Topic 18
Alternating Current

1 A sinusoidal current flowing in a 10 Ω resistor varies with time according to the equation I = 1.2 sin (100πt). Calculate the instantaneous current and the instantaneous power dissipated at

(a) t = 0.005 s
$I = 1.2\sin(100\pi \times 0.005) = 1.2$ A
$P = I^2R = 1.2^2R = 14.4$ W

(b) t = 0.010 s
$I = 1.2\sin(100\pi \times 0.010) = 0$ A
$P = 0$ W

(c) t = 0.018 s
$I = 1.2\sin(100\pi \times 0.018) = -0.705$ A
$P = I^2R = (-0.705)^2 10 = 4.98$ W

2 Determine the r.m.s. current in each case.

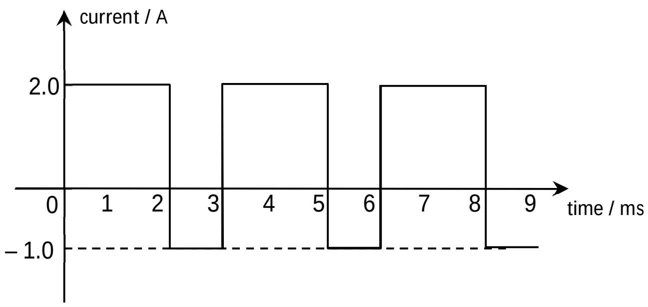
(a) A sinusoidal current of peak value 2.0 A.
For a sinusoidal current, $I_{r.m.s.} = \frac{I_o}{\sqrt{2}} = \frac{2}{\sqrt{2}} = 1.41$ A

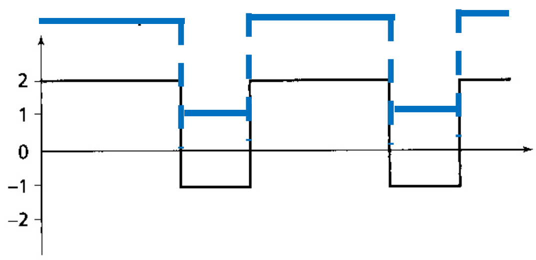
(b) A full-wave rectified sinusoidal current of peak value 3.0 A.
When a full-wave rectified sinusoidal current is squared, the power dissipated would be similar to the that of the regular sinusoidal current.
Hence, $I_{r.m.s.} = \frac{I_o}{\sqrt{2}} = \frac{3}{\sqrt{2}} = 2.12$ A

(c) A square-wave current with a frequency of 1 Hz which is 0.1 A for one half cycle and –0.1 A for the next half cycle.

I^2 would yield a constant value of 0.01 A^2. Hence $\langle I^2 \rangle$ = 0.01 A^2 and I_{rms} = 0.1 A.

(d) An uneven square wave current as shown below.





$$\langle I^2 \rangle = \frac{\int_0^T I^2 dt}{T} = \frac{4(2) + 1(1)}{3} = 3.0 \text{ A}^2$$
$$\therefore I_{rms} = \sqrt{\langle I^2 \rangle} = 1.73 \text{ A}$$

3 An alternating current of r.m.s. value 2 A and a steady direct current I flowing through identical resistors dissipate heat at equal rates. Determine the value of current I.
Since the r.m.s. value of an a.c. is the value of a steady d.c. which will dissipate energy at the same rate as the mean power dissipated by an a.c. in a given resistor, the value of I is 2 A.

4 A steady current I dissipates power P in a variable resistor. The resistance has to be halved to obtain the same power when a sinusoidal alternating current is used. Determine the r.m.s. value of the alternating current in terms of I.

$P_{dc} = I^2 R$ and $P_{ac} = I_{r.m.s.}^2 \frac{R}{2}$

Since $P_{dc} = P_{ac}$

$$I^2 R = I_{r.m.s.}^2 \frac{R}{2}$$

$$I_{r.m.s.} = \sqrt{2} I$$

5 A sinusoidal current described by the equation $I = 9.0 \sin \omega t$ flows through a 12.0 Ω resistor. Calculate

(a) the r.m.s. value of the current through and potential difference across the resistor.

$I_o = 9.0$ A

For a sinusoidal current, $I_{r.m.s.} = \frac{I_o}{\sqrt{2}} = \frac{9.0}{\sqrt{2}} = 6.36$ A

$V_{r.m.s.} = I_{r.m.s.} R = 6.36 \times 12 = 76.4$ V

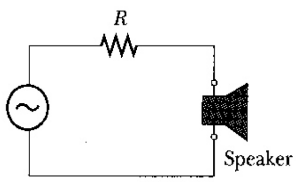
(b) the maximum instantaneous power dissipated in the resistor.

$P_o = I_o^2 R = 9^2 \times 12 = 972$ W

(c) the mean power dissipated in the resistor.

$P_{mean} = I_{r.m.s.}^2 R = 6.36^2 \times 12 = 486$ W

6 An audio amplifier, represented by an a.c. source and the resistor *R* delivers alternating voltages at audio frequencies to the speaker. If the source puts out an r.m.s. alternating p.d. of 15 V, resistance *R* is 8.20 Ω and the speaker is equivalent to a resistance of 10.4 Ω, calculate the average power delivered to the speaker.



By potential divider principle,

$$V_{speaker\ (r.m.s.)} = \frac{R_{speaker}}{R_{total}} V_{total\ (r.m.s.)} = \frac{10.4}{10.4 + 8.2}(15) = 8.39 \text{ V}$$

$$\langle P \rangle = \frac{V_{speaker\ (r.m.s.)}^2}{R_{speaker}} = \frac{8.39^2}{10.4} = 6.76 \text{ W}$$

7 **(a)** A power station generates 100 kW of power which is transmitted at a potential difference of 10 kV. Given that the connecting cables have a total resistance of 20 Ω, determine the turn ratio required for an ideal step-down transformer to bring electrical energy to the home at 240 V.

Current through transmission cables $I_{cable} = \frac{P_{transmitted}}{V_{transmission}} = \frac{100\times10^3}{10\times10^3} = 10$ A

Potential difference across transmission cable $V_{cable} = I_{cable}R = 10 \times 20 = 200$ V

Potential difference at step-down transformer = $10 \times 10^3 - 200 = 9800$ V

Hence turn ratio, $\frac{N_p}{N_s} = \frac{9800}{240} = \frac{245}{6}$

(b) Explain why, for the efficient transmission of electrical energy, it is necessary to

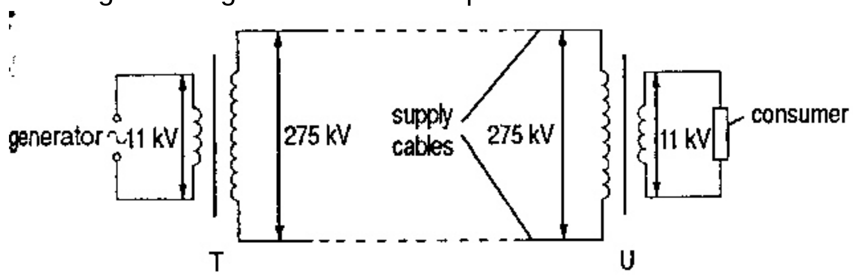
(i) use an alternating supply and

The process of stepping up (or down) utilizes the concept of electromagnetic induction which requires a changing magnetic linkage linking the secondary coil. This means that a changing magnetic flux has to be produced at the primary coil, which can only be achieved when an alternating supply is used.

(ii) transmit at high voltage.

When electrical energy is transmitted at high voltage, the transmission current can be reduced, which will result in less power loss through the transmission cables.

8 Electrical power of 4400 kW is supplied to an industrial consumer at a considerable distance from a generating station. This is represented below.



In order to do this, the electricity supply company makes use of a circuit containing two transformers T and U. The transformers can be considered to be ideal and the supply cables to have negligible resistance.

(a) The power is generated at 11 kV r.m.s. and is supplied to the consumer at 11 kV r.m.s. Calculate, for the current supplied to the consumer, its

(i) r.m.s. value, and

$$I_{r.m.s.} = \frac{P}{V_{r.m.s.}} = \frac{4400 \times 10^3}{11 \times 10^3} = 400 \text{ A}$$

(ii) peak value.

For a sinusoidal current, $I_o = I_{r.m.s.}\sqrt{2} = 400\sqrt{2} = 566$ A

(b) There is a potential difference of 275 kV r.m.s. between the supply cables. Calculate

(i) the ratio $\frac{N_s}{N_p}$ for each transformer

For transformer T, $\frac{N_S}{N_p} = \frac{V_S}{V_p} = \frac{275}{11} = 25$

For transformer U, $\frac{N_S}{N_p} = \frac{V_S}{V_p} = \frac{11}{275} = \frac{1}{25}$

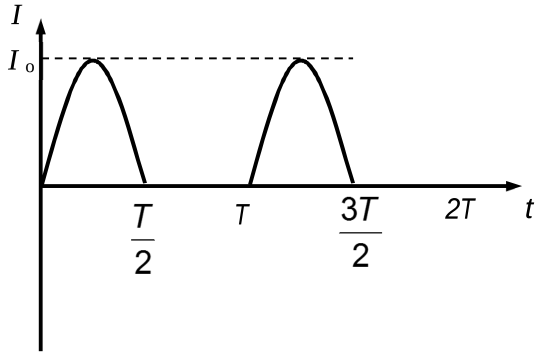
(ii) the r.m.s. value of current in the supply cables

$I_{r.m.s.} = \frac{P}{V_{r.m.s.}} = \frac{4400 \times 10^3}{275 \times 10^3} = 16 \text{ A}$

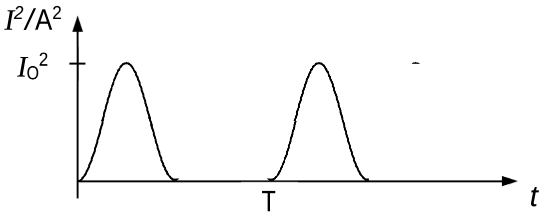
(c) Explain why, when the resistance of the supply cables cannot be neglected, this arrangement is preferable to a system which generates and transmits the power at the same voltage of 11 kV r.m.s.

If there is resistance in the supply cables, there would be power loss (I^2R) when transmitting electrical energy across the cables. With a step-up and step-down transformer, the electrical energy would be transmitted at a current lower than 400 A which would result in a much smaller power loss.

9 Determine the r.m.s. current I_{rms}, in terms of its peak current I_o, for a half-wave rectified sinusoidal current as shown below.



Squaring the current:



Hence, for half-wave rectification, $\langle I^2 \rangle = \frac{1}{4} I_o^2 \Rightarrow I_{rms} = \frac{1}{2} I_o$

Answer Key

1 **(a)** 1.2 A, 14.4 W **(b)** 0 A, 0 W **(c)** –0.705 A, 4.98 W

2 **(a)** 1.41 A **(b)** 2.12 A **(c)** 0.1 A **(d)** 1.73 A

3 2 A

4 $\sqrt{2}\,I$

5 **(a)** 6.4 A, 76.4 V **(b)** 972 W **(c)** 486 W

6 6.76 W

7 **(a)** 245:6

8 **(a)(i)** 400 A **(ii)** 566 A **(b)(i)** T:25, U: $\frac{1}{25}$ **(ii)** I_{rms}=16 A

9 I_o / 2