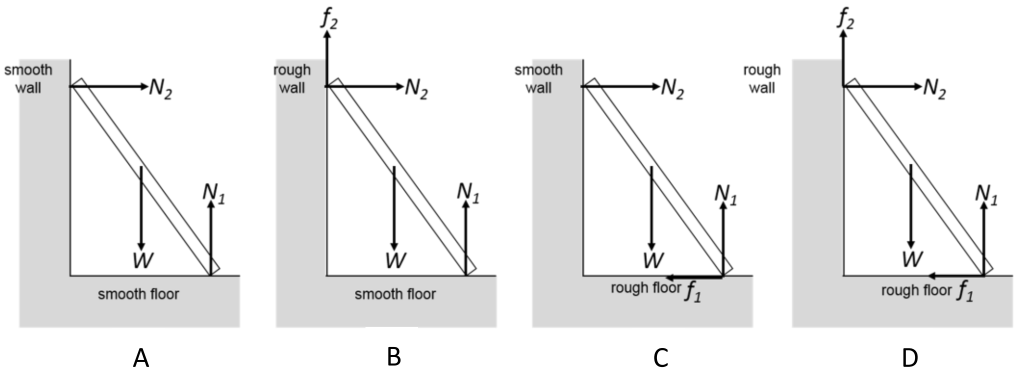
Tutorial 4 Forces

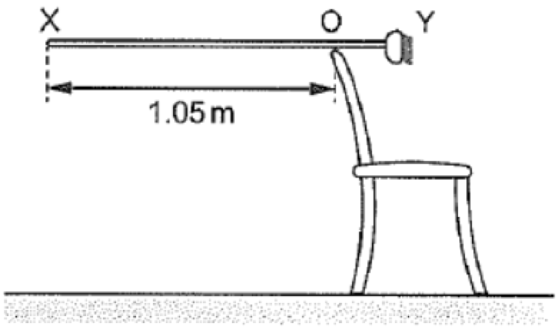
Discussion Questions (Suggested Solutions)

D1. On static equilibrium of a ladder



W : weight of ladder
N_1 : normal contact force by floor on ladder
N_2 : normal contact force by wall on ladder
f_1 : frictional force, by floor on ladder
f_2 : frictional force, by wall on ladder
In scenarios A and B, the ladder will definitely slip, as there is a net force: the horizontal forces are not balanced.

D2. Application of the Principle of Moments



Initially, the centre of gravity (c.g.) is at O. After the mass is tied to the handle, is 27 cm to the right of O, producing a clockwise moment about O.
Similarly, the mass is initially 95 cm to the left of O. After the broom is moved, it is 95 – 27 = 68 cm to the left of O, producing an anticlockwise moment about O.
Taking moments about O, by the principle of moments,
sum of anticlockwise moments =
sum of clockwise moments
(0.200 x 9.81) (0.68) = (9.81m) (0.27)
The mass of the broom m = 0.504 kg = 500 g (to 2 s.f.)

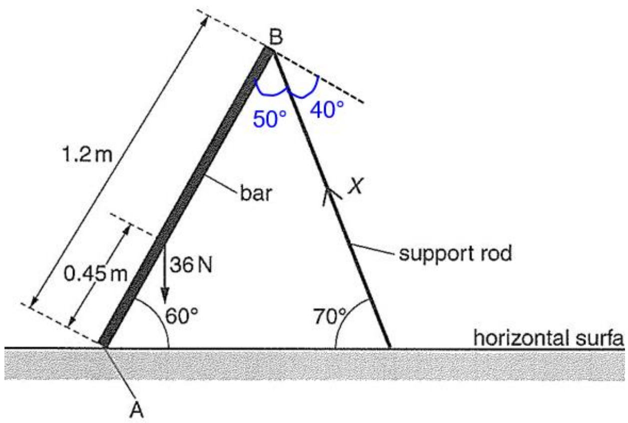
D3. On static equilibrium

(a) The component of X perpendicular to the bar is $X\cos 40°$.

Taking moments about A:
CW moment by 36 N = ACW moment by X

$$(36)(0.45\cos 60°) = (X\cos 40°)(1.2)$$

$$X = 8.81\ \text{N}$$



bi) Approach 1

For translational equilibrium, the net force acting on the bar must be zero.
The vector sum of the weight and X is a leftward and downward force. There must be a rightward and upward force at A.

Approach 2

For rotational equilibrium, the net moment acting on the bar about any point must be zero.

Taking moments about the C.G., X exerts an ACW moment. There must be a force at A that produces a CW moment.

bii) Method 1

Vertically, net force must be zero: $F_y + X\sin 70° = 36$ N

$F_y + (8.81)\sin 70° = 36$

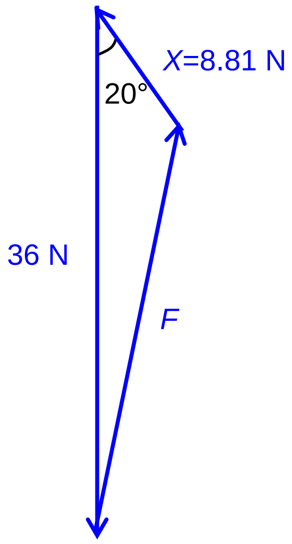
$F_y = 27.7$ N

Horizontally, net force must be zero: $F_x = X\cos 70°$

$F_x = \cos 70° = 3.01$ N

$$|F| = \sqrt{F_x^{\,2} + F_y^{\,2}} = \sqrt{3.01^2 + 27.7^2} = 27.9 \text{ N}$$

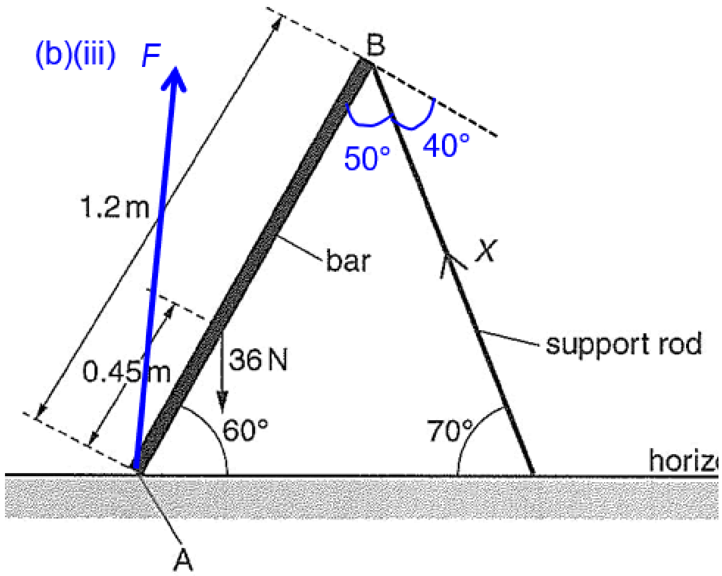
Method 2



Using cosine rule:

$$F^2 = 36^2 + 8.81^2 - 2(36)(8.81)\cos 20°$$

$$F = 27.9 \text{ N}$$

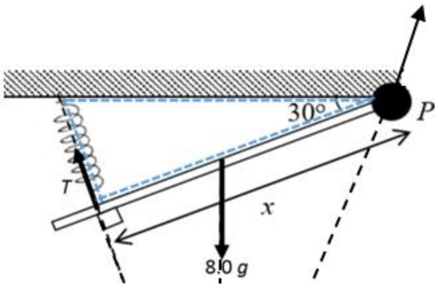


biii) See drawing.

COMMENT: Part (b)(ii) should be used to guide the drawing of F. The force's line of action must pass above the bar (because it is supposed to produce a CW moment about the CG to counter the ACW moment of X)

You may also recall that in a 3-force system, the lines of action of all 3 forces should intersect.

D4. Application of Hooke's Law. Static equilibrium of a rigid extended body.



Taking moments about P,

sum of clockwise moments =

sum of anticlockwise moments

$T\,x$ = (8.0 x 9.81) (0.25) cos 30°

(500 e) x = (8.0 x 9.81) (0.25) cos 30° ----- (1)

where e is the extension of the spring.

From the blue dashed triangle, using trigonometry,

tan 30° = (0.20 + e) / x

x tan 30° = 0.20 + e

e = x tan 30° – 0.20 ----- (2)

Combining (1) and (2),

16.99 = 500 (x tan 30° – 0.20) x

16.99 = 500 x^2 tan 30° – 100 x

$289x^2 - 100x - 16.99 = 0$

x = 0.47 m (accept) or x = -0.12 m (reject)

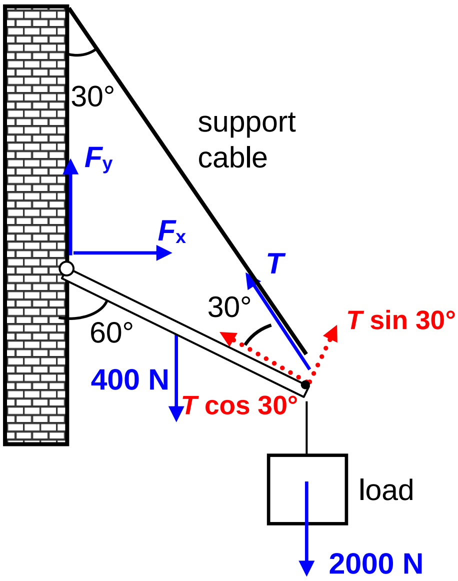
D5. On static equilibrium of a rigid extended body. [RJC/2009/Prelim/P3/Q1]

(a) The moment of a force about a point is the product of the (magnitude of the) force and the perpendicular distance from the line of action of the force to the point.

(b) (i) Taking moments about the hinge,

sum of clockwise moments =

sum of anticlockwise moments



$$(400)\left(\frac{L}{2}\sin 60^\circ\right)+(2000)(L\sin 60^\circ)=T\sin 30^\circ\,(L)$$

$$T = 3810\text{N} \quad (3\text{sf})$$

(ii) Analyse vertical components of forces:

$$\sum F_y = 0$$

$$F_y + T\cos 30^\circ = 2400$$

$$F_y = -900\text{N}$$

$$F_y = 900\text{N (downwards)}$$

Analyse horizontal components:

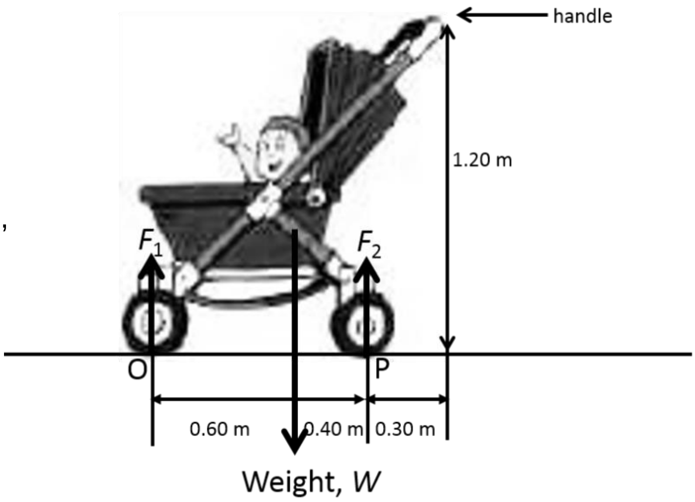
$$\sum F_x = 0$$

$$F_x = T\sin 30^0 \approx 1910\text{N (right)}$$

D6. On static equilibrium of a rigid extended body. [HCI/BT/05/P2/Q3(b)]

(i) Taking moments about the hind wheels,
sum of anticlockwise moments = sum of clockwise moments,
$(22.0)(9.81)(0.40) = F_1(1.00)$
$F_1 = 86.3$ N

Taking moments about the front wheels,
sum of anticlockwise moments = sum of clockwise moments,
$F_2(1.00) = (22.0)(9.81)(0.60)$
$F_2 = 129$ N



(ii) If the stroller topples, it will topple about point P. At the instant of toppling, the normal contact force from the ground acting on the front wheels (F_1) is zero.

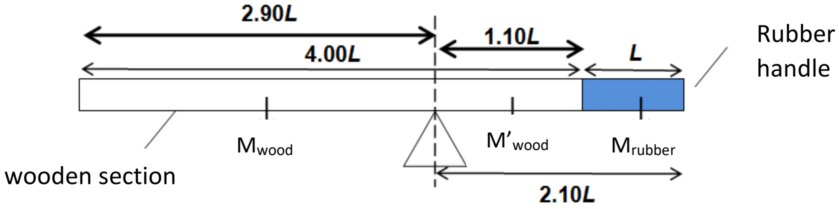
Taking moments about the hind wheels,
sum of anticlockwise moments = sum of clockwise moments,
$(22.0)(9.81)(0.40) = W(0.30)$
The weight of the groceries is $W = 288$ N.
Because the perpendicular distance from the handle to point P is 3/4 of the perpendicular distance from the centre of mass to the handle, the load can be 4/3 times as large as the combined weight of the stroller and the baby.

(iii) The baby may shift the centre of gravity towards P, which reduces the anticlockwise moments due to the baby about P.
Parents may lean on the handle and inadvertently press it down, creating an additional clockwise moment about P.
On an upward slope, the anticlockwise moment due to the weight of the stroller would decrease, but the clockwise moment due to the weight of the groceries would increase.

D7. Application of the principle of moments.

Answer: B

Method 1



Let A be the cross-sectional area of the rod.
Taking moments about the pivot,
$M_{wood}g(1.45L) = M'_{wood}g(0.55L) + M_{rubber}g(1.60L)$
$\rho_{wood}(2.90L)A(1.45L) = \rho_{wood}(1.10L)A(0.55L) + \rho_{rubber}(L)A(1.60L)$
$3.600\rho_{wood} = 1.60\rho_{rubber} \Rightarrow$ Ratio = 2.25

Method 2

Note that the centre of gravity of the rod is at the pivot. Hence, the pivot exerts an *upwards* force of magnitude W_{rod} at the pivot.
Taking moments about the left end of the rod,
W_{rod} (2.90L) = W_{wood} (2.00L) + W_{rubber} (4.50L)
$(4\rho_{wood} + \rho_{rubber})(2.90L) = (4\rho_{wood})(2.00L) + \rho_{rubber}(4.50L)$
$\rho_{rubber} / \rho_{wood} = 3.6 / 1.6 = 2.25$

D8. On upthrust.

Assume that in air, the upthrust by the air on the solid is negligible. Hence, W_1 is the weight of the solid.
When immersed in a liquid, let the upthrust by the liquid on the solid be U.
Thus, $W_2 = W_1 - U$
Since the solid is totally immersed in the liquid, by Archimedes' Principle, the upthrust is equal to the weight of the liquid displaced by the solid, $U = \rho V_{solid} g$.
Thus volume of the solid, $V_{solid} = (W_1 - W_2) / (\rho g)$

D9. On upthrust.

Answer: C

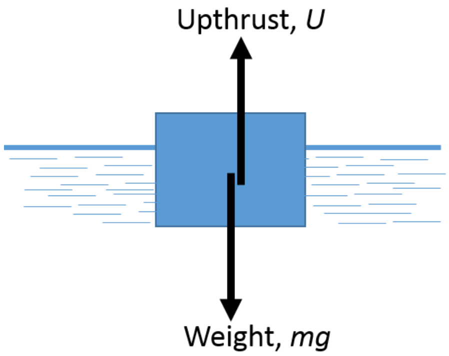
Upthrust acting on bubble = weight of air + viscous drag force
By Archimedes' principle, the upthrust acting on the bubble is equal to the weight of the water displaced,

weight of water displaced = weight of air + viscous drag force
$(1000)(2.370 \times 10^{-8})(9.81) = (1.290)(2.370 \times 10^{-8})(9.81)$ + viscous drag force

Hence, the viscous drag force is $F_{VD} = 2.322 \times 10^{-4}$ N

D10. On upthrust.

(a)



By the principle of flotation,

upthrust acting on wood = weight of wood

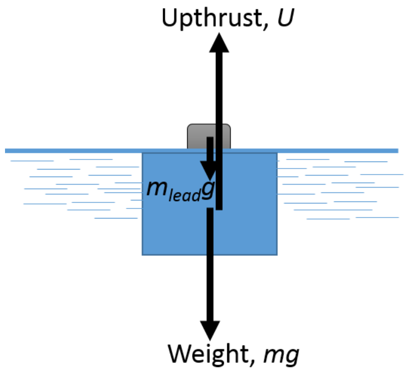
$$U = mg$$

$$\rho_{\text{water}} g A h_{\text{in water}} = \rho_{\text{wood}} g A h_{\text{wood}}$$

$$h_{\text{in water}} = \frac{\rho_{\text{wood}}}{\rho_{\text{water}}} h_{\text{wood}} = \frac{0.65 \times 10^3}{1.00 \times 10^3} 20 = 13 \text{ cm}$$

$$h_{\text{above water}} = 20 - 13 = 7.0 \text{ cm}$$

(b)



By the principle of flotation,

upthrust = weight of wood + weight of lead

$$U = m_{\text{wood}} g + m_{\text{lead}} g$$

$$\rho_{\text{water}}\, g\, A\, h = \rho_{\text{wood}}\, g\, A\, h + m_{\text{lead}} g$$

$$m_{\text{lead}} = 2.8 \text{ kg}$$

D11. On upthrust.

Answer: D

Identify the forces *X*, *Y*, *Z*.

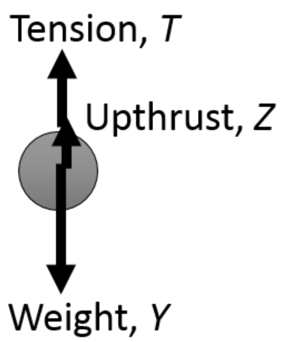
X: weight of the beaker and water

Y: weight of the solid

Z: weight of water displaced by object immersed in water = upthrust on object (by water)

Sketch the FBD of the object immersed in water.

T: tension by the spring balance on object (determine the spring balance reading)

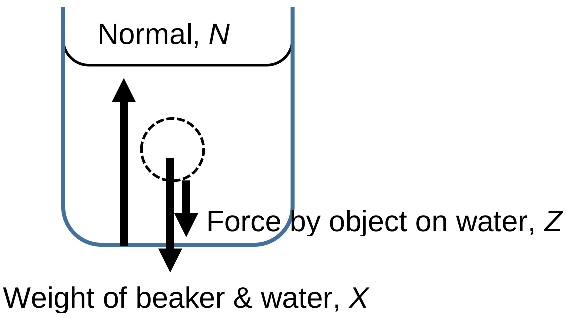


$$T = Y - Z$$

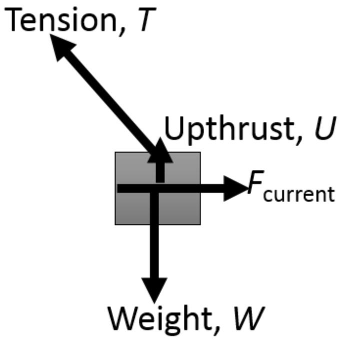
Sketch the FBD of the beaker & water.

N: normal force by weighing machine on Beaker & water (determine the weighing machine reading)

N = *X* + *Z*



D12. On upthrust.



$$\sum F_x = 0 \rightarrow F_{\text{current}} - T\sin 20 = 0$$

$$\rightarrow F_{\text{current}} = T\sin 20 \;\; (1)$$

$$\sum F_y = 0 \rightarrow T\cos 20 + U - W = 0$$

$$\rightarrow T = (W - U)\tan 20 \;\; (2)$$

Substituting equation (2) into equation (1),

$$F_{\text{current}} = (W - U)\tan 20$$

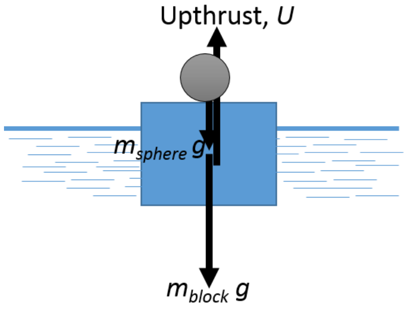
$$= \left(m_{\text{copper}}g - \rho_{\text{water}}gV_{\text{copper}}\right)\tan 20$$

$$= \left(0.50 \times 9.81 - 1 \times 10^3 \times 9.81 \times \frac{0.50}{9 \times 10^3}\right)$$

$$\rightarrow F_{\text{current}} = 1.59 \text{ N}$$

D13. On upthrust.

i.



ii. $$\sum F_y = 0 \rightarrow m_{\text{sphere}}g + m_{\text{block}}g = U$$

$$U = \rho_{\text{sphere}}V_{\text{sphere}}g + 0.050 \times g$$

$$\rightarrow U = 7.85 \times \frac{4}{3}\pi(0.70)^3 \times 9.81 \times 10^{-3} + 0.050(9.81)$$

Upthrust acting on the system (by the oil), $U = 0.601\ \text{N}$

iii. By Archimedes' Principle,

upthrust on system by oil = weight of oil displaced by block

$$U = \rho_{\text{oil}}gV_{\text{displaced}}$$

Hence, the volume of oil displaced by the wooden block $V_{\text{displaced}} = \frac{0.601\times10^3}{0.83\times9.81} = 73.8\ \text{cm}^3$

iv. Before the spherical ball dropped into the oil, the upthrust on the block was $U = m_{sphere}g + m_{block}g$. After it dropped into the oil, the upthrust on the block becomes $U = m_{block}g$.

The spherical ball dropped into oil is still displacing some oil, so it still experiences an upthrust. However, this upthrust is no longer supporting the ball's weight, so the upthrust acting on the ball is much smaller than the ball's weight. Hence, the total upthrust provided by the oil is smaller than before, which means that less oil is displaced than before: the oil level is lower than before.

Numerical solution:

	Upthrust (while the sphere is on the block)	Volume of oil displaced by block (while the sphere is on the block)	Upthrust (after the sphere fell into the oil)	Volume of oil displaced by sphere and block (after the sphere fell into the oil)
Sphere	0.11 N (weight of sphere)		$\rho_{\text{oil}} \times g \times 1.44$ = 0.012 N	$\frac{4}{3}\pi(0.7)^3$ = 1.44 cm^3
Block	0.4905 N (weight of block)		0.4905 N	$0.4905 = \rho_{\text{oil}} \times g \times V_{\text{disp}}$ $V_{\text{disp}} = 60.2$ cm^3
Total	**0.601 N (calculated earlier)**	**73.8 cm^3 (calculated earlier)**	**0.502 N**	**61.6 cm^3**