XINMIN SECONDARY SCHOOL
新民中学
SEKOLAH MENENGAH XINMIN
Preliminary Examination 2024

CANDIDATE NAME			
CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS 4049/01

Paper 1 **23 August 2024**

Secondary 4 Express **2 hour 15 minutes**

Setter : Mr Johnson Chua
Vetter : Ms Low Yan Jin
Moderator : Ms Pang Hui Chin

Candidates answer on the Question Paper.
No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your name, register number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is **90.**

Errors	**Qn No.**	**Errors**	**Qn No.**
Accuracy		Simplification	
Brackets		Units	
Geometry		***Marks Awarded***	
Presentation		***Marks Penalised***	

For Examiner's Use
90

Parent's/Guardian's Signature:

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1}b + \binom{n}{2} a^{n-2}b^2 + \ldots + \binom{n}{r} a^{n-r}b^r + \ldots + b^n \ ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)...(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} bc \sin A$$

1 A triangle has a base of length $\left(6+2\sqrt{7}\right)$ cm and an area of $\left(17+7\sqrt{7}\right)$ cm^2.

Find, **without using a calculator**, the perpendicular height to the base of the triangle, in cm, in the form $\left(a+b\sqrt{7}\right)$, where a and b are integers. [3]

2 Solve the equation $\sqrt{5-\sqrt{x+1}}=\sqrt{x}$. [4]

3 The equation of a curve is $y = \dfrac{4x+2}{\sqrt{x+1}}$, where $x > -1$.

(a) Find $\dfrac{dy}{dx}$, leaving your answer in the form $\dfrac{ax+b}{\sqrt{(x+1)^n}}$, where a, b and n are constants. [2]

(b) Explain why the curve is a increasing function. [2]

4 The line $2x+3y=12$ intersects the curve $y^2=4x-8$ at points A and B.

Find the value of p and q for which the length of AB can be expressed as $p\sqrt{q}$. [6]

5 The height, h m, of a baseball above ground t seconds after it has been hit is given by $h = c + 24t - 4t^2$, where c is a constant.

(a) If c = 1.65, express h in the form $h = p + q(t + r)^2$ where p, q and r are constants to be determined. Hence, state the maximum height attained by the baseball and the time at which this occurs. [4]

(b) Find the range of values of c if the baseball did not reach a height of 40 m. [2]

6 A curve is such that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=12e^{6x}+15e^{-3x}$.

The point $P(0,-2)$ lies on the curve and the normal to the curve at P is parallel to the y-axis. Find the equation of the curve. [6]

7 The equation of a curve is $y = kx^2 + kx + p$, where p and k are constants.

(a) Show that $p > \frac{k}{4}$ for which the curve lies completely above the x-axis. [3]

(b) In the case where $k = 2$ and $p = 4$, find the values of m for which the line $y = mx - 4$ is a tangent to the curve. [4]

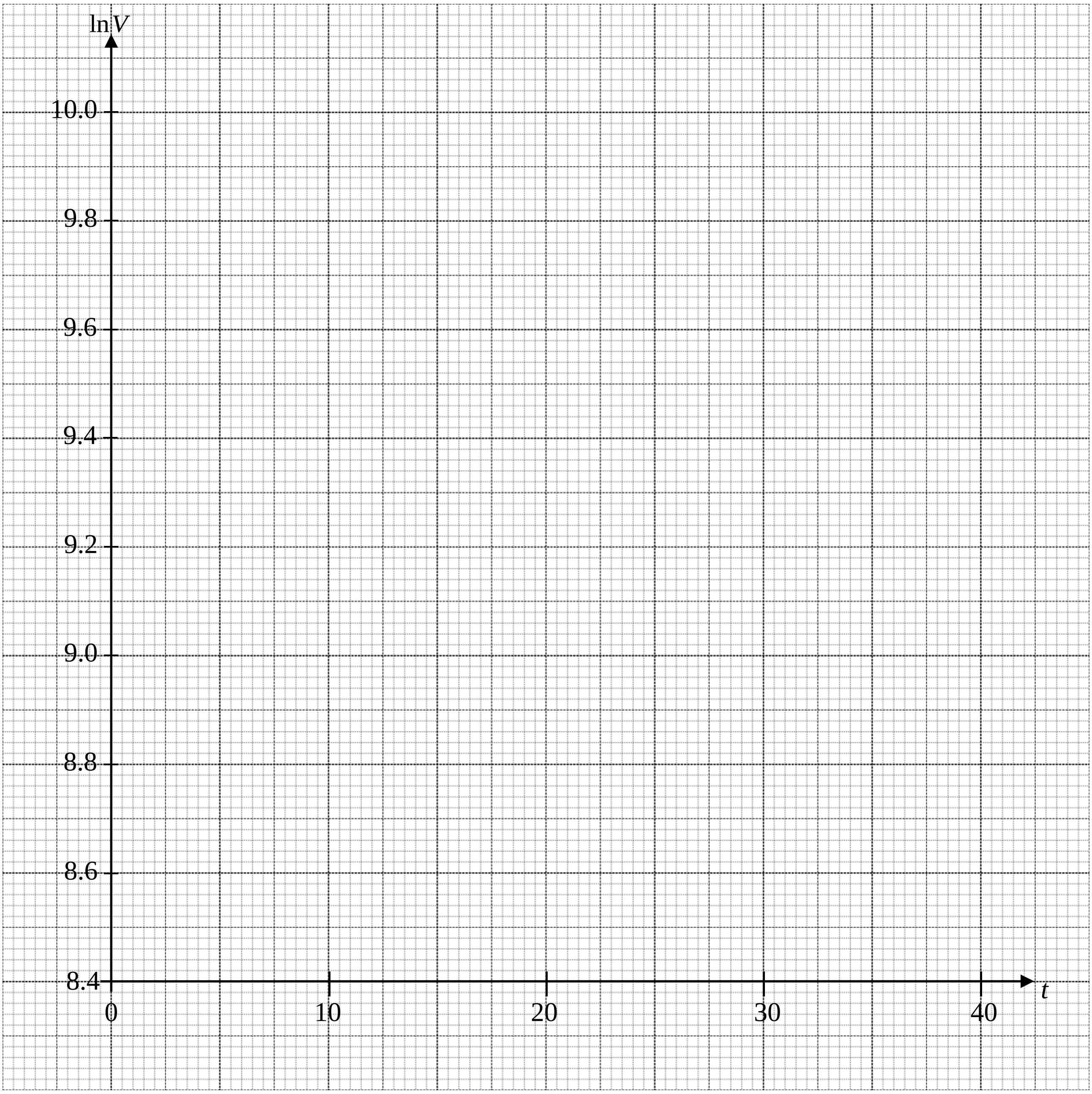
8 **(a)** Divide $2x^3 + 5x^2 + 5x + 9$ by $x^3 + 3x$. [1]

(b) Express $\dfrac{2x^3 + 5x^2 + 5x + 9}{x^3 + 3x}$ in partial fractions. [5]

9 The value, \$$V$, of a watch is related to t, the number of years since 1980.
The table below gives the value of the watch in 1990, 2000, 2010, 2020.

Year	1990	2000	2010	2020
t (years)	10	20	30	40
V (\$)	7200	9600	12 800	17 200

(a) Plot $\ln V$ against t and draw a straight line graph to illustrate the information. [2]



(b) Find the gradient and the intercept of the vertical axis of your straight line graph. Hence, express V in the form Ae^{kt}, where A and k are constants. [4]

(c) Explain what the constant A represents. [1]

10 **(a)** **(i)** Write down the first four terms in the expansion of $(3-2x)^7$. [2]

(ii) Find the coefficient of x^3 in the expansion of $\left(1-7x^2\right)(3-2x)^7$. [2]

(b) In the binomial expansion of $\left(x+\frac{k}{x}\right)^{11}$, where k is a positive constant, the coefficient of $\frac{1}{x^5}$ is 8 times the coefficient of x^3. Find the value of k. [5]

11 The triangle ABC is such that A is $(9,9)$, B is $(1,-3)$ and C is (p,q) where $q > p$.
C lies on the perpendicular bisector of AB and area of triangle ABC is 26 units^2.

(a) Find the equation of the perpendicular bisector of AB. [4]

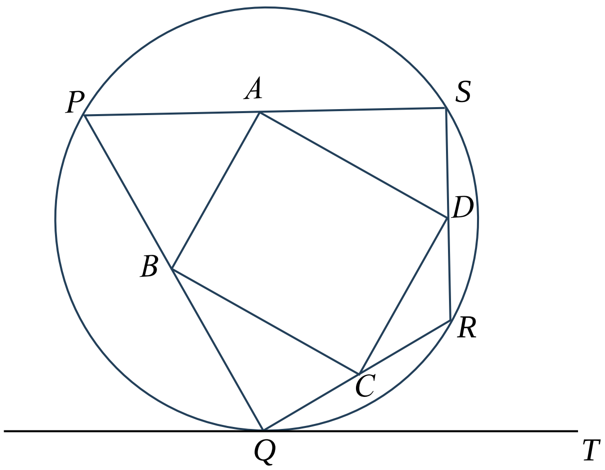
(b) Find the coordinates of C. [3]

12 (a) The graph of $y = a\sin bx + 2$ has one maximum point at $\left(\frac{5\pi}{4}, 7\right)$ and the next maximum point after this has coordinates $\left(\frac{9\pi}{4}, 7\right)$. Find the values of *a* and *b*. [2]

(b) (i) Sketch, on the same diagram, the graph of $y = -4\cos 4x$ and $y = -\frac{4}{\pi}x + 4$ for $0 \leq x \leq \pi$ radians. [4]

(ii) Hence, state the number of solutions to the equation $\frac{x}{\pi} - \cos 4x - 1 = 0$ for $0 \leq x \leq \frac{\pi}{2}$. [2]

13



In the diagram, *PQRS* is a kite, where all four points lie on the circumference of the circle.
RS = *RQ* and *PQ* = *PS*. *QT* is a tangent to the circle at *Q*.

(a) Show that $\angle QPS = 2 \times \angle RQT$. [3]

(b) A circle can be drawn passing through A, B, C and D, with BD as the diameter.
Given that A, B, C and D are midpoints of PS, PQ, QR and SR respectively, what can you deduce about quadrilateral $ABCD$? [4]

14 A particle travelling in a straight line, has a velocity, v m/s, at time t seconds, $t \geq 0$, given by $v = 3\sin 2t - 4\cos 2t$.

(a) Find the initial acceleration of the particle. [2]

(b) Find the total distance travelled by the particle in the first 1.5 seconds. [8]

Continuation of working space for question **14(b).**

END OF PAPER

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