CANDIDATE NAME	

CENTRE NUMBER	S					INDEX NUMBER				

MATHEMATICS   **9758/01**

Paper 1   **24 August 2023**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your name, centre number and index number on the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Write your answers in the spaces provided in the Question Paper.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

For Examiner's Use	
Question Number	Marks Obtained
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
Total Marks	

This document consists of **19** printed pages and 1 blank page.

[Turn over

1 **(a)** Find $\int x\tan^{-1} x \, dx$. [3]

    (b) Find $\int \sin 2x \cos 5x \, dx$. [2]

2 A curve C has equation $4x^2 - 24x + 9y^2 = 0$.
    (a) Sketch C, giving the exact coordinates of points where the curve meets the axes. [2]

    (b) Describe a sequence of transformations that transform the graph of C onto the graph

       of $\dfrac{x^2}{9} + \dfrac{(\alpha y - 1)^2}{4} = 1$ where α is a positive constant. [3]

    (c) State the value of α for which the resulting curve takes on the shape of a circle. [1]

3 The function g is defined by

$$g(x) = \frac{x}{\sqrt{4-x}}, \quad x \in \mathbb{R}, \quad 0 < x < 4.$$

It is given that $f(x) = ax + bx^2 + cx^3$ is the binomial expansion of $g(x) = \dfrac{x}{\sqrt{4-x}}$ in

ascending powers of x up to and including terms in x^3.

    (a) Find the exact value of a, b and c. [4]

    (b) Find the range of values of x such that the percentage error in using $f(x)$ to approximate $g(x)$ is less than 4%. [2]

4 A curve C has equation $y = \dfrac{-3x^2 + 8x - 5}{k(3-x)}$ where k is a positive real constant and $x \neq 3$.

    (a) Sketch C and give the equations of any asymptotes and the exact coordinates of the points where C crosses the axes. [4]

    (b) On the same axes, sketch the graph of $y = \dfrac{3}{k}x - \dfrac{7}{k}$. [1]

    (c) Hence solve the inequality $\dfrac{-3x^2 + 8x - 5}{3-x} \geq 3x - 7$. [2]

5 **(a)** James puts \$500 into an empty safe on 1 January 2023. On the first day of each subsequent month he puts \$10 more than in the previous month, so that he puts \$510 on 1 February 2023, \$520 on 1 March 2023, and so on. On what date will he first have over \$10,000 in total in his safe? [3]

(b) Nora puts \$$x$ on 1 January 2023 into a bank account which pays a compound interest at a rate of 0.5% per month on the last day of each month. She puts a further \$$x$ into the account on the first day of each subsequent month. Given that she does not withdraw any money from her account, find the least integer value of x for which her account will accumulate at least \$50,000 on 31 December 2027. [5]

6 The diagram shows the triangle PQR with $\angle QPR = 30°$, $PQ = (x+1)$ cm and $PR = (4-x)^2$ cm, where $-1 < x < 4$.
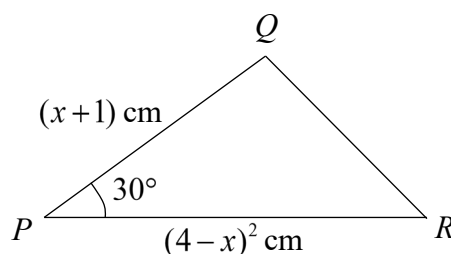
(a) Show that the area, A cm^2, of the triangle is given by $\dfrac{1}{4}(x^3 - 7x^2 + 8x + 16)$. [2]

(b) Using differentiation, find the length QR when A is a maximum. [6]

7 The curve C is defined by the parametric equations

$$x = \frac{1}{t} + 2, \quad y = \frac{4}{t^2} - t, \qquad \text{where } t \in \mathbb{R}, t \neq 0.$$

(a) Find the exact coordinates of the stationary point A. Hence write down the equation of the tangent of C at A. [4]

(b) The tangent at A cuts C again at the point B. Find the equation of the normal of C at B. [4]

(c) The normal of C at B cuts the y-axis at the point F. Find the area of the triangle ABF. [2]

8 **Do not use a calculator in answering this question.**

  (a) Solve the equation $(z+i)^* = 2iz+4$. [4]

  (b) The complex number z and w are such that $\left(\dfrac{z}{w^6}\right)^* = \dfrac{1}{16}e^{\frac{2}{3}\pi i}$ and $w = \sqrt{3}-i$.

   (i) By expressing w in exponential form, find the value of z, giving your answer in the cartesian form $a + ib$, where a and b are real numbers. [5]

   (ii) Sketch an Argand diagram, with origin O, showing the points Z, W and Q representing the complex numbers z, w and $z + w$ respectively. State the geometrical shape of $OZQW$. [2]

9 The curve C is defined by the parametric equations

$$x = 2\tan\theta \ , \ y = \cos^2\theta, \ \text{where} \ -\pi < \theta < \pi.$$

The finite region R is bounded by C, the x-axis and the lines $x = -a$ and $x = a$, where $a > 0$.

  (a) Sketch the curve C and shade the region R. [2]

  (b) Show that the area of R can be expressed as $\displaystyle\int_0^{\tan^{-1}\left(\frac{a}{2}\right)} k \, d\theta$ where k is an integer to be determined. Hence find the exact value of a if the area of R is $\dfrac{2\pi}{3}$ units^2. [4]

  (c) Show that the cartesian equation of C is $y = \dfrac{4}{x^2+4}$. [1]

  (d) For the case when $a = 2$, find the exact volume of the solid form when R is rotated π radians about the y-axis. [5]

10 Water is being pumped from reservoir into a filtration device at a constant rate of 8 gallons per hour. The filtration device processes the water and discharges it at a rate proportional to the volume of water currently in it. At time t hours, the volume of water in the device is v gallons. The filtration device is initially empty. After 10 hours, the volume of water in the device is 5 gallons.

**(a)** Find an expression for v in terms of t. [7]

(b) Hence find the volume of water in the filtration device in the long run. [1]

A fault in the filtration device is discovered when there are 4.5 gallons of water in the device. The device now discharges water at a rate that is equal to the square of the volume of water currently in it.

(c) Write down a differential equation to model the new situation. [1]

(d) Find the time (to the nearest minute) that elapsed from the moment the fault is discovered to the moment when the water in the device drops to 3 gallons. [4]

11 The diagram shows the cross-section of a room with a block of transparent material that is placed tilted to the floor. Points (x, y, z) are defined relative to a point O on the floor with coordinates $(0, 0, 0)$ where units are metres. The block has smooth top and bottom surfaces that are parallel to each other. It has a width of h m, where $h < 1$.
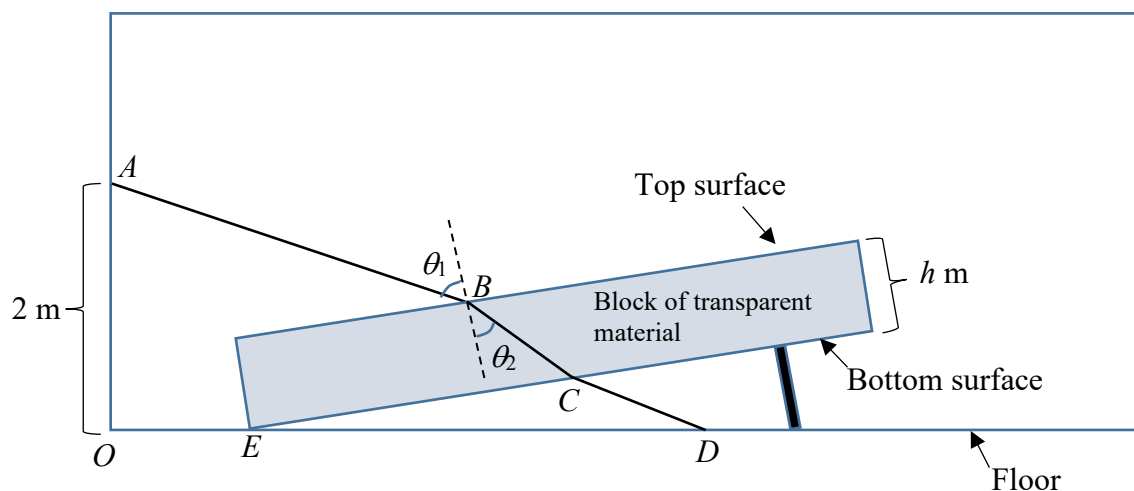
A light beam is projected from A with coordinates $(0, 0, 2)$. B with coordinates $(3, 0, 1)$ is a point on the top surface of the block and C is a point on the bottom surface of the block. The light travels from A to B, then from B to C in the block and finally from C to D which is a point on the floor. It is known that CD is parallel to AB.

The top surface of the block is a plane with equation $\mathbf{r} \cdot \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix} = 1$.

The normal of the top surface makes an acute angle of θ_1 and θ_2 with the lines AB and BC respectively. The angles θ_1 and θ_2 are related by the relation $\dfrac{\sin \theta_1}{\sin \theta_2} = k$.

It is given that $\theta_2 = \dfrac{\pi}{4}$.

(a) Find k. [3]

(b) Given that a vector parallel to BC is $\begin{pmatrix} t \\ 0 \\ -1 \end{pmatrix}$ where t is a positive constant, find the

value of t. Hence write down the equation of the line BC. [3]

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The point E has coordinates $(1, 0, 0)$,

(c) Find the equation of the bottom surface of the block. [1]

(d) Find the exact value of h. [2]

(e) Find the position vector of D. [5]