KUO CHUAN PRESBYTERIAN SECONDARY SCHOOL
2023 Preliminary Examination
Secondary 4 Express / 5 Normal Academic

NAME

CLASS INDEX NUMBER

ADDITIONAL MATHEMATICS 4049/01

Paper 1

28 August 2023

2 hours 15 minutes

Candidates answer on the Question Paper.
No Additional Materials are required.
Setter: Mr. Wilson Sitoh

READ THESE INSTRUCTIONS FIRST
Write your name, class and index number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non- exact numerical answers correct to three significant figures or 1 decimal place in the case of angles in degrees, unless a different degree of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.

The number of marks is given in brackets [ ] at the end of each question or part question.

Parent's Signature		
For Examiner's Use		
Paper 1 (50%)		/ 90 m
Paper 2 (50%)		/ 90 m
TOTAL %		/ 100

The total number of marks for this paper is 90.

Areas to Note		
	Accuracy	
	Presentation	

	Pencil	
	Units	

This document consists of **19** printed pages and 1 blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1.$$

$$\sec^2 A = 1 + \tan^2 A.$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A.$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae of ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

1 **(a)** Express $-2x^2 + 3x + 2$ in the form $a(x + b)^2 + c$ where a, b and c are constants. [2]

(b) Use your answer from part **(a)** to explain why the curves with equations

$y = -2x^2 + 3x + 2$ and $(x - 9)^2 + (y - 15)^2 = 25$ will not intersect. [4]

2 The table shows experimental values of two variables, x and y, which are connected by an equation of the form $y = \frac{k}{x^n}$, where n and k are constants.

x	1.58	2.00	5.01	13.80
y	20.89	15.14	4.37	1.10

(a) Plot lg y against lg x and draw a straight line graph to illustrate this information. [2]

(b) Use your graph to estimate the value of k and n. [3]

(c) By drawing another straight line on the same axes, estimate the value of x for which $\lg y = \lg x^2$. [2]

3 Express $\frac{4x^3-x^2+37x-50}{(x^2+9)(x-1)}$ in partial fractions. [6]

4 A curve has equation $y = x^2 - x - 12$. Points A and B lie on the curve and have x–coordinates of k and $2k$ respectively.

(a) Find the range of values of k for which the y–coordinate of B is lesser than y–coordinate of A . [3]

(b) Explain why the gradient of the curve at A is always lesser than the gradient of the curve at B. [2]

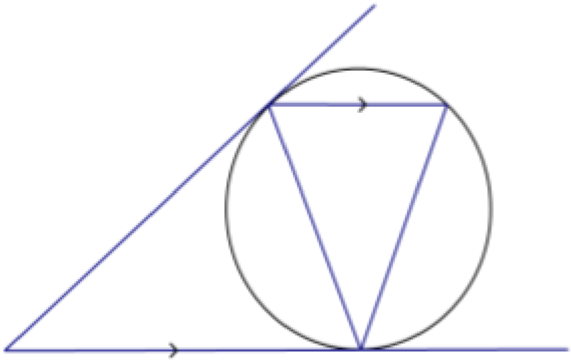
5 **(a)** Find the first 4 terms in the expansion of $\left(2-\frac{3}{x^2}\right)^7$ in descending powers of x, simplifying each term. [3]

(b) Hence, find the coefficient of x in the expansion of $(5x-4x^3)\left(2-\frac{3}{x^2}\right)^7$. [2]

(c) Given that the constant term in the binomial expansion of $\left(x^2 - \frac{k}{x}\right)^9$ is 61236, find the value of the positive constant *k*. [4]

6 A curve is such that $\frac{d^2y}{dx^2} = 4 - 3x$ and the point P (2,5) lies on the curve. The gradient of the curve at P is 2. Find the equation of the curve. [6]

7



In the diagram, TE and TC are tangents to the circle at A and B respectively.
BD is parallel to TE.

(a) Prove that $AB = AD$. [3]

(b) Prove that $(AB)^2 = TA \times BD$. [3]

8 The function f is defined, for $x > \frac{3}{2}$, by $\mathrm{f}(x) = \frac{e^{2x}}{1-x}$.

Explain clearly whether f is an increasing or a decreasing function. [5]

9 **(a)** Find the amplitude and period of $3 \sin sin\, 2x - 2$. [2]

(b) Sketch the curve $y = 3 \sin sin\, 2x - 2$ for $0° \leq x \leq 360°$ [2]

10 The normal to the curve $y = 2\tan 3x - 1$ at point $P(\frac{\pi}{3}, \pi)$ crosses the y-axis at point A and the x-axis at B. Find the exact area of triangle AOB. [6]

11 Find the coordinates of the points of intersection of the curve $x+3=2xy$ and the curve $y=\frac{2}{\sqrt{x}}$. [5]

12 **(a)** The equation of a curve is $y = -2x^2 - kx - 6 + k$, where k is a constant.

Find the range of values of k for which the curve lies completely below the x-axis. [4]

(b) Given that $2x^2 + bx + c$ is always positive, write down and simplify the relationship between b and c. Explain why c is never negative. [3]

13 **(a)** Solve $2x^3 - x^2 - 15x - 10 = 0$, showing all necessary working. [4]

(b) Solve the equation $x - 3x + 1 = 0$. [5]

14 A ferris wheel with a radius of 15 m completes one rotation every 3 minutes. Passengers get onto the ferris wheel at the lowest point of the wheel, which is 1 m above the ground. Suppose the ferris wheel starts to rotate once passengers board at the point. An equation that expresses how high a passenger is above the ground t minutes after boarding, h m can be represented by $h =- a \cos cos\, b\, t + c$.

(a) Find the values of a, b and c. [4]

(b) Obtain an expression for the velocity of the ferris wheel at time t and hence deduce the maximum speed. [3]

(c) Find the magnitude of the acceleration of the ferris wheel when $t = 1$. [2]

End of Paper

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