Topic 3: Dynamics

Part 1: Newton's Laws of Motion

1. A footballer accelerates a football horizontal from rest to a speed of 10 m s^{-1} during the time in which his toe is in contact with the ball for 0.20 s. If the football has a mass of 0.50 kg, what average force does the footballer exert on the ball? [25 N]

$$\langle F\rangle = m\langle a\rangle = m\left(\frac{\Delta v}{\Delta t}\right) = (0.50)\frac{(10-0)}{0.20} = 25\text{ N}$$

2. The air exerts a forward force of 10 N on the propeller of a 0.20 kg model plane. If the plane accelerates forward at 2.0 ms^{-2}, what is the magnitude of the resistive force exerted by air on the airplane? [9.6 N]

$$F = ma \Rightarrow (10-D) = (0.20)(2.0) \Rightarrow D = 10 - 0.40 = 9.6\text{ N}$$

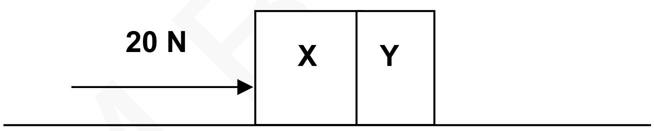
3. A 5.0 g bullet leaves the muzzle of a rifle with a speed of 320 ms^{-1}. What net force (assumed constant) is exerted on the bullet while it is moving down the 0.82 m long barrel? [312 N]

The bullet has constant acceleration *a* down the barrel due to constant net force.

$$v^2 = u^2 + 2aS \Rightarrow 320^2 = 0^2 + 2a(0.82) \Rightarrow a = 6.24\times10^4\text{ m s}^{-2}$$

$$F = ma = (5.0\times10^{-3})(6.24\times10^4) = 312\text{ N}$$

4. Two blocks, X and Y, of masses 3 kg and 2 kg respectively, are accelerated along a floor by a force 20 N applied to block X, as shown in the diagram. Given that the frictional forces that the floor acts on X and Y are 6 N and 4 N respectively,



(a) Draw free body diagrams showing how the forces act on X and Y.
(b) Base on Newton's second law, write three equations that govern the motion of the objects
(c) What are their acceleration(s)? [2 ms^{-2}]
(d) What is the magnitude of the force exerted by block Y on block X? [8 N]

Contact forces

Two solids are actually subjected to a pressure over the area where they are in contact. It is hard to calculate the pressure distribution, so instead we replace the pressure by a statically equivalent force. If the two contacting surfaces are flat, then

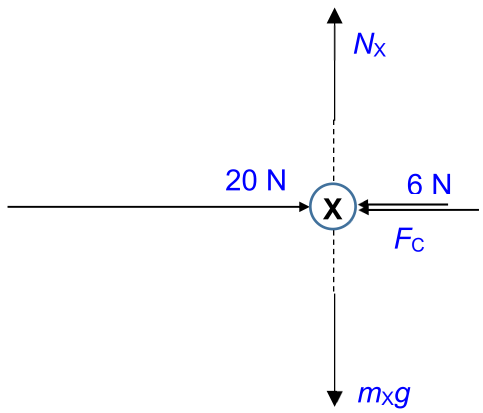
(1) The reaction force can be modelled as a single force, with no moment

(2) The force can act anywhere within the area of contact (its actual position is determined by force and moment balance)

(3) The force must be perpendicular to the two surfaces

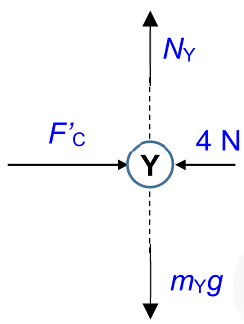
(4) The force acts to repel the two solids.

(a)



N_X is the normal reaction acted by the floor on X.

(F_C is the force exerted by Y on X,
F'_C is the force exerted by X on Y.
F_C and F'_C is a third-law pair.)



N_Y is the normal reaction acted by the floor on X.

(b), (c) & (d)

Isolating X,

$20 - 6 - F_C = (3)a$ (1)

$\Rightarrow F_C = 14 - 3a$

Isolating Y,

$F_C - 4 = (2)a$ (2)

$\Rightarrow F_C = 4 + 2a$

Isolating both X & Y,

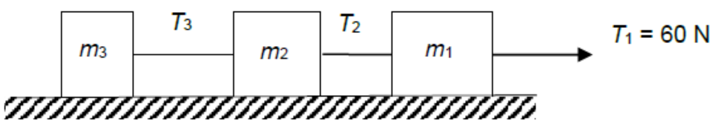
$20 - 6 - 4 = (3 + 2)a$ (3)

$\Rightarrow a = \dfrac{10}{5} = 2 \text{ m s}^{-2}$

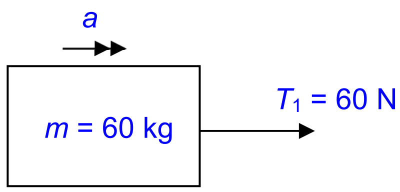
Substitute value of a into (1), we have

$F_C = 14 - 3a = 8 \text{ N}$

5. Three blocks are connected on a horizontal frictionless table and pulled to the right with a force T_1 = 60 N. If m_1 = 30 kg, m_2 = 20 kg and m_3 = 10 kg, find the tensions T_2 and T_3. [30 N, 10 N]

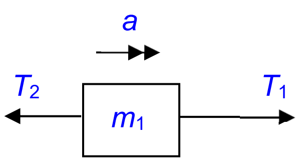


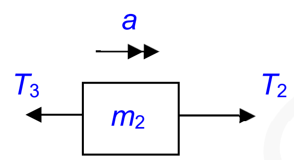
Considering the whole system, i.e. consider m_1, m_2, and m_3 as one object of mass m:



Acceleration of system $a = \frac{F}{m} = \frac{60}{60} = 1.0 \text{ m s}^{-2}$

Taking rightward direction as positive,





Isolating m_1,
$T_1 - T_2 = m_1 a$ (1)
$= 30\ (1.0) = 30$

Isolating m_2,
$T_2 - T_3 = m_2 a$ (2)
$= 20\ (1.0) = 20$

Isolating m_3,
$T_3 = m_3 a$
$= 10\ (1.0) =$ <u>10 N</u> (3)

Sub T_3 = 10 N into (2) gives

T_2 = <u>30 N</u>

6. Determine the force that an 80 kg man exerts on the floor of an elevator when it

(a) is at rest;
(b) rises with a constant velocity of 2.0 m s^{-1};
(c) descends with a constant velocity of 2.0 m s^{-1};
(d) rises with a constant acceleration of 2.0 m s^{-2};
(e) descends with a constant acceleration of 2.0 m s^{-2}
(f) accelerating downwards at 9.81 m s^{-2}

[785 N, 785 N, 785 N, 945 N, 625 N, 0 N]

Isolating the man of mass m and taking upward direction as positive,
Newton's second law gives $N - mg = ma$

N is the 'measured' weight of the man.
It is also called 'apparent' weight or effective weight.

(a) $v = 0,\ a = 0,$
$N - mg = m(0) \Rightarrow N = mg = 80 \times 9.81 = 785\text{ N}$

(b) $v = 2.0\text{ m s}^{-1},\ a = 0,$
$N - mg = m(0) \Rightarrow N = mg = 80 \times 9.81 = 785\text{ N}$

(c) $v = -2.0\text{ m s}^{-1},\ a = 0,$
$N - mg = m(0) \Rightarrow N = mg = 80 \times 9.81 = 785\text{ N}$

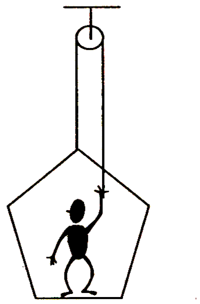
(d) $a = 2.0\text{ m s}^{-2},$
$N - mg = m(2.0) \Rightarrow N = m(9.81 + 2.0) = 80 \times 11.81 = 945\text{ N}$

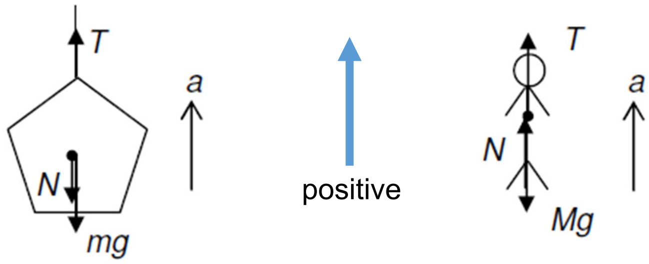
(e) $a = -2.0\text{ m s}^{-2},$
$N - mg = m(-2.0) \Rightarrow N = m(9.81 - 2.0) = 80 \times 7.81 = 625\text{ N}$

(f) $a = -9.81\text{ m s}^{-2},$
$N - mg = m(-9.81) \Rightarrow N = m(9.81 - 9.81) = 0\text{ N}$

7. The figure shows a painter in a crate which hangs alongside a building. When the painter pulls on the rope, the force he exerts on the floor of the crate is 300 N. If the mass of the painter and the crate is 75 kg and 25 kg respectively, find the acceleration of the crate.

[2.2 m s^{-2} upwards]





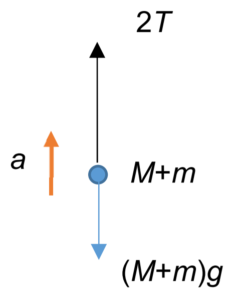
Considering forces on the crate, $T - N - mg = ma \quad \cdots(1)$

Considering forces on the painter, $T + N - Mg = Ma \quad \cdots(2)$

Subtracting equation (1) from equation (2),

$$2N - (M - m)g = (M - m)a$$

$$a = \frac{(2 \times 300) - (75 - 25) \times 9.81}{75 - 25} = 2.2 \text{ m s}^{-2} \text{ (upward)}$$



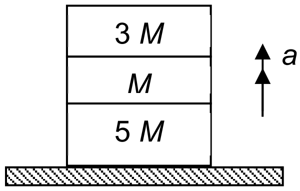
Alternatively, we can treat the man and the crate as one system and draw its free body diagram. Therefore,

$2T - (M + m)g = (M + m)a$ ... (3)

The unknowns a and T can be solved by using any pair of the three equations (1), (2) and (3).

8. Three crates of masses 3*M*, *M* and 5*M* are stacked on top of one another on the floor of a lift as shown on the right.

When the lift is accelerating upwards with an acceleration *a*, the magnitude of the force mass *M* exerts on the mass 5*M* is given by



A $4Mg - 5Ma$

B $4Mg - 4Ma$

C $4Mg + 4Ma$

D $4Mg + 5Ma$

Answer: **C** (There are a few ways to solve the problem.)

Let N to be the normal force acted by the lift floor on the system of three crates.
Apply Newton's 2nd law to the system of 3 crates:

$N-(3M+M+5M)g=(3M+M+5M)a$

$\Rightarrow N=9Ma+9Mg$

Isolating $5M$, Newton's 2nd law:

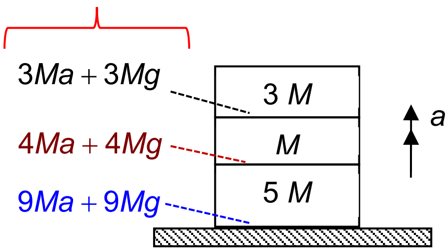
$N-F_C-5Mg=(5M)a$ where F_C is the (contact) force mass M exerts on mass $5M$

$\Rightarrow F_C=N-5Mg-5Ma$

$=(9Ma+9Mg)-5Mg-5Ma$

$=\underline{4Ma+4Mg}$

Note the magnitude of contact force between the surfaces:



OR

Let F_1 be the contact force between mass $3M$ and mass M, and F_2 the contact force between mass M and $5M$.

Isolating $3M$, Newton's 2nd law:

$F_1-3Mg=3Ma \Rightarrow F_1=3Ma+3Mg$

Isolating M, Newton's 2nd law:

$F_2-F_1-Mg=Ma \Rightarrow F_2=F_1+Mg+Ma=\underline{4Ma+4Mg}$

OR

Let F_C be the contact force between mass M and mass $5M$.

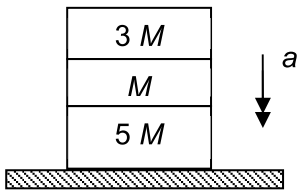
Isolating the masses (3*M*+*M*), Newton's 2nd law:

$F_C-(3M+M)g=(3M+M)a$

$F_C=4Mg+4Ma$

<u>Practice</u>

When the lift is accelerating <u>downwards with an acceleration *a*</u>, what will be the magnitude of the contact forces between the surfaces?



[$9Mg - 9Ma$, $4Mg - 4Ma$, $3Mg - 3Ma$]

Part 2: Linear Momentum and Impulse

9. A 0.10 kg ball is thrown straight up into the air with an initial speed of 15 m s^{-1}. Find the momentum of the ball,

(a) at its maximum height [0 kg m s^{-1}]
(b) halfway to its maximum height [1.06 kg m s^{-1}]

(a) $v = 0$ at its maximum height,
$mv = (0.10)(0) = 0$ kg m s^{-1}

(b) Let the maximum height be H, and velocity at $(H/2)$ be v_1.
Using $v^2 = u^2 + 2gS$, & taking upward as positive,
at maximum height, $0^2 = u^2 + 2(-9.81)H$ (1)
halfway the maximum height, $v_1^2 = u^2 + 2(-9.81)(H/2)$ (2)

$$(1) \Rightarrow H = \frac{u^2}{2(9.81)}$$

$$(2)-(1) \Rightarrow v_1^2 = 2(9.81)(H/2) = 2(9.81)\left(\frac{u^2}{4(9.81)}\right) = \frac{u^2}{2}$$

$$\Rightarrow v_1 = \frac{u}{\sqrt{2}} = \frac{15}{\sqrt{2}} = 10.6 \text{ m s}^{-1}$$

Momentum $= mv_1 = (0.10)(10.6) = 1.06$ kg m s^{-1}

10. A car is stopped by a traffic signal. When the light turns green, the car accelerates, increasing the speed uniformly from 0 to 5.20 m s^{-1} in 0.832 s. For a 70.0 kg passenger, what is,

(a) the magnitudes of the impulse and [364Ns]
(b) the average force experienced by the passenger in the car during this time? [438N]

(a) Magnitude of the impulse over the time interval Δt is
$\langle F \rangle \Delta t = m(v-u) = (70.0)(5.20-0) = 364$ N s
The direction of the impulse is the direction of the change in momentum.

(b) Average force experienced by the passenger

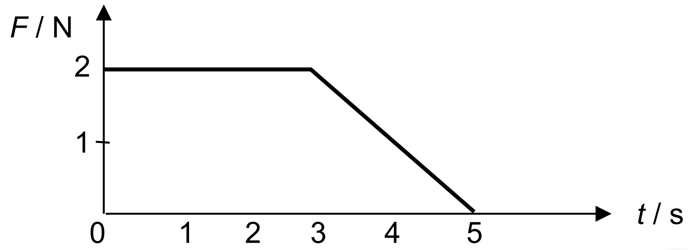
$$\langle F \rangle = \frac{\text{impulse}}{\Delta t} = \frac{364}{0.832} = 438 \text{ N}$$

The direction of average force is the direction of impulse.

lim.boonsiong@dhs.edu.sg

11. The force shown in the force-time graph acts on 1.5 kg mass in the positive *x*-axis direction. Calculate,

(a) the impulse of the force [8 N s]
(b) the final velocity of the mass if it is initially at rest and [5.3 m s^{-1}]
(c) the final velocity of the mass if it is initially moving along *x*-axis with a velocity of –2.00 m s^{-1}. [3.33 m s^{-1}]



(a) impulse of the force = area between the graph and time axis

$$= \frac{(3.0 + 5.0)(2.0)}{2}$$

$$= 8.0 \text{ N s}$$

(b) impulse = change in momentum

$$8.0 = \text{final momentum} - \text{initial momentum}$$

$$\text{final momentum} = 8.0 + \text{initial momentum} = 8.0 + 0.0 = 8.0$$

$$\text{final velocity} = \frac{\text{final momentum}}{\text{mass}} = \frac{8.0}{1.5} = 5.3 \text{ m s}^{-1}$$

(c) final momentum = 8.0 + initial momentum

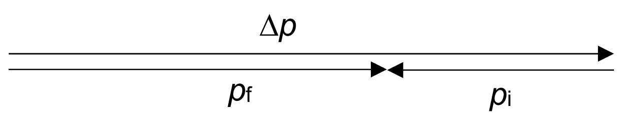
$$= 8.0 + (1.5)(-2.00)$$

$$= 5.0$$

$$\text{final velocity} = \frac{\text{final momentum}}{\text{mass}} = \frac{5.0}{1.5} = 3.3 \text{ m s}^{-1}$$

Vector equation: $p_f = \Delta p + p_i$

In terms of vector diagram:



12. Explain why, when catching a fast moving ball, the hands are drawn back while the ball is being brought to rest. Discuss whether your explanation has any bearing on the use of crushable boxes for packing eggs.

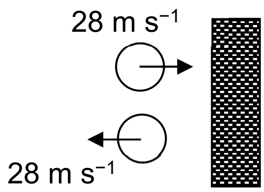
A fast moving ball has a certain momentum, mv, all of which is lost as it comes to rest. The slowing force F must therefore act for a time t where $Ft = mv$ (Newton's 2nd law of motion), so the longer t the smaller F, and the long t is achieved by drawing back the hands. [Alternatively, a fast moving ball has a certain kinetic energy, $\frac{1}{2}mv^2$, all of which is lost as it comes to rest. The slowing force must therefore act for a distance d where $Fd = \frac{1}{2}mv^2$. Work done on the ball is the same but there's a greater distance d, therefore less force F is applied.] Exactly the same argument applies to egg boxes; if an egg box is dropped then t is increased by the box crushing, hence the decelerating force F is smaller.

13. Two objects are known to have the same momentum. Do they have the same kinetic energy?

$$\text{Kinetic energy } K = \frac{1}{2}mv^2 = \frac{1}{2}\frac{(mv)^2}{m} = \frac{p^2}{2m}$$

K depends on p and m. If two objects have the same p, the one with a larger mass will have a smaller kinetic energy.

14. Consider a ball of mass 0.20 kg traveling with a velocity 28 m s^{-1} directly towards a wall. It hits the wall and bounces off in the opposite direction with a velocity of 28 m s^{-1}. Calculate the impulse due to the force by the wall. [11 kg m s^{-1} to the left away from the wall]



Take rightward direction as positive,

impulse of force on ball = change in momentum of ball

$= m(v_f - v_i)$

$= (0.20)(-28 - 28)$

$= -11$ kg m s^{-1}

The impulse due to the force by the wall is 11 kg m s^{-1} to the left.

Part 3: Principle of Conservation of Momentum

15. A 730 N man stands in the middle of a frozen pond of radius 5.0 m. He is unable to get to the other side because of the lack of friction between his shoes and the ice. To overcome this difficulty, he throws his 1.2 kg physics textbook horizontally towards the north shore, at a speed of 5.0 m s^{-1}. How long does it take him to reach the south shore? [62 s]

Conservation of momentum:

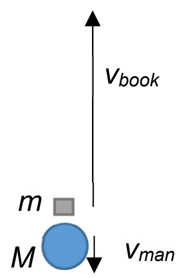
$$0 = Mv_{man} - mv_{book}$$

$$v_{man} = \frac{m}{M}v_{book}$$

$$= \frac{1.2}{730/9.81}(5.0) = 0.0806 \text{ m s}^{-1}$$

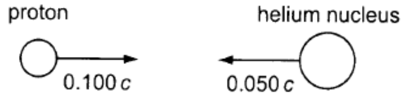
Time to move 5.0 m to the south shore with velocity v_{man},

$$t = \frac{5.0}{v_{man}} = 62 \text{ s}$$



(Plan View)

16. A proton (mass 1*u*) travelling with velocity of +0.100*c* collides elastically head-on with a helium nucleus (mass 4*u*) travelling with velocity −0.050*c*.



(Before)

What are the velocities of each particle after the collision?

[proton: −0.140c; helium nucleus: +0.010c]

(After)

Assume after the collision the proton moves to the left with speed v_1 and helium nucleus to the right with speed v_2.

Conservation of momentum:

$$m_p(0.100c) + m_{He}(-0.050c) = m_p(-v_1) + m_{He}(v_2)$$

$$\Rightarrow m_p(0.100c + v_1) = m_{He}(v_2 + 0.050c) \quad \text{... (1)}$$

Elastic collision:

$$0.100c + 0.050c = v_2 + v_1$$

$$\Rightarrow 0.150c = v_2 + v_1$$

$$\Rightarrow v_2 = 0.150c - v_1 \quad \text{... (2)}$$

(2) and (1):

$$m_p(0.100c + v_1) = m_{He}(0.150c - v_1 + 0.050c)$$

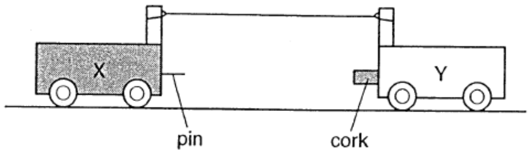
$$v_1 = \frac{m_{He}(0.200c) - m_p(0.100c)}{(m_p + m_{He})} = \frac{4u(0.200c) - u(0.100c)}{(u + 4u)} = 0.140c$$

Substituting v_1 into (2):

$$v_2 = 0.150c - 0.140c = 0.010c$$

Therefore, after the collision, proton moves to the left with speed 0.140*c* and helium nucleus moves to the right with speed 0.010*c*.

17. The diagram shows two trolleys X and Y held stationary and connected by an extended elastic cord. The mass of X is twice that of Y.



The trolleys are released at the same instant. They move towards each other and stick together on impact. Just before the collision, the speed of X is 20 cm s^{-1}. What is the speed of Y after the collision?

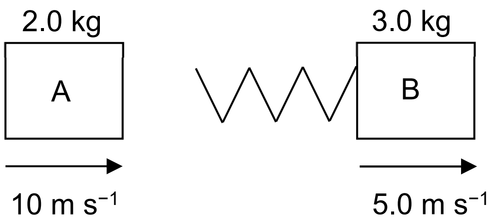
[0 m s^{-1}]

Considering the two trolleys as one system, the force of the elastic cord as well as the force of impact are both internal forces.
Since there is no external force (no friction, no air resistance) acting on the system, the total momentum of the system remains constant, i.e., zero.
Upon collision, the rightward momentum of X $\left(2m \times 20 \text{ m s}^{-1}\right)$ cancels out the leftward momentum of Y $\left(m \times 40 \text{ m s}^{-1}\right)$ and both trolleys come to rest.

OR

momentum of system before the release of the elastic cord =
momentum of system when they stick together on impact
$\Rightarrow 0 = \left(m_X + m_Y\right)v \Rightarrow v = 0.$ (v is the common speed of both X and Y after collision)

18. Block A of mass 2.0 kg moves with a velocity of 10 m s^{-1} on a smooth horizontal table. Block B of mass 3.0 kg moves with a velocity of 5.0 m s^{-1} in front of A in the same direction. A light spring of force constant, k = 1000 N m^{-1} is attached to B as shown in the following figure.



When A collides with B, there will be an instant when the spring experiences maximum compression.

(a) Calculate the maximum compression of the spring, x. (Elastic potential energy stored in a spring = ½ kx^2) [0.173 m]

(b) Calculate the velocity of A and B after they are separated. [4.0 m s^{-1}, 9.0 m s^{-1}]

(a) Smooth surface means that there is no friction.

When the spring experiences maximum compression, the 2 blocks are moving together with the same velocity v, similar to a completely inelastic collision.

By the principle of conservation of momentum,

Total initial momentum = Total final momentum at maximum compression

$$(2.0)(10) + (3.0)(5.0) = (2.0 + 3.0)\, v$$
$$v = 7.0 \text{ m s}^{-1}$$

By the principle of conservation of energy,

Total energy of the system before collision = Total energy of the system when the spring experiences maximum compression

KE of blocks before collision = KE of blocks when spring experiences maximum compression + EPE stored in the spring

$$\tfrac{1}{2}(2.0)(10)^2 + \tfrac{1}{2}(3.0)(5.0)^2 = \tfrac{1}{2}(2.0 + 3.0)(7.0)^2 + \tfrac{1}{2}(1000)x^2$$
$$x = 0.173 \text{ m}$$

(b) When the spring extends to its original length, all the EPE stored in it is transformed into the KE of the blocks, like in an elastic collision.

Assuming after they are separated both A and B move off to the right with speed v_1 and v_2 respectively, by the principle of conservation of momentum,

$$(2.0)(10) + (3.0)(5.0) = 2.0v_1 + 3.0v_2 \quad \text{----- (1)}$$

Using relative speed of approach = Relative speed of separation,

$$10 - 5.0 = v_2 - v_1 \rightarrow v_1 = v_2 - 5 \quad \text{----- (2)}$$

Sub (2) into (1): $35 = (2.0\, v_2 - 10) + 3.0\, v_2$

$45 = 5.0\, v_2$

$v_2 = 9.0$ m s^{-1} (to the right)

Hence, $v_1 = 4.0$ m s^{-1} (to the right)

19. A ball of clay is thrown against a wall. The clay sticks to the wall and stops. Explain how the principle of conservation of momentum applies in this case.

Initially the clay has momentum directed towards the wall. When it collides and sticks to the wall, it *appears* that the final momentum is zero. It is tempting to conclude that momentum is not conserved. In reality, the momentum of the clay is transferred to the wall and Earth, causing both to move although the speed is too small to be observable due to the enormous mass of wall and Earth.

Mathematically, let the mass of the clay be m, and that of the (wall and Earth) be M.
If no external forces acting on the system consisting of the clay and (wall and Earth), then the system's momentum along the direction of the clay's initial motion is conserved:

$mv + M(0) = (m + M)v'$ (assuming wall and Earth to be stationary initially)
where v' is the common velocity of the system after the clay sticking to the wall.

$v' = \dfrac{mv}{m+M} \approx \dfrac{m}{M}v$ which is not zero but very small because $M >> m$.

20. A fast moving neutron with an initial velocity ***u*** has a head-on elastic collision with a stationary proton. After the collision, the velocity of the neutron is ***v*** and that of the proton is ***w***. Taking the masses of the neutron and proton to be equal, which one of the following statements is false?

(a) $u + v = w$ (**True**; as relative speed of approach = relative speed of separation: $u – 0 = w – v$
➔ $u + v = w$)

Assuming:

(before) (after)

(b) $u^2 = v^2 + w^2$ (**True**; conservation of kinetic energy: $\frac{1}{2} mu^2 + 0 = \frac{1}{2} mv^2 + \frac{1}{2} mw^2$)

(c) The speed of the proton after the collision is the same as that of the neutron before the collision. (**True**; solving $u – v = w$ and $u^2 = v^2 + w^2$ gives $v = 0$ and $w = u$.)

(d) The proton and the neutron move off in opposite directions with equal speeds. (**False**; the neutron becomes stationary after the collision, as shown in part (c) that $v = 0$ and $w = u$.)

End