a row matrix $\mathbf{D}$.

Answer $\mathbf{D} = \begin{pmatrix} 7 & 3 & 2 \end{pmatrix}$ [1]

(e) Evaluate $\mathbf{E = DC}$.

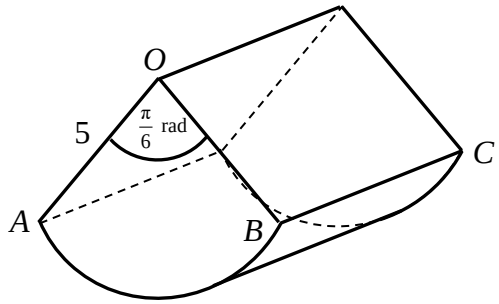
$$\mathbf{E} = \mathbf{DC} = \begin{pmatrix} 7 & 3 & 2 \end{pmatrix} \begin{pmatrix} 48 \\ 64 \\ 78 \end{pmatrix} = \begin{pmatrix} 684 \end{pmatrix}$$

Answer $\mathbf{E} = \begin{pmatrix} 684 \end{pmatrix}$ [1]

(f) State what the element of matrix **E** represents.

The element of matrix **E** represents the total cost price of Hadi's purchase of the 3 different pastry packages. [1]

20



The diagram shows the cross-section of a prism which is a sector of a circle with centre *O* and radius 5 cm. Angle $AOB = \frac{\pi}{6}$ radians. It is given that the volume of the prism is 25π cm^3.

Find

(a) the length *BC*,

Volume of prism $= 25\pi$ cm^3

$\frac{1}{2} \times (5)^2 \times \frac{\pi}{6} \times BC = 25\pi$

$\frac{25}{12} \times BC = 25$

$BC = 12$ cm

Answer $BC =$ 12 cm [2]

(b) the total surface area of the prism.

Total surface area of the prism

$= 2 \times \left[\frac{1}{2}(5)^2\left(\frac{\pi}{6}\right)\right] + 2(5 \times 12) + \left(5 \times \frac{\pi}{6}\right) \times 12$

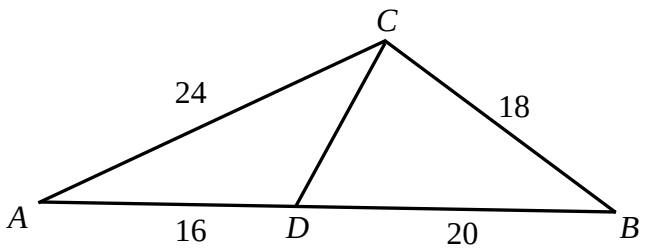
$= \frac{85}{6}\pi + 120$

≈ 164.5059

≈ 165 cm^2

Answer 165 cm^2 [3]

21



In the diagram, $AD = 16\,\text{cm}$, $AC = 24\,\text{cm}$, $BC = 18\,\text{cm}$ and $BD = 20\,\text{cm}$.

(a) Name a pair of similar triangles and show that they are similar.

ΔDAC and ΔCAB are similar.

$\frac{DA}{CA} = \frac{16}{24} = \frac{2}{3}$ and $\frac{AC}{AB} = \frac{24}{16+20} = \frac{24}{36} = \frac{2}{3}$

Hence, $\frac{DA}{CA} = \frac{AC}{AB}$.

$\angle DAC = \angle CAB$ (common $\angle$)

Hence, ΔDAC is similar to ΔCAB. (SAS Similarity Test)

[3]

(b) Find the length of CD.

Since ΔDAC is similar to ΔCAB,

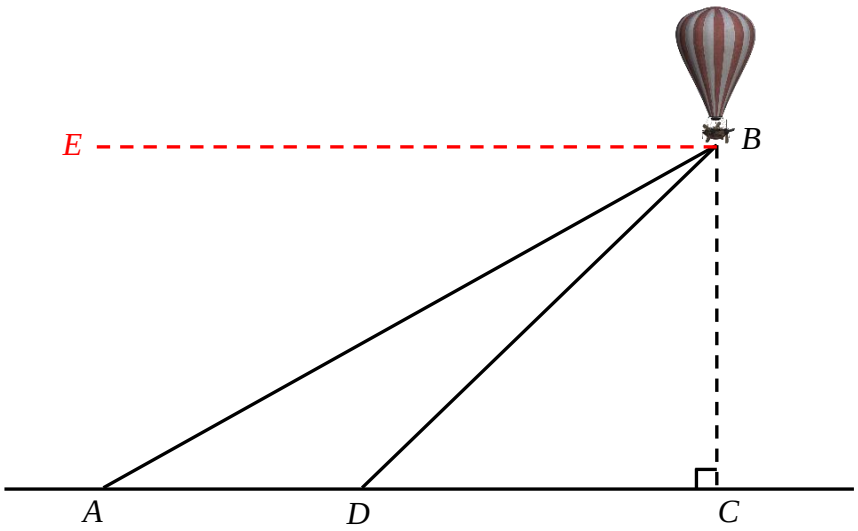
$\frac{CD}{BC} = \frac{AC}{AB}$

$\frac{CD}{18} = \frac{2}{3}$

$CD = 12$ cm

Answer $CD =$ 12 cm [2]

22



In the diagram, A and D are two points on the ground, 350 m apart.
B is a hot-air balloon above a point C on the ground.
The angle of elevation of B from A is $32°$.
The angle of depression of D from B is $55°$.

Calculate the height of the hot-air balloon above the ground.

$\angle EBA = \angle BAD$ (Alt. $\angle$s, $EB // AC$)

$= 32°$

$\angle ABD = 55° - 32°$

$= 23°$

In ΔABD, using sine rule,

$$\frac{BD}{\sin 32°} = \frac{350}{\sin 23°}$$

$$BD = \frac{350 \sin 32°}{\sin 23°}$$

≈ 474.6787

$\angle BDC = \angle EBD$ (Alt. $\angle$s, $EB // AC$)

$= 55°$

$$\sin \angle BDC = \frac{BC}{BD}$$

$BC = BD \sin 55°$

$= 474.6787 \sin 55°$

≈ 388.834

≈ 389 m

$\therefore$ the height of the hot-air balloon above ground ≈ 389 m

Answer 389 m [5]

23 The scale of a map of a housing estate is $1:50\,000$.

(a) A road on the map is 9 cm long. Calculate the actual length of the road, giving your answer in metres.

$\text{Scale} = 1:50\ 000$

$= 1 \text{ cm} : 50\ 000 \text{ cm}$

$= 1 \text{ cm} : 500 \text{ m}$

$\therefore$ actual length of the road $= 9 \times 500$

$= 4500 \text{ m}$

Answer 4500 m [1]

(b) The housing estate occupies an area of 200 cm^2 on the map. On another map of scale $1:100\,000$, what is the area that will be occupied by the same housing estate? Give your answer in cm^2.

Area scale of 1st map $= 1 \text{ cm}^2 : 0.25 \text{ km}^2$

$\therefore$ actual area of the housing estate $= 200 \times 0.25$

$= 50 \text{ km}^2$

Scale of the 2nd map $= 1:100\ 000$

$= 1 \text{ cm} : 1000 \text{ m}$

$= 1 \text{ cm} : 1 \text{ km}$

Area scale of the 2nd map $= 1 \text{ cm}^2 : 1 \text{ km}^2$

$\therefore$ map area of the housing area on the 2nd map $= \frac{50}{1} = 50 \text{ cm}^2$

Answer 50 cm^2 [2]

24 **(a)** $\mathscr{E} = \{x : x \text{ is an integer between 20 and 30}\}$

$A = \{x : x \text{ is a multiple of 8}\}$

$B = \{x : x \text{ divided by 6 leaves a remainder of 3}\}$

(i) List the elements of A and B.

Answer $A = \{24\}$

$B = \{21, 27\}$ [2]

(ii) Find $\text{n}(A \cup B)'$.

$A \cup B = \{21, 24, 27\}$

$(A \cup B)' = \{22, 23, 25, 26, 28, 29\}$

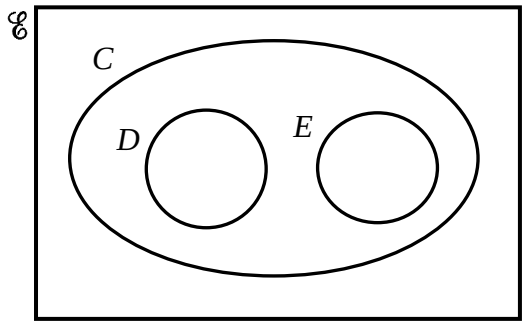
$\therefore \text{n}(A \cup B)' = 6$

Answer 6 [1]

(b) The sets C, D and E satisfy the following three conditions:
$D \subset C$, $D \cap E = \varnothing$ and $C \cup E = C$.

Represent these sets on a Venn diagram.

Answer



[2]

25 In a sequence, the difference between the consecutive terms is constant.
The first five terms of the sequence are

$$-3 \qquad f \qquad g \qquad h \qquad 17.$$

(a) Find the values of f, g and h.

Let the difference between consecutive terms be d.

$-3+4d=17$

$d=5$

$\therefore f=-3+5=2$

$g=2+5=7$

$h=7+5=12$

Answer $f=$ 2

$g=$ 7

$h=$ 12 [3]

(b) Write down an expression for the nth term of the sequence.

nth term in the sequence $=-8+5n$

Answer $-8+5n$ [1]

(c) Determine if 217 is a term of this sequence.

$-8+5n=217$

$\therefore n=45$

Since n is a positive integer, 217 is a term of this sequence.

[1]

End of Paper