INNOVA JUNIOR COLLEGE
JC 2 PRELIMINARY EXAMINATION
in preparation for General Certificate of Education Advanced Level
Higher 2

CANDIDATE
NAME

CIVICS GROUP

INDEX NUMBER

Mathematics **9740/02**

Paper 2 **12 September 2017**

3 hours

Additional materials: Answer Paper
Cover Page
List of Formulae (MF 15)

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number on all the work you hand in.

Write in dark blue or black pen on both sides of the paper. You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages.

Section A: Pure Mathematics [40 marks]

1 A graphic calculator is **not** to be used in answering this question.

    (i) Solve the equation

$$w^5 = 243,$$

    giving the roots in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \le \pi$.   [3]

    (ii) The roots represented by w_1 and w_2 are such that $-\pi < \arg(w_1) < \arg(w_2) < 0$. On an Argand diagram, sketch the locus $|z - w_1| = |z - w_2|$.   [2]

2 The function f is given by $f : x \mapsto 3 + \dfrac{1}{x-2}$ for $x \in \square$, $x > 2$.

    (i) Find $f^{-1}(x)$ and state the domain of f^{-1}.   [3]

    (ii) Explain why the composite function f^2 exists.   [1]

    (iii) Find the value of x for which $f^2(x) = x$. Explain why this value of x satisfies the equation $f(x) = f^{-1}(x)$.   [3]

3 It is given that a curve C has parametric equations

$$x = t^2 - t, \quad y = \frac{1}{t^2 + 1} \quad \text{for } -2 \le t < 2.$$

    (i) Sketch C, indicating clearly the coordinates of the end points and the points where C cuts the y-axis.   [4]

    (ii) Find the equation of the tangent to C that is parallel to the y-axis.   [4]

    (iii) Express the area of the region bounded by C, the tangent found in part **(ii)** and both axes, in the form

$$\int_a^b f(t) \, dt,$$

    where the function f and the constants a and b are to be determined. Hence find this area, leaving your answer in exact form.   [5]

4 A farmer owns a plot of farmland. To prepare for wheat planting, the farmer has to plough the farmland before sowing wheat seeds. At the start of the first week, 300 m^2 of the farmland is ploughed. The farmer ploughs another 100 m^2 of the farmland at the beginning of each subsequent week. To sow wheat seeds, the farmer is considering two different options.

(a) In the first option, the farmer sows wheat seeds on 60% of the **unsown** ploughed land at the end of each week.

  (i) Find the area of **unsown** ploughed land at the end of the second week. [1]

 (ii) Show that the area of **unsown** ploughed land at the end of the nth week is given by

$$\left[0.4^n (300) + k \left(1 - 0.4^{n-1} \right) \right] \text{ m}^2,$$

where k is an exact constant to be determined. [3]

(iii) Find the number of complete weeks required for the area of **unsown** ploughed land to first fall below 70 m^2. [3]

(b) In the second option, the farmer sows 80 m^2 of the **unsown** ploughed land at the end of the first week. At the end of each subsequent week, he sows 20 m^2 of the **unsown** ploughed land more than in the previous week. This means that the area of sown ploughed land is 100 m^2 in the second week, 120 m^2 in the third week, and so on.

  (i) Find, in terms of n, the area of **unsown** ploughed land at the end of the nth week. [4]

 (ii) Find the number of complete weeks required for the farmer to finish sowing all the ploughed farmland in this option. Deduce the area of ploughed land to be sown in the final week. [4]

4

Section B: Statistics [60 marks]

5 A group of twelve people consists of six married couples. Each couple consists of a husband and a wife.

    (i) The twelve people are to stand in a straight line. Find the number of different arrangements if each husband must stand next to his wife. [2]

    (ii) The group of twelve people finds a round table with ten chairs. Assuming only ten people are to be seated, find the probability that five married couples are seated such that each husband sits next to his wife and husbands and wives alternate. [3]

6 Seven red counters and two blue counters are placed in a bag. All the counters are indistinguishable except for their colours. Clark and Kara take turns to draw a counter from the bag at random with replacement. The first player to draw a blue counter wins the game and the game ends immediately.

If Clark draws first, find the probability that

    (i) Clark wins the game at his third draw, [2]

    (ii) Kara wins the game. [3]

7 The number of calls received at an office per 10 minutes is modelled by a Poisson distribution with mean 4.

    (i) Find the length of time, to the nearest minute, for which the probability that no calls are received is 0.1. [2]

    (ii) Using a suitable approximation, find the probability that there are at most 50 calls in a period of 2 hours. [3]

Two assumptions are required for a Poisson distribution to be a suitable model for the number of calls received at an office. They are as follows.

- Calls arrive at a uniform average rate.

- Calls arrive independently of each other.

    (iii) Comment briefly on the validity of each of these assumptions if the office is the civil defence emergency control room. [2]

8 At a hospital, records show that 84.5% of patients turn up for their appointments. It is known that on any day, the doctor has time to see 20 patients.

On one particular day, there are 20 patients who make appointments to see the doctor.

   (i) State, in this context, one condition that must be met for the number of patients who turn up for their appointments to be well modelled by a binomial distribution. [1]

For the remainder of this question, assume that the condition stated in part **(i)** is met.

  (ii) Find the probability that more than 15 patients turn up for their appointments. [2]

 (iii) Given that at least 12 patients turn up for their appointments, find the probability that more than 2 patients fail to turn up for their appointments. [3]

 (iv) In a particular month, there are a total of 300 patients who make appointments to see the doctor. Using an approximation, find the probability that the number of appointments for which the patient fails to turn up during this period is between 40 and 50, inclusive. [3]

9 In order to recruit the best possible employees, a large corporation has designed an entrance test that consists of three components, namely Logical Reasoning, Personality and Communication. The scores obtained by candidates in each of the three components are independent random variables L, P and C which are normally distributed with means and standard deviations as shown in the table.

	Mean	Standard deviation
Logical Reasoning, L	35.2	5.2
Personality, P	24.6	3.8
Communication, C	29.3	4.3

   (i) For a particular role in the corporation, the Logical Reasoning and the Personality scores of a candidate is valued and hence a special score of $3L+2P$ is computed.

     (a) Find the special score that is exceeded by only 1% of candidates taking the test. Leave your answer in 1 decimal place. [4]

     (b) Five candidates are selected randomly. Find the probability that three of them obtained a special score of more than 150, and the other two obtained less than 140. [3]

  (ii) For another role in the corporation, a candidate must achieve a result such that his special score of $3L+2P$ differs from $5C$ by less than 25. Find the percentage of candidates who will be able to achieve this. [4]

10 The following table shows the mass (m) of a foetus, in grams, taken at various weeks (t).

t	12	16	20	24	28	32	36
m	14	100	300	600	1005	1702	2622

   (i) Draw a scatter diagram to illustrate the data, labelling the axes clearly. [1]

   (ii) Calculate the product moment correlation coefficient between t and m, giving your answer correct to 5 decimal places. Explain why this value does not necessarily mean that the linear model is the best model for the relationship between t and m. [2]

It is proposed that the mass of the foetus at week t can be modelled by

$$m = at^b,$$

where a and b are positive constants.

   (iii) By using logarithm to transform $m = at^b$ into a linear equation, calculate the value of the product moment correlation coefficient and give two reasons why this model may be a better model. [4]

   (iv) Calculate the values of a and b. [2]

   (v) Using the equation of a suitable regression line, estimate the mass of the foetus at 26 weeks, giving your answer to the nearest grams. Comment on the reliability of the estimate. [2]

11 The mass of strawberry jam in a randomly chosen jar follows a normal distribution and has a mean mass of 200 grams. A retailer suspects that the mean mass of the strawberry jam is being overstated. He takes a random sample of 30 jars of strawberry jam and weighs the content, x grams, in each jar. The results are summarized as follows.

$$\sum(x - 200) = -66 \quad \text{and} \quad \sum(x - 200)^2 = 958$$

   (i) Test at 2% significance level, whether the retailer's suspicion is justifiable. [6]

   (ii) Explain, in this context, the meaning of 'at 2% significance level'. [1]

   (iii) Suppose the retailer now decides to test whether the mean mass differs from 200 grams at 2% significance level. Without carrying out the test, explain whether the conclusion would change in part **(i)**. [1]

The manufacturing process has now been improved and the population standard deviation is 3.5 grams. The retailer selects a new random sample of 20 jars of strawberry jam and the sample mean is found to be k grams. Find the range of possible values of k so that the retailer's suspicion that the mean mass differs from 200 grams is not justified at the 2% significance level. Give your answer correct to one decimal place. [4]