VICTORIA JUNIOR COLLEGE

JC2 COMMON TEST 2023

CANDIDATE NAME	

CLASS		INDEX NUMBER	

H2 MATHEMATICS **9758/01**
Paper 1 **3 hours**

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)
Writing paper

READ THESE INSTRUCTIONS FIRST

Write your class and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Write your answers in the spaces provided in the question paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved graphing calculator is expected, where appropriate.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

For Examiner's Use	
Question Number	Marks Obtained
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
Total Marks	

Section A: Pure Mathematics (60 marks)

1 Express $\dfrac{x-7}{x^2-3x+2}-1$ as a single simplified fraction. [1]

Hence, without using a calculator, solve the inequality $\dfrac{x-7}{x^2-3x+2}<1$. [4]

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2 The curve C is given by the equation $\mathrm{e}^y = 1 + 3^{x-a}$, where a is a real constant.

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\ln k}{1 + 3^{a-x}},$$

where k is a constant to be determined. [3]

(b) Find the value of a such that T, the tangent to C at $x = 0$, makes an angle of 45° with the positive x-axis and hence find the equation of T. [4]

3 Using the standard series from the List of Formulae (MF26), show that the series expansion for $\ln(\sec x)$ in ascending powers of x, up to and including the term in x^4, is $\dfrac{x^2}{2} + \dfrac{x^4}{12}$. [4]

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3 [Continued]

By putting $x = \frac{\pi}{4}$, show that an approximate value for ln 2 is given by $\frac{\pi^2}{m} + \frac{\pi^4}{n}$, where m and n are integers to be found. [2]

Deduce the series expansion for $\tan x$, up to and including the term in x^3. [2]

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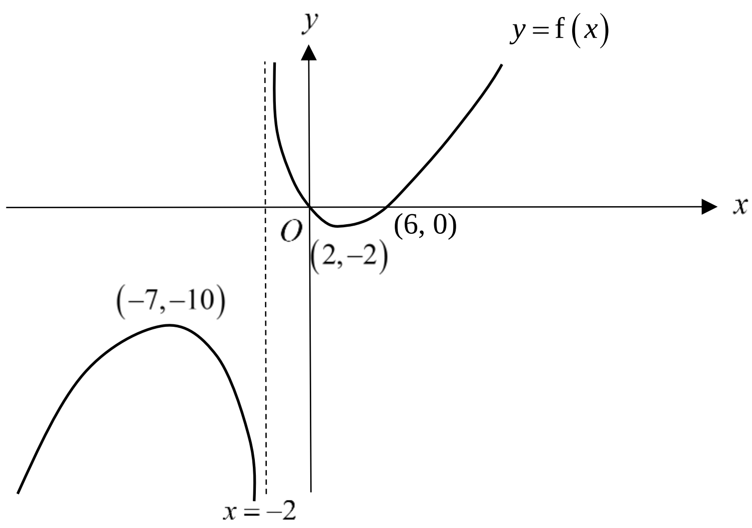
4 The diagram below shows a sketch of the curve $y = \mathrm{f}(x)$. The curve cuts the x-axis at the origin and $(6, 0)$. It has turning points at $(-7, -10)$ and $(2, -2)$, and an asymptote with the equation $x = -2$.



(a) Sketch including the points of intersection with the axes, turning points and equations of asymptotes, if any, the following graphs

(i) $y = \mathrm{f}(3 - x)$, [3]

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4 **[Continued]**

(ii) $y = \dfrac{1}{\mathrm{f}(x)}$. [3]

(b) It is given that

$$a(x-2)^2 = a - \frac{1}{\left[\mathrm{f}(x)\right]^2},$$

where a is a positive constant. By sketching a suitable graph on the diagram in part **(a)(ii)**, find the range of values of a such that the equation has 2 real distinct roots. [3]

5 Plane π_1 contains the line with equation $x+1=\frac{z-2}{3}$, $y=3$ and the point A with position vector $-3\mathbf{i}+\mathbf{j}$.

(a) Show that π_1 is perpendicular to $\begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$ and find the cartesian equation of π_1. [3]

(b) The perpendicular to π_1 from the point B with position vector $5\mathbf{i}-6\mathbf{j}+10\mathbf{k}$ meets π_1 at the point N. Find the position vector of N. [3]

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5 [Continued]

(c) Find a vector equation of the line which is a reflection of the line *BA* in the line *BN*. [3]

[Turn over

6 The function f is defined by

$$\mathrm{f}: x \rightarrow \frac{x^2 - x + 19}{x + 2}, \; x \in \mathbb{R}, \; x \neq -2.$$

(a) Sketch the graph of $y = \mathrm{f}(x)$, stating the equation of any asymptotes and the coordinates of any points where $y = \mathrm{f}(x)$ crosses the axes and of any turning points. [4]

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6 **[Continued]**

The function g is defined by

$$g : x \to \left|\frac{3}{x+1}\right| - 1,\ x \in \mathbb{R},\ x \neq -1\,.$$

(b) Show that gf exists and find the range of gf. [3]

(c) It is given that gg exists and

$$\mathrm{gg}(x) = \begin{cases} \mathrm{p}(x) & , \quad x < a \\ \mathrm{q}(x) & , \quad x > a \end{cases}$$

where p(x) and q(x) are polynomials and a is a constant. Find p(x), q(x) and the value of a. [3]

7 Mr Eccles initially weighs 115 kg. He participates in a new healthy keto diet programme. Under the programme, his calorie intake is C calories per day. The calories are used partly to meet his daily energy needs. A particular study found that an average male requires 36 calories per kg of his body mass to meet his daily energy needs. The rate of change of the body mass is proportional to the number of excess calories per day which is calculated by subtracting the calories to meet his daily energy needs from his calorie intake per day.

(a) Justify why $C = 4140$ if Mr Eccles' body mass is maintained constant at 115 kg. [1]

Mr Eccles' body mass at time t days after the start of the healthy keto diet programme is m kg.

(b) Mr Eccles targets to bring his body mass down to 85 kg by reducing his daily calorie intake to 3200 calories per day. Write down a differential equation relating m and t. Given that he loses 4 kg after 20 days, solve the differential equation to obtain m as a function of t. [6]

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7 **[Continued]**

(c) Sketch the graph of m against t. Hence explain whether Mr Eccles is able to meet his target body mass of 85 kg. [3]

(d) Deduce the greatest integer value for Mr Eccles' daily calorie intake if he wishes to meet his target of 85 kg body mass in the long run. [2]

Section B: Statistics (40 marks)

8 In the game "Break the Code", a player has to guess the mystery code that starts and ends with a letter with three digits in between them. Each of the two letters is equally likely to be any of the twenty-six letters of the alphabet A – Z while each of the three digits is equally likely to be any of the nine digits 1 – 9.

Find the probability that the code

(a) starts and ends with the same letter or has exactly two digits the same or both, [4]

(b) consists of exactly one E and exactly one odd digit. [3]

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9 'V&Vs' cookies are produced in large quantities and are sold in boxes of 12. Each box is made up of randomly chosen cookies of different flavours. On average 20% of V&Vs are coffee-flavoured.

(a) Explain why a binomial distribution is appropriate for modelling the number of coffee-flavoured cookies in a box. [2]

(b) Find the probability that a randomly chosen box of V&Vs contains at least four but no more than eight coffee-flavoured cookies. [2]

(c) Fifty boxes of cookies are randomly selected. Find the probability that the mean number of coffee-flavoured cookies per box is at least 2.5. [3]

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(d) On average the proportion of V&Vs that are matcha-flavoured is p. It is known that the modal number of matcha-flavoured cookies in a box is 3. Use this information to find the exact range of values that p can take. [4]

10 A four-sided biased die has its faces marked with the numbers 2, 3, 5 and 8. When the die is tossed, the probability of getting 2, 3, 5 and 8 are $\frac{1}{6}$, a, $\frac{2}{5}$, b respectively. The die is tossed repeatedly and the mean number obtained is $\frac{43}{10}$.

(a) Find the values of a and b, leaving your answers as exact fractions in their simplest form. [3]

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10 [Continued]

A game is played where a player tosses two such dice simultaneously. The number on each die is recorded and the player's score X is defined as the absolute difference of the two numbers.

(b) The table below shows the probability distribution of X, where q, r and s are constants.

x	0	1	2	3	5	6
$\mathrm{P}(X = x)$	q	$\frac{1}{10}$	r	s	$\frac{2}{25}$	$\frac{2}{45}$

Find the exact values of q, r and s. [3]

(c) Find $\mathrm{P}(X < 3.5)$. [1]

Tom pays $\$k$ to play this game and he receives an amount (in dollars) corresponding to twice his score obtained. Determine the range of values of k if Tom is expected to win some money in this game. [3]

11 **In this question you should state clearly the values of the parameters of any normal distribution you use.**

The mass of the fruit in a randomly chosen tin has a normal distribution with mean 300 grams and standard deviation 10 grams.

(a) Sketch the distribution for the mass of the fruit between 260 grams and 340 grams. [2]

(b) The random variable $\overline{F}$ denotes the mean mass (in grams) of fruit per tin in a random sample of n tins. Given that $P(\overline{F} > 301) = 0.233$, find the value of n. [3]

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11 [Continued]

The total mass of the fruit and the juice in a tin has a normal distribution with mean 400 grams and the standard deviation 8 grams.

(c) Find the probability that the total mass of fruit in 4 randomly chosen tins differ from three times the total mass of the fruit and the juice in a randomly chosen tin by at most 10 grams. [3]

(d) Find the probability that the mass of the juice in a randomly chosen tin is greater than 110 grams. [3]

State an assumption needed for your calculations in parts **(c)** and **(d)**. [1]

[Turn over

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