2024 H2 FM 9649 Prelim Paper 2 (ACJC_EJC_NJC_RVHS)

Section A: Pure Mathematics [50 marks]

P2	Prove by induction that $2(7^n) + 3(5^n) - 5$ is divisible by 24 for all positive integers <i>n</i> .	[5]
Q1		
P2 Q2	Show that the substitution $x = \sin t$, where $-\frac{\pi}{2} < t < \frac{\pi}{2}$, reduces the differential equation	
	$(1 - x^2)\frac{d^2 y}{dx^2} - x\frac{dy}{dx} + 4y = 0$	
	to	
	$\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + 4 y = 0.$	[4]
	Hence find the general solution for y in terms of x , giving your answer in non-trigonometric form	.[4]

P2 Q3	(a)	Determine the eigenvalues and corresponding eigenvectors of the matrix $\mathbf{M} = \begin{pmatrix} 1 & 1 \\ 3 & -1 \end{pmatrix}$.	[4]
	(b)	By expressing M in the diagonalised form \mathbf{QDQ}^{-1} , find \mathbf{M}^n for positive integers <i>n</i> .	[4]
	(c)	Hence, or otherwise, show that \mathbf{M}^n is a diagonal matrix for all even values of n .	[1]





have stated, write down the condition satisfied by a, b and c. [3]

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P2	A psychologist is conducting an experiment to determine if there is a difference in the reaction times								on times				
Q6	of participants under two different conditions: before taking a stimulant (Condition A) and after taking										er taking		
	a stimulant (Condition B). The reaction times are recorded in seconds as shown below.										U		
	Participant 1 2 3 4 5 6 7 8 9 G West 24 24 25<												
		Condition A	21	24	20	22	25	24	23	27	28	32	
		Condition B	18	22	19	24	23	24	18	25	27	31	
	 (a) Carry out a sign test at the 2% significance level for the above investigation. [5] (b) Explain if the conclusion of the test would change if the investigation is about whether the 										5.57		
											[5]		
											ther the		
	reaction times of participants are shorter after taking the stimulant.[2](c)Give a reason why it is more appropriate to conduct a sign test than Wilcoxon matched pair									[2]			
										hed pair			
	signed rank test in this context. [1]												
P2	The	webpage of a con	nany re	eceives :	an avera	ge of 5	views n	er hour	and the	views	are inde	nendent	
Q7	ofea	ch other	ipuny it		an avera	.50 01 5	views p	er nour,	, and the			pendent	
	(a)	Find the probab	ility tha	t in a 3-1	hour du	ation t	ne wehn	age rece	eives mo	ore than	20 view	vs [2]	
		ality view is one	where	the view	ver snen	ds more	than 3	minute	s on the	webna	ge On s	average	
	100r	when the second second	uality y	iews L	et V ren	resent ti	ne numb	er of vi	ews of	the web	nage in	n to and	
	inclu	iding the first qual	lity view	,							puge, uj	p to und	
	(h)	State in the co	ntext o	r. f the au	estion	two ass	umption	ns neede	ed to m	odel V	hv a of	ometric	
	(b) State, in the context of the question, two assumptions needed to model Y by a geometric distribution									[2]			
	aistribution. [2]									[2]			
	(c)	Find the probab	ility the	t the fire	t quality		cours of	ter the P	5 th view			[3]	
		Find the probab	ility the	t the 10	th view :	e the se	cond au	ality via		n that th	a 3rd via	[J]	
	(u) Find the probability that the 10 view is the second quality view given that the 3 view is the												
	Inst quanty view. [2]												

Section B: Probability and Statistics [50 marks]

P2 Q8	(a)	Shov	v that $\int_{0}^{1} \frac{1}{\sqrt{x-x^{2}}} dx = \pi$.	[2]
	(b)	The	continuous random variable X has probability density function	
			$f(x) = \begin{cases} \frac{k}{\sqrt{x - x^2}}, & 0 < x < 1, \\ 0, & \text{otherwise,} \end{cases}$	
		wher	k = k is a constant.	
		(i)	State the value of <i>k</i> .	[1]
		(ii)	Without the use of a calculator, find the exact value of $E(X)$.	[3]
		(iii)	Find the lower quartile of <i>X</i> .	[3]

P2 Two brands of a health supplement, Brand A and Brand B, are known to enhance short-term cognitive Q9 focus in users after consumption. A pharmaceutical laboratory would like to investigate the effectiveness of the two brands of supplement and conducted clinical trials where cognitive focus of participants from a random sample, as measured by a standardised cognitive focus scale, before and after consuming a dosage of the supplement was assessed.

The increase in cognitive focus scores, x and y, of participants after consuming Brand A and Brand B respectively are recorded as two sets of data for analysis, with 6 readings belonging to participants for the clinical trials of Brand A and 6 readings belonging to participants for the clinical trials of Brand B.

x	7.8	7.3	7.2	7.4	7.5	7.0
У	6.4	7.9	7.1	6.8	6.6	7.2

 ⁽a) Carry out an appropriate test, at the 2% level of significance, to determine whether there is a difference between the effects of the two brands of supplement. State any other assumptions made in applying the test.

The statistician later found out that the two sets of data above were collected from 6 participants, with each column of data coming from a particular participant. Each participant had gone through two rounds of clinical trials, one for Brand A and the other for Brand B, with sufficient wait time between the trials for the effects of the supplement to wear off.

- (b) Construct a 98% confidence interval for the mean difference in the increase of cognitive focus after consuming the two brands of supplement. [2]
- (c) Hence, explain whether there is evidence, at the 2% significance level, that there is a significant difference between the effects of the two brands of supplement. [2]
- (d) Discuss whether the test in part (a) or part (c) is more suitable for the purpose of the investigation. [2]

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P2 An analyst suggests that the number of emergency calls received at a particular fire station in a day Q10 follows a Poisson distribution. The number of emergency calls received in each day over a span of 60 days has the following frequency distribution. No. of emergency 0 2 3 4 5 1 ≥ 6 calls in a day 7 0 7 4 Frequency 14 16 12 Carry out an appropriate test to determine whether the analyst's claim is valid. (a) [7] **(b)** Suggest a possible reason for your conclusion in part (a). [1] (c) The analyst decides to conduct the test, at 10% significance level, with the number of emergency calls received in each day spanning over 1200 days. Assume that the proportion for the sample in each category remains the same, state the new value of the test statistic and determine whether the analyst's claim is valid. [4]