Answer all the questions

1 The right angled triangle *PQR* is such that its height, *PQ* is $(\sqrt{2}+4)$ cm and the area is $(5\sqrt{2}-1)$ cm². *Without using a calculator*, find

(a) the base, QR, in cm in the form of $a + b\sqrt{2}$, where a and b are integers. [4]

(b) in expression, in cm², for PR^2 in the form of $c + d\sqrt{2}$, where c and d are [3] integers.

4

2 (a) (i) Find the range of values of x for which $2x^2 - 8x + 6 > 2x - 2 - x^2$ [3] for all real values of x.

5

(ii) Hence, explain the significance of the answer. [1]

A cup of hot milo was left in a room to cool. The temperature T^{*C} at time t minutes is given by $T = 27 + Pe^{kt}$, where P and k are constants. It is given that when t = 0, T = 75, and when t = 3, T = 55.

6

(i) Find the values of P and k. [3]

(ii) Find the time t when T = 35.

[2]

(b)

(iii) Explain why the minimum temperature of the cup of milo is 27 C if [2] the cup of milo is left in the room for a very long time.

3 (a) The mass, *m* grams, of a piece of radioactive element **X** after *t* days is recorded in the following table.

t	2	4	6	8
т	5.47	3.74	2.56	1.75

- (i) On the grid on page 9, by plotting ln *m* against *t*, draw a straight line to illustrate this data. [3]
- (ii) Use your graph to estimate the time when m is 5. [2]

It is known that the correct formula connecting t and m is $m = Be^{-gt}$ where B and g are constants.

(iii) Use your graph to estimate values of *B* and *g*. [3]

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(b)

$$y = \frac{ax^2}{1}$$

The variables x and y are related by the equation x+b, where a and b are constants. Explain clearly how the values of a and b could be obtained by plotting a straight line using values of x and y. [3]

4 (a) Without using a calculator, show that $\csc^2 75$ can be expressed in the form of $a + b\sqrt{3}$ where a and b are both integers. [3]

(b) The equation of a curve is
$$y = 4\cos 3x - 1$$
.

(i) State the maximum and minimum values of *y*. [2]

(ii) Sketch the graph of
$$y = 4\cos 3x - 1$$
 for $0 \le x \le 3\pi$ [3]

(iii) Hence, find the range of values of *b* for which $1+b=4\cos 3x$ has [2] no solution.

5 A factory owner manufactures an open hollow right circular cone of radius r cm and vertical height h cm using plastic as shown in the diagram below. The cone is designed to hold 100 cm³ of liquid.



(a) Show that the curved surface area, S, of the cone is $\frac{1}{3}\sqrt{\frac{\pi^2 r^6 + 90000}{r^2}}$. [4]

(b) Given that the radius, r cm, can vary, find the minimum amount of material to make the cone. [4]

6 (a) Show that
$$\frac{2^{x}3^{x+1}-2(6^{x})}{3^{(1-x)}18^{x-3}}$$
 is always independent of x. [3]

(b) Given that
$$\int_{0}^{4} f(x) dx = \int_{4}^{5} f(x) dx = 8$$
, evaluate
(i) $\int_{4}^{0} f(x) dx$ [1]

(ii)
$$\int_0^5 f(x) \, dx$$
 [2]

(c) A graph
$$y = f(x)$$
 is always symmetrical about the line $y = x$ for $x > 0$.
Explain why $\int_{2}^{4} f(x) dx + \int_{4}^{2} f(y) dy = 0$ [2]

7 (a)
$$\frac{\sin 2x - 1}{\cos 2x} = \frac{\tan x - 1}{\tan x + 1}$$
 [3]

(b) Solve the equation $3\cos 2x + 5\tan 2x = 5\sec 2x$ for $0 \le x \le \pi$. [5]

- 8 A particle A, moving in a straight line, passes through a fixed point O with a speed of 5 m/s. The velocity, v m/s, is given by $v = t^2 6t + p$ where t is the time in seconds from passing O.
 - (a) Explain why p = 5. [1]

- (b) Calculate
 - (i) the total distance travelled in 5 seconds, [3]

(ii) the distance travelled in 4^{th} seconds.

[2]

Another particle *B* also passes *O* with a velocity, $v = 3t^2 - 16t + 23$ and *t* is the time in seconds from passing *O*. Both particle *A* and *B* passes through *O* at the same instant.

(c) Did any overtaking takes place? Show your workings clearly. [3]

9 In the diagram below, the rod *AC* of 10.5 cm long hinged at *A* is held by two strings *DC* and *BC*. The string *DC* is attached horizontally at a point on the wall at 12 cm away from *A* while the string *BC* is attached vertically at a point on the ceiling 8 cm away from *A*. The rod *AC* forms an angle of *x* with the horizontal.



(a) Show that the area *ABCD* formed by the rod, strings and ceiling and wall is $S = 42 \sin x + 63 \cos x$ [3]

(b) Express S in the form of
$$R\cos(x-\alpha)$$
 where $R > 0$ and $0^{\circ} \le \alpha \le 90^{\circ}$. [3]

(c) Show that $x = 56.3^{\circ}$ when area of triangle *ABC* is equal to the area of triangle *ADC*. [2]

10 The diagram shows a trapezium RSTU where R is (1, 6), S is (-11,12), T is (-10,3). RS is parallel to TU and U lies on x-axis.



(i) Show that the coordinates of U is (-4, 0). [2]

(ii) Find the acute angle that the line RU makes with x – axis. [2]

(iii) Given that the point *A* lies on the line *RS* such that 3SA = 2RS, determine the coordinates of *A*. [3]

(iv) Find the area of triangle RAU. Hence, in exact form, find the shortest distance from U to AR. [3]

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