Candidate Index Number

Anglo - Chinese School (Independent)

FINAL EXAMINATIONS 2013
YEAR 3 INTEGRATED PROGRAMME
CORE MATHEMATICS
PAPER 1

FRIDAY **4th OCTOBER 2013** **1 h 30 min**

Additional Material
Graph Paper (1 sheet)

INSTRUCTIONS TO CANDIDATES

- Write your index number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Answer all questions in the spaces provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- The maximum mark for this paper is 80.

For Examiner's Use

This paper consists of 13 printed pages.

[Turn over

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

*Answer **all** the questions in the spaces provided.*

1 *[Maximum mark: 3]*

Ann buys y erasers at \$0.50 each and $y-4$ pens at \$1.20 each. She wishes to spend less than \$40.

(a) Form an inequality in y.

[1 mark]

(b) Solve the inequality and hence, find the largest possible value of y.

[2 marks]

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

2 *[Maximum mark: 6]*

(a) (i) Given that $\sqrt{\frac{m-3}{m+5}} = y$, express m in terms of y .

(ii) Hence, find the value of m when $y = -3$.

[4 marks]

(b) Simplify and express $\frac{1+\frac{3}{x^2}}{2x+\frac{6}{x}}$ as a single fraction.

[2 marks]

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

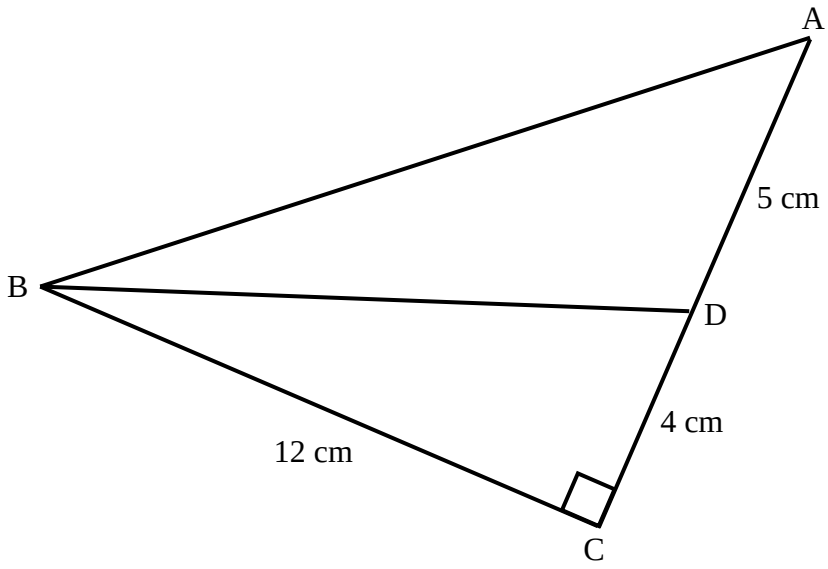
……

……

……

3 *[Maximum mark: 6]*

In the diagram, BC = 12 cm, CD = 4 cm , DA = 5 cm, $\angle ACB = 90^o$ and ADC is a straight line.



(a) Calculate AB.

[2 marks]

(b) Write down, as a fraction, the value of

(i) $\sin \angle CBD$.

(ii) $\cos \angle ADB$.

[4 marks]

……

……

……

……

……

……

……

……

……

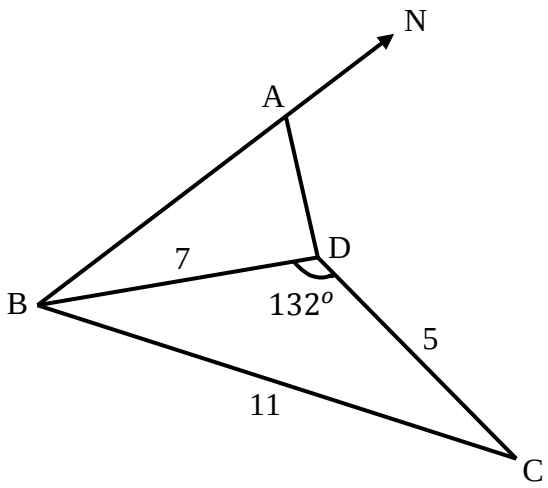
……

……

……

4 *[Maximum mark: 7]*

In the diagram, A, B, C and D are points on level ground. A is north of B. The bearing of D from A is 115^o and the bearing of D from B is 25^o. $\angle BDC = 132^o$, $BD = 7$ m, $CD = 5$ m and $BC = 11$ m.



(a) Find the value of $\angle ADB$.

[2 marks]

(b) Find the bearing of *C* from *D*.

[2 marks]

(c) Calculate the value of $\cos 132^o$, leaving your answer in fraction.

[3 marks]

……

……

……

……

……

……

……

……

……

……

……

……

……

……

5 *[Maximum mark: 7]*

Jacob bought x calculators, each at the same price, for a total cost of \$258.

(a) Write down an expression for the cost of each calculator in terms of x.

[1 mark]

(b) He kept 3 calculators for himself and sold the rest at a profit of \$3 per calculator. Write down an expression, in terms of x, the total amount, in dollars, he received for all the calculators sold.

[1 mark]

(c) Given that Jacob made a profit of \$102 altogether, form an equation in x and show that it reduces to $x^2 - 37x - 258 = 0$.

[2 marks]

(d) Solve the equation in part (c) and hence find the selling price of each calculator.

[3 marks]

……

……

……

……

……

……

……

……

……

……

……

……

……

……

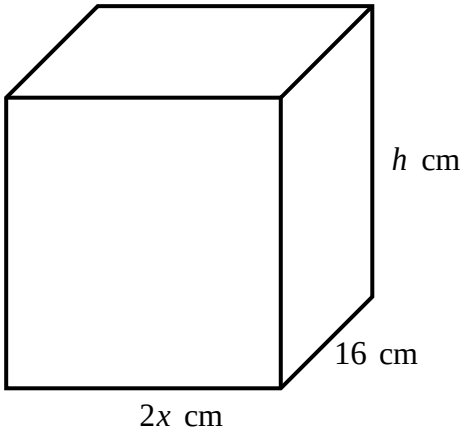
……

……

……

6 *[Maximum mark: 7]*

A metal wire, of length 128 cm is to be bent to form the frame of a rectangular box as shown below.



Given that the dimensions of the box are $2x$ cm $\times$ 16 cm $\times$ h cm,

(a) express h in terms of x.

[2 marks]

(b) express the volume V cm^3 of the box in term of x.

[2 marks]

(c) hence, by completing the square, find the maximum volume of the box.

[3 marks]

……

……

……

……

……

……

……

……

……

……

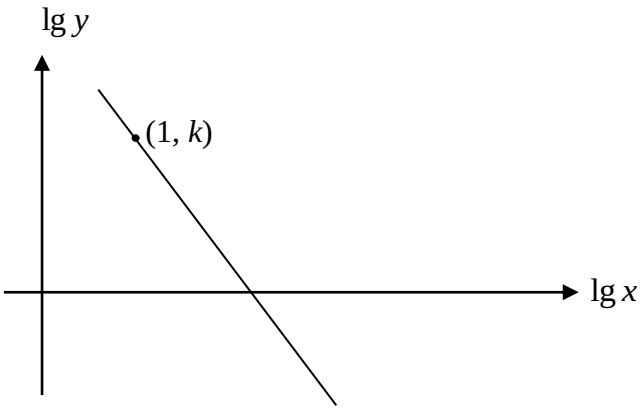
……

……

……

7 *[Maximum mark: 3]*

The diagram below shows the straight line obtained by plotting $\lg y$ against $\lg x$.



Given that $y^2 = \frac{10^3}{x^2}$, calculate the value of k.

[3 marks]

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

8 *[Maximum mark: 8]*

Given that $\sin A = \frac{2}{5}$ and $90^o \le A \le 180^o$, find the value of

(a) $\cos A$,

[2 marks]

(b) $\tan A$,

[2 marks]

(c) $\frac{5\sin A - 5\sqrt{7}\cos A}{2 - \sqrt{7}\tan A}$, leaving your answer in the form $a + b\sqrt{3}$ where *a* and *b* are constants.

[4 marks]

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

9 *[Maximum mark: 9]*

(a) Solve the equation $2^{4x+3} + 7\left(2^{2x}\right) = 1$.

[3 marks]

(b) Solve the simultaneous equations

$$\log_8(y^2 + 17) = \frac{1}{3} + \frac{2}{\log_x 8}$$

$$25\left(5^{\frac{y}{2}}\right) = \sqrt{5^x}$$

[6 marks]

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

……

10 *[Maximum mark: 7]*

(a) Find the range of values of k for which the curve $y = 4 + 2x + x^2$ meets the line $y - kx = 3$ at two distinct points.

[4 marks]

(b) Find the least integer value of c for which $x^2 + 6 + 8x + 5c$ is always positive.

[3 marks]

11 *[Maximum mark: 9]*

Solutions to this question by accurate drawing will not be accepted.

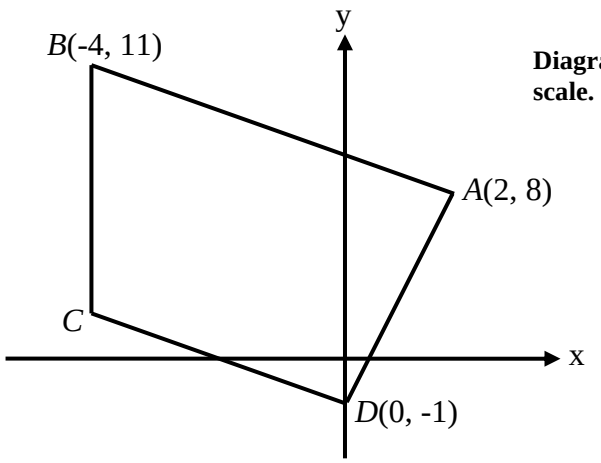


Diagram not drawn to scale.

ABCD is a trapezium where *AB* is parallel to *CD* and *BC* is parallel to the *y*-axis. The coordinates of *A*, *B* and *D* are (2, 8), (-4, 11) and (0, -1) respectively. Find

(a) the equation of *AB*,

[2 marks]

(b) the equation of *CD*,

[2 marks]

(c) the coordinates of *C*,

[2 marks]

(d) the equation of the perpendicular bisector of *AB*.

[3 marks]

……

……

……

……

……

……

……

……

……

……

……

12 *[Maximum mark: 8]*

Answer the whole of this question on a sheet of graph paper.

The perimeter, P, in metres, of a rectangle is given by the equation $P = 2x + \frac{24}{x^2} + 3$.

The table of values below is for the curve $P = 2x + \frac{24}{x^2} + 3$.

x	1	2	3	4	5	6	7	8
P	29.0	13.0	11.7	12.5	14.0	15.7	17.5	19.4

(a) Using a scale of 2 cm to represent 1 metre on the horizontal axis and 4 cm to represent 4 metres on the vertical axis, draw the graph of $P = 2x + \frac{24}{x^2} + 3$ for values $1 \le x \le 8$ and $10 \le P \le 30$.

[4 marks]

(b) Use your graph to find the minimum perimeter of the rectangle.

[1 marks]

(c) Using the graph, solve the equation $3x + \frac{24}{x^2} - 15 = 0$

[3 marks]

********** END OF PAPER 1 **********