

# **1. Quantities & Measurement Problem Set**

# PHYSICAL QUANTITIES AND SI UNITS

# **Guided Exercises**

For this section of the problem set, you should try the exercises without looking at the guided solutions. The guided solutions are there if you get stuck and you should also use it to understand how to present your work.

- Perform the following conversion of units: 1
  - 50 km h<sup>-1</sup> to  $m s^{-1}$ . 320 m s<sup>-1</sup> to km h<sup>-1</sup>. (i)
  - (ii)

[13.9 m s<sup>-1</sup>] [1152 km h<sup>-1</sup>]

## Step-by-step guide

Convert km to m and h to min

50 km h<sup>-1</sup> = 
$$\frac{50 \text{ km}}{1 \text{ h}} = \frac{50\ 000\ m}{60\ \min} = \frac{50\ 000\ m}{60\ \times 60\ s} = 13.9 \text{ m s}^{-1}$$
  
Convert min to s

320 m s<sup>-1</sup> =  $\frac{320 \text{ m}}{1 \text{ s}} = \frac{1000}{\frac{1}{60 \times 60} \text{ h}} = 1152 \text{ km h}^{-1}$ 



- 2 Express the units of the following quantities in SI base units:
  - (i) Gravitational constant *G*, where  $F = \frac{GM_1M_2}{r^2}$ , *F* is force,  $M_1$  and  $M_2$  are masses and *r* is separation distance between the masses. [m<sup>3</sup> kg<sup>1</sup> s<sup>2</sup>]
  - (ii)  $\mu$ , if  $v = \sqrt{\frac{T}{\mu}}$  where v is the speed of the wave on a string of length L held at a tension of T, a type of force. [kg m<sup>-1</sup>]

Look at Examples 1.1 and 1.2 in your lecture notes for a step-by-step guide, it will be useful to rearrange the equations to write G and  $\mu$  in terms of other quantities before determining the units.

Please also be mindful of the presentation of your working!

(i)

$$G = \frac{Fr^2}{M_1M_2}$$
 and  $F = ma$ 

Units of G = 
$$\frac{(kg m s^{-2})(m^2)}{(kg)(kg)} = m^3 kg^{-1} s^{-2}$$

(ii)

$$v^2 = \frac{T}{\mu} \implies \mu = \frac{T}{v^2}$$
 and  $T = ma$ 

Units of 
$$\mu = \frac{\text{kg m s}^{-2}}{(\text{m s}^{-1})^2} = \frac{\text{kg m s}^{-2}}{\text{m}^2 \text{ s}^{-2}} = \text{kg m}^{-1}$$

Presentation style:

Always write "units of" in front of the variable you are finding units for. E.g  $\mu = \frac{\text{kg m s}^{-2}}{(\text{m s}^{-1})^2}$  would be marked wrong for presentation.



- 3 The critical flow speed  $v_c$  is related to the width *a* of the obstacle, the density  $\rho$  liquid, and its viscosity  $\eta$  by one of the equations:
  - (i)  $V_c = \frac{A\eta a}{\rho}$  (ii)  $V_c = \frac{B\eta}{a\rho}$  (iii)  $V_c = \frac{C\rho a}{\eta}$

A, B and C are constants with no units and unit of  $\eta$  is Pa s. Determine the correct equation.

You are given 3 equations. A dimensionally correct equation is one where the units of each term has the same base units. (pg 8 of your lecture notes) Note that a dimensionally correct equation need not be correct! Hence you can't actually determine the correct equation, just the dimensionally correct one! Step-by-step guide using (i) Unit of  $v_c = m s^{-1}$ Unit of  $\frac{A\eta a}{\rho} = \frac{Pa s (m)}{(kg m^{-3})} = \frac{\binom{kgms^{-2}}{m^2} s (m)}{(kg m^{-3})} = \frac{s^{-1}}{(m^{-3})} = m^3 s^{-1}$ LHS  $\neq$  RHS, (i) is not dimensionally correct

- Option (ii) is the dimensionally correct equation.
- 4 Bernoulli's equation, which applies to fluid flow, states that  $p + h\rho g + \frac{1}{2}\rho v^2 = k$ Where p is a pressure, h a height,  $\rho$  a density, g an acceleration, v a velocity and k a constant.

Show that the equation is homogenous and state an SI base unit for k.

[kg m<sup>-1</sup> s<sup>-2</sup>]

Note: An equation where each term has the same base units (dimensionally consistent) is known as a homogeneous equation.

To determine whether an equation is homogenous, you will need to check whether all the terms in the equation have the same base units. Hence you will need to check whether units of p = units of  $h\rho g =$  units of  $\frac{1}{2}\rho v^2$  and the units for k will be the same.

Units of pressure = N  $m^{-2}$  = kg m  $s^{-2} m^{-2}$  = kg  $m^{-1} s^{-2}$ 

Units of  $h\rho g = m \text{ kg m}^{-3} \text{ m s}^{-2} = \text{ kg m}^{-1} \text{ s}^{-2}$ 

Units of  $\frac{1}{2}\rho v^2 = \text{kg m}^{-3} (\text{m s}^{-1})^2 = \text{kg m}^{-1} \text{s}^{-2}$ 

Since units of p = units of  $h\rho g =$  units of  $\frac{1}{2}\rho v^2$  equation is homogenous if units of k = kg m<sup>-1</sup> s<sup>-2</sup>



5 The activity *A* of a radioactive sample is given by the equation,  $A = A_o e^{-\lambda t}$  where  $A_o$  is the initial activity of the sample,  $\lambda$  is the decay constant and *t* is the time taken for the sample to decay. Determine the unit of  $\lambda$ . Hint: you do not need to know the units of A.

[s<sup>-1</sup>]

All exponents are dimensionless. The unit of A = unit of  $A_o$  and  $-\lambda t$  has no units.

Units of  $(-\lambda t)$  = units of  $\lambda \times$  units of t

1 = units of  $\lambda \times s$ 

Units of  $\lambda = s^{-1}$ 

6 Express the following quantities using the required prefixes:

- (i) 800,000 W with prefix kW, MW and TW;
- (ii) 0.0000325 m with prefix mm and  $\mu$ m;
- (iii)  $2.65 \times 10^{-10}$  s with prefix ps

[800 kW, 0.8 MW, 8 x 10<sup>-7</sup> TW] [ 0.0325 mm, 32.5 μm] [265 ps]

Common prefixes required for A levels can be found in Pg 7 of your lecture notes.

(i)

$$\begin{split} &800,000 \text{ W} = 800 \times (10^3 \text{ W}) = 800 \text{ kW} \\ &800,000 \text{ W} = 800,000 \times \frac{1 \times 10^6}{1 \times 10^6} \text{ W} = \frac{800,000}{1 \times 10^6} \times (1 \times 10^6 \text{ W}) = 0.8 \text{ MW} \\ &800,000 \text{ W} = 800,000 \times \frac{1 \times 10^{12}}{1 \times 10^{12}} = \frac{800,000}{1 \times 10^{12}} \times (1 \times 10^{12} \text{ W}) = 8 \times 10^{-7} \text{ TW} \end{split}$$
 (ii) (ii)  $0.0000325 \text{ m} = 0.0000325 \text{ x} 1 \text{ m} = 0.0000325 \text{ x} 1000 \text{ mm} = 0.0325 \text{ mm} \\ 0.0000325 \text{ m} = 0.0000325 \text{ x} 1 \text{ m} = 0.0000325 \text{ x} 1 \times 10^6 \text{ \mum} = 32.5 \text{ \mum} \end{cases}$  (iii)

 $2.65 \times 10^{-10} \times 1$  second =  $2.65 \times 10^{-10} \times 1 \times 10^{12}$  ps = 265 ps



## Problems

P1 The behaviour of many real gases can be represented quite closely over certain ranges of temperature and pressure by an equation of the form

$$(p + \frac{a}{V^2_m})(V_m - b) = RT$$

in which the values of *a* and *b* are characteristics constants of the particular gas. *p* is the pressure of gas,  $V_m$  is the volume of gas per mole and *T* is the thermodynamic temperature of gas. Express the units of *a*, *b* and *R* in SI base units.

P2 Express the unit of  $\varepsilon_0$  the permittivity of vacuum, in terms of base units from the equation

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_o r^2}$$

where  $Q_1$  and  $Q_2$  are the charge of two particles separated by a distance *r*. The unit of  $\mu_o$ , the permeability of a vacuum, is kg m s<sup>-2</sup> A<sup>-2</sup>.

Use the unit of  $\mu_o$ , and unit of  $\varepsilon_o$ , to decide which one of the following relations between  $\varepsilon_o$ ,  $\mu_o$  and *c*, the speed of light in a vacuum, is possible.

$$\varepsilon_o\mu_o=c^2$$
;  $\varepsilon_o\mu_o=c$ ;  $\varepsilon_o\mu_o=c^{-1}$ ;  $\varepsilon_o\mu_o=c^{-2}$ 

- P3 Suggest how to check whether a hypothetical equation is possible and explain why your suggested method of checking does not give confirmation that an equation is physically correct.
- P4 N13/1/2

What is the order-of-magnitude of the mass of a single sheet of A4 paper?

A 0.01 g B 0.3 g C 1 g D 30 g

P5 N08/1/2

Which estimate is realistic?

A The kinetic energy of a bus travelling on an expressway is 30 000 J.

- B The power of a domestic light is 300 W
- C The temperature of a hot oven is 300 K.
- D The volume of air in a car tyre is 0.03 m<sup>3</sup>.



# **Errors and uncertainties**

#### **Exercises**

1 *N* represents the number of data collected, *x* represents the possible values of the results and  $x_0$  represents the actual value. Comment on the accuracy and precision of the data shown on each graph.



Refer to Pg 14 of your lecture notes					
Accurate and precise	Accurate but not precise	Not accurate but precise	Not accurate and not precise		

Four students each made a series of measurements of the acceleration of free fall g, where  $g = 9.81 \text{ m s}^{-2}$ . The table shows the results obtained. Which student obtained a set of results that could be described as precise but not accurate?

Student	F	Results,	g / m s	8 <sup>-2</sup>	Average	<i>Spread</i> (max – min)	Precise	Accurate
Α	9.81	9.79	9.84	9.83	9.82			
В	9.81	10.12	9.89	8.94	9.69			
С	9.45	9.21	8.99	8.76	9.10			
D	8.45	8.46	8.50	8.41	8.46			

Refer to pg 13 of your lecture notes.

Accuracy is a measure of how close a measurement or result is to the <u>true</u> value. Precision is a measure of how close the repeated measured values are to each other regardless of true value. (this can also mean how close each value is to the mean/ average value)

Student	Spread (max – min)	Precise	Accurate
Α	0.05	Yes	Yes
В	1.18	No	No
С	0.69	No	No
D	0.09	Yes	No
The actual value of g is 9.81 m s <sup>-2</sup> . Best choice of answer is <b>D</b>			



3 Four student measures the time *t* for a ball to fall from rest through a vertical height, *h*. The measurements was repeated for different values of *h*. Since *h* and *t* are related by  $h = \frac{1}{2}gt^2$ . Graphs of *h* against  $t^2$  were plotted. Comment on the accuracy and precision of the data obtained by the students.



You have been provided with the equation of the graph of h vs t<sup>2</sup>. After arranging it in the linear form of y = mx+c, you will see that the y intercept c = 0 and the gradient is  $\frac{1}{2}g$ .

If the graph cuts at origin, c is zero and hence the graph is accurate. Conversely, if the graph does not cut at origin, c is not zero and there exist a systemic error. The graph is not accurate.

What about precision?

If the data points are close to the best fit line, all the data is close to the mean value and the graph is precise. Conversely, if the data points are scattered far from best fit line, the graph is not precise and there is more random error!

Note: From statistics (Linear Regression), the best fit line is a line that passes through the estimated average values of y corresponding to each value of x. So drawing a best fit line for your data is a form of averaging (if you know that a linear relation exist between y and x).





4 A student carries out an experiment to determine the speed of sound in air. His measurements give the speed as 347.785 m s<sup>-1</sup> with an uncertainty of ±2.56 m s<sup>-1</sup>. What is the correct statement of the result of the experiment?

А	(347.78 ± 2.56) m s <sup>-1</sup>	B $(347.7 \pm 2.6)$ m s <sup>-1</sup>
С	$(348 \pm 3) \text{ m s}^{-1}$	D $(350 \pm 5)$ m s <sup>-1</sup>

[C]

Important details to remember when determining correct statement of results

- 1) Absolute uncertainty should only have 1 s.f.
- 2) The value of result should follow the precision of the absolute uncertainty

Therefore, in this question 2.56 to 1 sf is 3 m s<sup>-1</sup> and therefore 247.785 will be written as 248.

- 5 An instrument gives a numerical reading of 0.00160 ± 0.00005. Which statement is correct?
  - A The actual uncertainty is 5.
  - B The fractional uncertainty is  $5 \times 10^{-5}$ .
  - C The fractional uncertainty is 5/16.
  - D The percentage uncertainty is 3%.

[D]

Refer to pg 17 for the equations for fractional and percentage uncertainty. Choice A is wrong, the actual uncertainty is 0.00005 Choice B is wrong, the fractional uncertainty  $=\frac{0.00005}{0.00160} = 3.1 \times 10^{-2}$ Choice C is wrong, the fractional uncertainty  $=\frac{0.00005}{0.00160} = \frac{5}{160}$ Choice D is correct, the percentage uncertainty  $=\frac{0.00005}{0.00160} \times 100\% = 3.1\%$ 



#### 6 N17/1/2

After taking measurements of the quantities in the expression  $\frac{xy^2}{z}$ , the total uncertainty is calculated as 6%.

Which individual percentage uncertainties of x, y and z when combined give this total of 6%?

	x/%	y1%	z/%
Α	1	1	4
в	2	1	2
с	3	2	2
D	4	1	1

[B]

#### Step-by-step guide

Let  $Q = \frac{xy^2}{z}$ 

$$\frac{\Delta Q}{Q} \times 100\% = \left[\frac{\Delta x}{x} + 2\frac{\Delta y}{y} + \frac{\Delta z}{z}\right] \times 100\% = 6\%$$

Use trial and error

e.g.

Option A:  $\frac{\Delta Q}{Q} \times 100\% = 1\% + 2(1\%) + 4\% = 7\% \neq 6\%$  (Option A is incorrect)

Only option B gives 6%



## **Problems**

P1 N11/1/2

In an experiment to measure the viscosity  $\eta$  of a liquid, the following equation was used.

$$\eta = \frac{kr^2}{v}$$

where  $r = (0.83 \pm 0.01)$ mm  $v = (0.065 \pm 0.002)$ m s<sup>-1</sup> and k is a constant of value 93.7 N m<sup>-3</sup>.

How should the value of  $\eta$  be expressed?

A  $(9.93 \pm 0.54) \times 10^{-4} \text{ N s m}^{-2}$ 

B  $(9.9 \pm 0.6) \times 10^{-4} \text{ N s m}^{-2}$ 

- C  $(9.93 \pm 0.42) \times 10^{-4} \text{ N s m}^{-2}$
- D  $(9.9 \pm 0.4) \times 10^{-4} \text{ N s m}^{-2}$

## P2 N08/1/1.

The manufacturers of a digital voltmeter give its specification below,

'accuracy  $\pm$  1% with an additional uncertainty of  $\pm$  10 mV'

The meter reads 4.072 V.

How should this reading be recorded, together with its uncertainty?

A	$(4.07 \pm 0.01)V$	В	$(4.072 \pm 0.052)V$
С	$(4.07 \pm 0.04)V$	D	$(4.07 \pm 0.05)V$

P3 A student used a meter ruler to measure a row of identical blocks as shown below. The reading of the length of the row of identical blocks was given as  $(5.6 \pm 0.1)$  cm. The student suggested that the length of one block and the associated uncertainty is  $(1.1 \pm 0.1)$  cm. Explain whether you agree with the suggested length and uncertainty of one block.





P4 Given the following quantities and their corresponding uncertainties:

 $r_1 = 2.2 \pm 0.1 \text{ cm}$   $r_2 = 3.5 \pm 0.1 \text{ cm}$  $m = 0.800 \pm 0.001 \text{ kg}$ 

Determine the value of S and its associated uncertainty for  $S = \frac{3m}{4\pi(r_2^2 - r_1^2)}$ .

## P5 N13/1/1.

An object falls freely from rest and travels a distance *s* in time *t*. The equation relating *s* and *t* is given by  $s = \frac{1}{2}gt^2$ . A graph of  $t^2$  against *s* is plotted and used to determine the acceleration of free fall *g*.



The gradient of the graph is found to be 0.204 s<sup>2</sup> m<sup>-1</sup>. Which statement about the value obtained for *g* is correct?

- A It is accurate but not precise
- B It is both precise and accurate
- C It is neither precise nor accurate
- D It is precise but not accurate

P6 N07/2/1.

A student times the fall of a small metal ball. Data for the time *t* taken for the ball to fall a vertical distance *h* from rest are given below.

$$h = 266 \pm 1 \text{ cm}$$
  
 $t = 0.740 \pm 0.005 \text{ s}$ 

(a) Given that  $h = \frac{1}{2}gt^2$ , use the data to determine

- (i) a value, to three significant figures, of the acceleration of free fall g,
- (ii) the percentage uncertainty, to two significant figures, of
  - 1. the distance *h*,
  - 2. the time t.



- (b) Use your answers in (a) to determine the actual uncertainty in the value of *g*. Hence give a statement of *g*, with its uncertainty, to an appropriate number of significant figures.
- (c) Suggest two reasons why, in this experiment, although the value of *t* is precise, it may not be accurate.

#### P7 N13/3/6(a)

A student takes measurements to determine the acceleration of a ball as it rolls down a slope. He uses the apparatus illustrated in the figure below.



The student measures the time t for the ball to roll a distance s down the slope after the ball has been released from the electromagnet. Given that  $s = \frac{1}{2}at^2$ . The variation with t<sup>2</sup> of the distance s is shown below.





- (a) State the feature of the data shown in above figure that indicates the presence of
  - (i) Random error,
  - (ii) Systematic error
- (b) State why, by drawing a line of best fit for the data points on above figure the effect of random error is reduced.

## Additional Problem

C1 The equation connecting object distance *u*, image distance *v* and focal length *f* for a lens is  $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$ 

A student measures values of u and v, with their associated uncertainties. These are

 $u = (50 \pm 3)mm$  $v = (200 \pm 5)mm$ 

Calculate the value of *f* and its associated uncertainty.



# **Scalars and Vectors**

#### **Exercises**

1 Two forces  $F_1$  and  $F_2$  act on a body.  $F_1$  is 2.00 N due east and  $F_2$  is 4.00 N acting 20° East of North. Find the resultant of these two forces.

[5.05 N, 48.1° North of East]





- 2 The initial velocity of a car is  $12 \text{ m s}^{-1}$  due east. Its final velocity is  $12 \text{ m s}^{-1}$  due south.
  - (a) Sketch a vector diagram showing the change in velocity
  - (b) Calculate the change in velocity.

[17 m s<sup>-1</sup> 45° west of south]

Refer to pg 30 to 32 of your lecture notes for examples on how to draw a vector diagram for change in velocity.

Hint: change in velocity = v - u = v + (-u)

You can think of it as the resultant vector of the vectors v and (-u)





3 Write the equations of the x and y components of the following forces.





(b)	<i>x</i> component: $-15 \cos 20 = -14.1$	<i>y</i> component: $15 \sin 20 = 5.13$
(C)	<i>x</i> component: $25 \sin 30 = 12.5$	<i>y</i> component: $-25 \cos 30 = -21.7$
(d)	<i>x</i> component: $-10 \cos 15 = -9.66$	<i>y</i> component: $-10 \sin 15 = -2.59$



4 The diagram shows forces  $F_1$ ,  $F_2$ ,  $F_3$  and  $F_4$  acting of a point. By resolving the forces in the x and y directions, determine the resultant of the four forces F.







5 In the diagram on the right, a body S of weight W hangs vertically by a thread tied at Q to the string PQR. If the system is in static equilibrium (net force = 0), what is the tension T in the section PQ in terms of W?







6 Resolve the components of the weight of the object, *W* along and perpendicular to the slope.



Important note: Components of any vector should never be longer than the vectors themselves.

It is useful to draw the perpendicular axis in dotted lines as shown below then use the axis to 'box' up the vector that you are find components of.





7 A pendulum is swung around in a vertical circle as shown in the diagram on the right. At the position P, determine the components of the weight of the pendulum bob along the radial and tangential directions. (*radial means in the direction along the radius towards the center. Tangential is the direction perpendicular to radial*)







## Problems

P1 N10/1/2.

A boat changes its velocity from 8 m s<sup>-1</sup> due north to 6 m s<sup>-1</sup> due east. What is its change in velocity?

- A 2 m s<sup>-1</sup> at a direction of 37 ° east of north
- B 2 m s<sup>-1</sup> at a direction of 53 ° east of north
- C 10 m s<sup>-1</sup> at a direction of 37 ° east of south
- D 10 m s<sup>-1</sup> at a direction of 53 ° west of south
- P2 A picture frame is hung by a cord from a nail.

Which vector triangle represents the resultant R of the tension forces T in the cord?









D

P3 A box of weight *W* is suspended from a fixed point O by a fixed thread. A force *F* is applied such that the thread makes an angle  $\theta$  to the vertical when box is at rest. Express the horizontal and vertical component of the tension, *T* in the thread in terms of *F* and *W* respectively.

