H2 Mathematics (9758)
Topic 18: Permutations & Combinations
Tutorial Questions

Suggested Anchor Questions: Q4, 7, 8, 11

Section 1: Discussion Questions (Students are to attempt all questions in this section)

1 A delegation of 7 people is to be selected from a group of 10 men and 8 women. Find the number of such delegations that contain at least one man.

Concepts Tested

1) Combination using Complement method

Method 1: Complement

No. of delegations that contain at least one man

= Total w/o restrictions – Delegations with all women

= ${}^{18}C_7 - {}^{8}C_7 = 31816$

Method 2: Direct

$\left({}^{10}C_1{}^{8}C_6\right)+\left({}^{10}C_2{}^{8}C_5\right)+\left({}^{10}C_3{}^{8}C_4\right)+\left({}^{10}C_4{}^{8}C_3\right)+\left({}^{10}C_5{}^{8}C_2\right)+\left({}^{10}C_6{}^{8}C_1\right)+\left({}^{10}C_7{}^{8}C_0\right)=31816$

Common mistake: Fix 1 man, choose remaining 6 people. ${}^{10}C_1{}^{17}C_6 = 123760$

This is an overcount: for example, $M_1, M_2, W_1, W_2, W_3, W_4, W_5$ and $M_2, M_1, W_1, W_2, W_3, W_4, W_5$ can both happen, but they are the same selection of people actually.

2 **(a)** In how many ways can five different books be distributed among 10 people if each person can get any number of books?

(b) Mary has seven cousins altogether. In how many ways can she invite some or all of them to her birthday party next Saturday?

Concepts Tested

1) Multiplication Principle

(a) Each book can be distributed in ${}^{10}C_1$ ways, so

total $= {}^{10}C_1 \times {}^{10}C_1 \times {}^{10}C_1 \times {}^{10}C_1 \times {}^{10}C_1 = 100000$ ways

(b) **Method 1:**

Each cousin can either be invited or not invited — i.e 2 ways.

So total no. of ways $= 2^7 - 1 = 128 - 1 = 127$

Method 2:

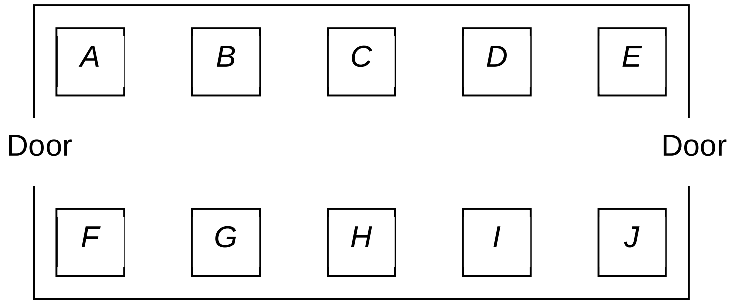
No. of ways $= {}^{7}C_1 + {}^{7}C_2 + {}^{7}C_3 + {}^{7}C_4 + {}^{7}C_5 + {}^{7}C_6 + {}^{7}C_7 = 127$

Common mistake:

(a) Consider the 10 people rather than the 5 books to be distributed.

(b) $2^7 = 128$ [Not removing the case where none is invited.]

3 **[N2004/II/4]**



A rectangular shed, with a door at each end, contains ten fixed concrete bases marked *A*, *B*, *C*, …, *J*, five on each side (see diagram). Ten canisters, each containing a different chemical, are placed with one canister on each base. In how many different ways can the canisters be placed on the bases? Find the number of ways in which the canisters can be placed

(i) if 2 particular canisters must not be placed on any of the 4 bases *A*, *E*, *F* and *J* next to a door,

(ii) if 2 particular canisters must not be placed next to each other on the same side of the shed.

Concepts Tested
(i) Permutation with restrictions
(ii) Permutation using Complement Method

No. of ways in which canisters can be placed in = $10! = 3628800$

(i) **<u>Method 1: Consider the 2 particular canisters first</u>**

No. of ways = ${}^{6}C_2 \times 2! \times 8! = 1209600$

<u>Method 2: Put 4 other canisters at A, E, F, J first</u>

No. of ways = ${}^{8}C_4 \times 4! \times 6! = 1209600$

From the remaining 8 canisters, pick 4 to put at the 4 corners then fill the 6 bases in the centre with 6 canisters.

<u>Method 3: Complement</u>

No. of ways without restriction – both canisters are near the doors – only 1 canister near the doors= $10! - \left({}^{4}C_2 \times 2! \times 8!\right) - \left({}^{4}C_1 \times {}^{6}C_1 \times 2! \times 8!\right) = 1209600$

(ii) No. of ways = $10! - {}^{8}C_1 \times 2! \times 8! = 2983680$

Explanation:

No. of ways without restriction – (Choose the positions for the 2 canisters x Permutate the the positions for the canisters x Arrange the other 8 canisters)

<u>Method 2</u>: (If Can 1 is on A, on B, on C, …, J)

$(8 \times 8!) + (7 \times 8!) + (7 \times 8!) + (7 \times 8!) + (8 \times 8!) +$

$(8 \times 8!) + (7 \times 8!) + (7 \times 8!) + (7 \times 8!) + (8 \times 8!) = 2983680$

4 **[N2008/II/10]**

A group of diplomats is to be chosen to represent three islands, *K*, *L* and *M*. The group is to consist of 8 diplomats and is chosen from a set of 12 diplomats consisting of 3 from *K*, 4 from *L* and 5 from *M*. Find the number of ways in which the group can be chosen if it includes

(i) 2 diplomats from *K*, 3 from *L* and 3 from *M*,

(ii) diplomats from *L* and *M* only,

(iii) at least 4 diplomats from *M*,

(iv) at least 1 diplomat from each island.

Concepts Tested

1) Splitting into mutual exclusive cases
2) Complement method

(i) No. of ways = ${}^{3}C_{2} \times {}^{4}C_{3} \times {}^{5}C_{3} = 120$

(ii) **<u>Method 1:</u>**

No. of ways = ${}^{9}C_{8} = 9$

<u>Method 2: Cases</u>

No. of ways = $\left({}^{4}C_{4}\,{}^{5}C_{4}\right) + \left({}^{4}C_{3}\,{}^{5}C_{5}\right) = 9$

<u>Method 3: Complement</u>

No restriction – (1 K) – (2 K) – (3 K) = ${}^{12}C_{8} - \left({}^{3}C_{1}\,{}^{9}C_{7}\right) - \left({}^{3}C_{2}\,{}^{9}C_{6}\right) - \left({}^{3}C_{3}\,{}^{9}C_{5}\right) = 9$

(iii) **<u>Method 1: Cases</u>**

<u>Case 1</u>: 4 from M — No. of ways = ${}^{5}C_{4} \times {}^{7}C_{4} = 175$

<u>Case 2</u>: 5 from M — No. of ways = ${}^{5}C_{5} \times {}^{7}C_{3} = 35$

$\therefore$ Total no. of ways = 175 + 35 = 210

<u>Method 2: Complement</u>

No restriction – (0 M) – (1 M) – (2 M) – (3 M) =

${}^{12}C_{8} - (0) - \left({}^{5}C_{1} \times {}^{7}C_{7}\right) - \left({}^{5}C_{2} \times {}^{7}C_{6}\right) - \left({}^{5}C_{3} \times {}^{7}C_{5}\right) = 210$

(iv) No. of ways

= No. of ways with no restrictions – No. of ways if no *K* – No. of ways if no *L*

= ${}^{12}C_{8} - {}^{9}C_{8} - {}^{8}C_{8} = 485$

*Note: There is no case for No. of ways if no *M* since the other 2 islands only make up 7 diplomats (i.e., there must always be at least 1 diplomat from *M*)

Common mistake:

<u>(iii) Fix 4 M, choose remaining 4 people.</u> ${}^{5}C_{4}\,{}^{8}C_{4}$ This is an overcount.

Example: M1, M2, M3, M4, M5

M1, M2, M3, M4 — M5, L1, L2, K1

M1, M2, M3, M5 — M4, L1, L2, K1

(iv) Consider cases of (1) 1K, (2) 2K, (3) 3K, (4) 1L, (5) 2L, (6) 3L… [overcount]

Fix 1K, 1L, 1M, choose remaining 5 people. ${}^{3}C_{1}\,{}^{4}C_{1}\,{}^{5}C_{1}\,{}^{9}C_{5}$ [overcount]

Example:

K1, L1, M1 — K2, L2, L3, L4

K2, L1, M1 — K1, L2, L3, L4

5 [N14/II/6]

A team in a particular sport consists of 1 goalkeeper, 4 defenders, 2 midfielders and 4 attackers. A certain club has 3 goalkeepers, 8 defenders, 5 midfielders and 6 attackers.

(i) How many different teams can be formed by the club?

One of the midfielders in the club is the brother of one of the attackers in the club.

(ii) How many different teams can be formed which include exactly one of the two brothers?

The two brothers leave the club. The club manager decides that one of the remaining midfielders can play as either a midfielder or as a defender.

(iii) How many different teams can now be formed by the club?

Concepts Tested

1) Multiplication Principle
2) Splitting into mutual exclusive cases

(i) Number of teams = ${}^3C_1\,{}^8C_4\,{}^5C_2\,{}^6C_4 = 31500$

(ii) Case 1: Number of teams that has the midfielder brother = ${}^3C_1\,{}^8C_4\,{}^4C_1\,{}^5C_4 = 4200$ Case 2: Number of teams that has the attacker brother = ${}^3C_1\,{}^8C_4\,{}^4C_2\,{}^5C_3 = 12600$ Total number of teams = 4200 + 12600 = 16800

(iii) **Method 1**

Case 1: Number of teams if one of the midfielders (particular) plays as midfielder = ${}^3C_1\,{}^8C_4\,{}^3C_1\,{}^5C_4 = 3150$

Case 2: Number of teams if one of the midfielders (particular) plays as defender = ${}^3C_1\,{}^8C_3\,{}^3C_2\,{}^5C_4 = 2520$

Case 3: Number of teams if one of the midfielders (particular) does not play = ${}^3C_1\,{}^8C_4\,{}^3C_2\,{}^5C_4 = 3150$

Total = 3150 + 2520 + 3150 = 8820

Method 2

${}^3C_1\,{}^8C_4\,{}^4C_2\,{}^5C_4 + {}^3C_1\,{}^9C_4\,{}^3C_2\,{}^5C_4 - {}^3C_1\,{}^8C_4\,{}^3C_2\,{}^5C_4 = 8820$

(player as Midfielder) + (player as Defender) – (without the player)

6 A committee of 5 people is to be chosen from 6 married couples. Find how many ways this committee can be chosen if

(a) all are equally eligible,

(b) the two youngest women and at most one of the oldest two men are to be included,

(c) the committee must consist of at least one woman and at least one man,

(d) no husband and wife can serve in the same committee?

Concepts Tested

1) Combination with and w/o restrictions
2) Complement method
3) Splitting into mutual exclusive cases

(a) Total no. of ways w/o restrictions = ${}^{12}C_5 = 792$

(b) **Method 1: Cases**

Case 1:

2 youngest women in and the oldest two men not in

No. of ways = ${}^{2}C_2 \times {}^{8}C_3 = 56$

Case 2:

2 youngest women in and one of the oldest two men in

No. of ways = ${}^{2}C_2 \times {}^{2}C_1 \times {}^{8}C_2 = 56$

Hence, total no. of ways = 112

Method 2: Complement

No. of ways = ${}^{2}C_2 \times {}^{10}C_3 - {}^{2}C_2 \times {}^{2}C_2 \times {}^{8}C_1 = 112$

Explanation:

(2 youngest women choose 2) (remaining 10 choose 3) – (2 youngest women choose 2) (2 oldest men choose 2)(remaining 8 choose 1)

(c) **Method 1: Complement**

Required no. of ways = ${}^{12}C_5 - {}^{6}C_5 - {}^{6}C_5 = 780$ (all – all male – all female)

Method 2: Cases

$\left({}^{6}C_1 \times {}^{6}C_4\right) + \left({}^{6}C_2 \times {}^{6}C_3\right) + \left({}^{6}C_3 \times {}^{6}C_2\right) + \left({}^{6}C_4 \times {}^{6}C_1\right) = 780$

(d) Required no. of ways = ${}^{6}C_5 \times \underbrace{2 \times 2 \times 2 \times 2 \times 2}_{} = 192$

${}^{6}C_5$: Choose any 5 married couples out of 6.

$2 \times 2 \times 2 \times 2 \times 2$: Each couple choose either the husband or wife

Method 2: Cases

${}^{6}C_5 + \left({}^{6}C_4 \times {}^{2}C_1\right) + \left({}^{6}C_3 \times {}^{3}C_2\right) + \left({}^{6}C_2 \times {}^{4}C_3\right) + \left({}^{6}C_1 \times {}^{5}C_4\right) + \left({}^{6}C_5\right) = 192$ (5M, 4M& 1W from remaining untouched 2 couples, 3M & 2W from remaining untouched 3 couples…)

Common mistake: <u>(c) Fix 1 M, 1W, choose remaining 3 people.</u> ${}^{6}C_1 {}^{6}C_1 {}^{10}C_3$ [Overcount]

Example: W1, M2, W2, W3, M4
W2, M2, W1, W3, M4
W2, M4, W1, W3, M2

7 A six-digit number is to be formed from the digits 1, 2, 3, 4, 5, 6, 7, 8 and 9. For each of the following cases, find how many different ways the six-digit number can be formed.

(i) The even and odd digits of the number must alternate and any digit may appear more than once.

(ii) The number must be odd and is less than 600 000 and no digit may appear more than once.

(iii) The number is formed using only four different digits, eg. 621313, 255567.

Concepts Tested

1) Multiplication Principle and identifying mutually exclusive cases

(i) 2 possible arrangements: O E O E O E or E O E O E O

Total = (5 x 4 x 5 x 4 x 5 x 4) x 2 = 16 000

(ii) **Case 1:** last digit = 7, 9

1,2,3,4,5					7 or 9

7 numbers left

5 x (7 x 6 x 5 x 4) x 2 = 8400

Case 2: last digit = 1, 3, 5

2, 4 and two of the odd digits					1, 3, 5

7

numbers left

4 x (7 x 6 x 5 x 4) x 3 = 10080

Total = 8400 + 10080 = 18480

(iii) Select 4 numbers : ${}^9C_4$

Case 1 : 1 digit appear 3 times e.g. 666789

$${}^4C_1 \times \frac{6!}{3!}$$

Case 2 : 2 digits each appear twice e.g. 667789

$${}^4C_2 \times \frac{6!}{2!2!}$$

Total = ${}^9C_4 \times \left({}^4C_1 \times \frac{6!}{3!} + {}^4C_2 \times \frac{6!}{2!2!} \right) = 196\ 560$

8 Find the number of different arrangements of the word EVERMORE.
Calculate also the number of these arrangements that

(i) begin with R and end with E,

(ii) all three letters E are consecutive,

(iii) all the vowels are next to each other,

(iv) letters V and M are separated,

(v) no two E's are next to each other,

(vi) letters V and M are separated, and the three Es are not all together.

Three letters are chosen from the word EVERMORE to form a codeword. Find the number of codewords that can be formed.

Concepts Tested
Permutation with **(i)** restrictions, **(ii)** grouping of identical objects, **(iii)** grouping of distinct objects, **(iv)** slotting method/complement method, **(v)** Slotting method

EEE RR V M O

Total no. of arrangements without restriction = $\frac{8!}{3!2!} = 3360$

(i) R_ _ _ _ _ _ E
Letters remaining: EEVRMO.

No. of different arrangements that begin with R and end with E = $\frac{6!}{2!} = 360$

(ii) **<u>Method 1: Group</u>**

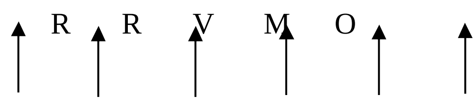
(EEE), R, R, V, M, O
Group the 3 E's as one unit.
Hence, no. of arrangements with all the letters E are consecutive

$= \frac{6!}{2!} \times 1 = 360$

Only 1 way to arrange EEE

<u>Method 2: Slotting</u>

$\frac{5!}{2!} \times {}^{6}C_{1} = 360$



(iii) (EEEO), V, R, R, M

Group all the vowels as one unit.
Hence, no. of arrangements with all the vowels next to each other

$= \frac{5!}{2!} \times \frac{4!}{3!} = 240$

(iv) **<u>Method 1: Slotting</u>**

No. of arrangements with letter V and M separated $= \frac{6!}{3!2!} \times {}^{7}C_{2} \times 2! = 2520$

<u>Method 2: Complement</u>

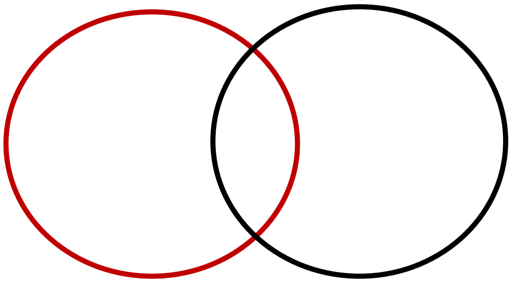
No. of arrangements with letter V and M together $= \frac{7!}{3!2!} \times 2! = 840$

Hence, no. of arrangements with letter V and M separated
= 3360 – 840 = 2520

(v) No. of arrangements with no 2 E's next to each other $=\frac{5!}{2!}\times {}^{6}C_{3}=1200$

(*Using Slotting Method)

(vi)



We first need to find the no of ways such that 3Es are together and V,M are separated.

Treat 3Es as one unit. No. of ways to arrange [EEE], R, R, O $=\frac{4!}{2!}=12$

No of ways to slot in V,M in 5 slots= ${}^{5}C_{2}\times 2!=20$

No of ways such that 3Es are together and V,M are separated $=12\times 20=240$

From Venn diagram, we can see that

No of ways letters V and M are separated, and the three Es are not all together
= No of ways V,M separate - No of ways 3Es are together and V,M are separated
$=2520-240=2280$

Last Part

Since there are repeated letters in EVERMORE, we need to split into cases:

Case 1: All 3 letters are identical
No. of arrangements = 1 (EEE)

Case 2: 2 identical, 1 different (either EE_ or RR_)

No. of arrangements = $2\times 4\times \frac{3!}{2!}=24$

Case 3: All 3 letters are different

No. of arrangements = ${}^{5}C_{3}\times 3!=60$

Total no. of possible codewords $=1+24+60=85$

Common mistake:

(i) ${}^{2}C_{1}\,{}^{3}C_{1}\frac{6!}{2!}$ [The Rs and Es are identical]

(ii) $\frac{6!}{2!}\times 3!$ [There's no need to permutate the Es as they are identical]

9 Five men, two women and a child sit at a round table with eight chairs. Find the number of ways this can be done if

(i) the child is seated between two men,

(ii) three particular men are all separated.

Two more chairs are added to the table such that there are now 10 chairs. Find the number of ways that the five men, two women and the child can be seated.

Concepts Tested

1) Circular Permutation
2) Grouping method vs Slotting method

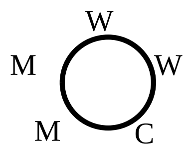
Possible Extension:

Find the number of possible arrangements if the seats are numbered?

(i) No. of ways that the child is seated between two men = $\frac{6!}{6}\times {}^{5}C_{2}\times 2! = 2400$

(ii) **<u>Method: Slot the 3 particular men</u>**

No. of ways that 3 particular men are separated = $\frac{5!}{5}\times {}^{5}C_{3}\times 3! = 1440$



Common mistake:

(ii) No restrictions – 3 particular men together [For case where 3 particular men together, it can mean 3 men together, or 2 men together + 1 man separate]

No of ways to arrange 8 distinct people and 2 empty spaces $=\frac{(10-1)!}{2!}=181440$

[We need to divide by 2! since the empty seats are considered identical.]

10 **(a)** In how many ways can a party of 10 children be divided into

(i) three groups consisting of 2, 3, 5 children,

(ii) two groups consisting of 5 children each,

(iii) three groups consisting of 3, 3, 4 children?

(b) A child was given four boxes of toys. In the first box, there were three identical toy cars. In the second box, there were four identical toy vans. In the third box, there were two identical toy motorcycles. In the last box, there was a toy garbage truck. Find the number of ways in which the child can choose at least one toy from any of these boxes.

Concepts Tested

1) Dividing people into equal sized groups
2) Box method with complement

(a) (i) No. of ways = ${}^{10}C_2 \times {}^{8}C_3 = 2520$

(ii) No. of ways = $\dfrac{{}^{10}C_5}{2!} = 126$

(Whenever one group of 5 children is selected, the remaining 5 children automatically form the other group. E.g. if C_1, C_2, C_3, C_4, C_5 are selected, then the other group is $C_6, C_7, C_8, C_9, C_{10}$. But if $C_6, C_7, C_8, C_9, C_{10}$ are selected, then the other group is C_1, C_2, C_3, C_4, C_5. These two selections are identical and thus repeated.)

(iii) No. of ways = $\dfrac{{}^{10}C_3 \times {}^{7}C_3}{2!} = 2100$

ABC/DEF/GHIJ $\equiv$ DEF/ABC/GHIJ, hence the need to divide by 2!

(b)

3 Cars (4ways)	4 Vans (5 ways)	2 MTCs (3ways)	1 Truck (2ways)

Since the toys are identical, the only choice is on the number of cars/vans/motorcycles/truck.

No. of ways child can choose at least one toy = $5 \times 4 \times 3 \times 2 - 1 \leftarrow (\text{child chooses 0 toys}) = 119$

11 [N2007/II/9]

A group of 12 people consists of 6 married couples.

(i) The group stand in a line.

(a) Find the number of different possible orders.

(b) Find the number of different possible orders in which each man stands next to his wife.

(ii) The group stand in a circle.

(a) Find the number of different possible arrangements.

(b) Find the number of different possible arrangements if men and women alternate.

(c) Find the number of different possible arrangements if each man stands next to his wife and men and women alternate.

(d) Suppose the group is asked to sit at a round table with 11 white chairs and one black chair such that a particular couple must be seated together. Find the number of different seating arrangements.

Concepts Tested

1) Permutation in a row with groupings
2) Circular Permutation with restrictions using slotting method

Possible Extension:

Find the number of possible arrangements if the seats are numbered?

(i) (a) No. of arrangements = 12! = 479001600

(b) **Method 1:** No. of arrangements = $6!\times 2^6 = 46080$

Method 2: No. of arrangements = ${}^{12}C_1\,{}^{10}C_1\,{}^{8}C_1\,{}^{6}C_1\,{}^{4}C_1\,{}^{2}C_1 = 46080$

(ii) (a) No. of arrangements = $\frac{12!}{12} = 39916800$

(b) No. of arrangements = $\frac{6!}{6}\times 6! = 86400$

(c) No. of arrangements = $\frac{6!}{6}\times 2 = 240$

(d) Treat the particular couple as 1 unit. No. of ways to seat 11 units around a circle = 10!
Also, there are 2 ways to arrange the couple.

Every seat is different if we consider its position relative to the black chair. Therefore, we can treat the seats as numbered seats.
No. of arrangements $= 10! \times 2\times 12 = 87,091,200$

Section 2: Supplementary Questions

You may try these questions after the tutorial for extra practice.

12 The number 25725 can be expressed in prime factors $3\times5^2\times7^3$.

Excluding 1 and 25725, how many positive integers are factors of 25725?

[Hint: one factor of 25725 is 245, which can be expressed as 3^0 x 5^1 x 7^2.]

Concepts Tested

1) Multiplication Principle

Number	**3**	$\mathbf{5^2}$	$\mathbf{7^3}$
Factors	3^0 3^1	5^0 5^1 5^2	7^0 7^1 7^2 7^3

Total number of factors

= 2 × 3 × 4 – 2 = 22 (exclude 1 and 25725)

13 **[N2006/I/4]**

A box contains 8 balls, of which 3 are identical (and so are indistinguishable from one another) and the other 5 are different from each other. 3 balls are to be picked out of the box; the order in which they are picked out does not matter. Find the number of different possible selections of 3 balls.

Concepts Tested

1) Combination with identical objects

Method 1: Cases

No. of possible selections (All identical + 2 identical + 1 identical + None identical)

$= {}^5C_0 + {}^5C_1 + {}^5C_2 + {}^5C_3 = 26$

Method 2: Cases (1 identical and None identical) as one case (i.e. 6 different balls)

No. of possible selections

$= {}^5C_0 + {}^5C_1 + {}^6C_3 = 26$

14 **(a)** Four girls and three boys stand in a queue in random order. Find the number of ways where all four girls stand next to each other.

(b) A code consists of blocks of ten digits, four of which are zeros and six of which are ones, e.g. 1011011100. Calculate the number of such blocks in which the first and last digits are the same as each other.

Concepts Tested

1) Permutation with grouping
2) Permutation with restrictions (splitting into distinct cases)

(a) Group the 4 girls as one unit.

$G_1G_2G_3G_4$, B1, B2, B3

Total no. of units = 4

Thus, no. of ways where all 4 girls stand next to each other = 4! × 4! = 576

(b) **<u>Method 1</u>: (Arranging the digits)**

<u>Case 1</u> : First and last are digit zero (left with 6 ‘1’s and 2 ‘0’s)

No. of such blocks = $\dfrac{8!}{6!2!} = 28$

<u>Case 2</u> : First and last are digit one (left with 4 ‘1’s and 4 ‘0’s)

No. of such blocks = $\dfrac{8!}{4!4!} = 70$

Total no. of blocks in which the first and last digits are the same as each other = 98

<u>Method 2</u>: (Choosing the positions to place the digits, put zeros then ones)

<u>Case 1</u> : First and last are digit zero (left with 6 ‘1’s and 2 ‘0’s)

No. of such blocks = ${}^{8}C_{2} \times {}^{6}C_{6} = 28$

<u>Case 2</u> : First and last are digit one (left with 4 ‘1’s and 4 ‘0’s)

No. of such blocks = ${}^{8}C_{4} \times {}^{4}C_{4} = 70$

Total no. of blocks in which the first and last digits are the same as each other = 98

15 **(a)** Five people, of whom three are women and two are men, are to form a queue. Find the number of different arrangements there if no two people of the same gender are to stand next to each other.

(b) How many even numbers between 3000 and 7000 can be formed using the digits 1, 3, 6 and 8 if

(i) no digit can occur more than once,

(ii) repetition of digits is allowed?

Concepts Tested

(a) Permutation with restrictions/ slotting method

(b)(i) Permutation with restriction and no repetition of digits

(b)(ii) Permutation with restriction and repetition of digits

(a) WMWMW

No. of different arrangements = 3! × 2! = 12

(b) (i) (Method shown: Fix the first digit/Alternative: Fix the last digit)

<u>Case 1</u>

Digit 3 (1 way)	(2 ways)	(1 way)	Digit 6 (1 ways)

<u>Case 2</u>

Digit 3, 6 (2 way)	(2 ways)	(1 way)	Digit 8 (1 way)

So no. of even numbers that can be formed $=(1\times 2\times 1\times 1)+(2\times 2\times 1\times 1)=6$

(b) (ii) Repetition of digits allowed

No. of even numbers that can be formed = 2 × 4 × 4 × 2 = 64

Questions to ask- Why not? / Common mistake:

(b) $2\times {}^{4}C_{2}\times 2!\times 2$ [does not allow for repetition]

$2\times {}^{8}C_{2}\times 2!\times 2$ (see it as 1, 1, 3, 3, 6, 6, 8, 8)[overcount by saying that 2^{nd} place has 8 choices instead of just 4].

16 **[N2009/II/8]**

Find the number of ways in which the letters of the word ELEVATED can be arranged if

(i) there are no restrictions,

(ii) T and D must not be next to each other,

(iii) Consonants (L, V, T, D) and vowels (E, A) must alternate,

(iv) Between any two Es there must be at least 2 other letters.

Concepts Tested

(i) Permutation

(ii) Slotting Method/ Complement Method

(iii)(iv) Permutation with restrictions

(i) **Method 1:** Number of ways $=\frac{8!}{3!}=6720$ (8 letters, 3 repeated)

Method 2: Number of ways $={}^{8}C_{3}\times 5!=6720$

(ii) **Method 1: Slotting** Number of ways $=\frac{6!}{3!}\times {}^{7}C_{2}\times 2!=5040$

Method 2: Complement Number of ways $= 6720-\frac{7!}{3!}\times 2!=5040$

(iii) Number of ways $=2\times 4!\times\frac{4!}{3!}=192$

(iv)

	No of ways
Case 1: E _ _ E _ _ E _	5!
Case 2: E _ _ E _ _ _ E	5!
Case 3: E _ _ _ E _ _ E	5!
Case 4: _ E _ _ E _ _ E	5!

Number of ways $=4\times 5!=480$

Questions to ask- Why not? / Common mistake:

(iii) L V T D

$4!\times {}^{5}C_{4}\times\frac{4!}{3!}$ (Arrange 4 consonants, choose 4 slots for vowels, arrange vowel)

[This includes the case where **v** L **v** V T **v** D **v**]

17 [N2005/I/4]

A board of directors consists of 9 men and 4 women, one of whom is Mrs Lee. A committee consisting of 4 people is to be formed from this board of directors and it has been decided that it must contain at least one woman.

(i) How many different committees can be formed?

(ii) How many different committees can be formed that have Mrs Lee as a member?

Concepts Tested

1) Combination with restrictions
2) Combination using Complement Method

(i)

Method 1: Complement

No. of different committees without any woman $= {}^{9}C_{4} = 126$

No. of different committees with at least 1 woman $= {}^{13}C_{4} - {}^{9}C_{4} = 589$

Method 2: Cases

$\left({}^{9}C_{3}\,{}^{4}C_{1}\right)+\left({}^{9}C_{2}\,{}^{4}C_{2}\right)+\left({}^{9}C_{1}\,{}^{4}C_{3}\right)+\left({}^{9}C_{0}\,{}^{4}C_{4}\right)=589$

(ii)

Method 1:

No. of different committees with Mrs Lee as a member $= {}^{12}C_{3} = 220$

Method 2: Complement

No restriction – Mrs Lee not in $= {}^{13}C_{4} - {}^{12}C_{4} = 220$

Questions to ask- Why not? / Common mistake:

(ii) No restriction – Mrs Lee not in – Case with no woman

[Case with no woman: this is already accounted for within the case where Mrs Lee not in]

Tutorial Questions

1. 31 816

2. **(a)** 100 000 **(b)** 127

3. 3 628 800, **(i)** 1 209 600, **(ii)** 2 983 680

4. **(i)** 120 **(ii)** 9 **(iii)** 210 **(iv)** 485

5. **(i)** 31 500 **(ii)** 16 800 **(iii)** 8820

6. **(a)** 792 **(b)** 112 **(c)** 780 **(d)** 192

7. **(i)** 16 000 **(ii)** 18 480 **(iii)** 196 560

8. 3360, **(i)** 360, **(ii)** 360, **(iii)** 240, **(iv)** 2520, **(v)** 1200, **(vi)** 2280, **(last part)** 85

9. **(i)** 2400, **(ii)** 1440, 181 440

10. **(a)** 2520; 126; 2100 **(b)** 119

11. **(i) (a)** 479 001 600 **(b)** 46 080 **(ii) (a)** 39 916 800, **(b)** 86 400, **(c)** 240, **(d)** 87 091 200

Supplementary Questions

12. 22

13. 26

14. **(a)** 576, **(b)** 98

15. **(a)** 12, **(b)(i)** 6, **(ii)** 64

16. **(i)** 6720, **(ii)** 5040, **(iii)** 192, **(iv)** 480

17. **(i)** 589, **(ii)** 220