H2 Topic 13 – Electric Field

"Faraday Cage" demonstration at the Singapore Science Centre. *An innocent audience-volunteer can often be found sitting unharmed inside the cage, as high-voltage, high-current electrical arcs sends lethal amounts of charges coursing down the metallic cage structure.*

Content

- Concept of an electric field
- Electric force between point charges
- Electric field of a point charge
- Uniform electric fields
- Electric potential

Learning Outcomes

Candidates should be able to:

(a) show an understanding of the concept of an electric field as an example of a field of force and define electric field strength at a point as the electric force exerted per unit positive charge placed at that point

(b) represent an electric field by means of field lines

(c) recognise the analogy between certain qualitative and quantitative aspects of electric field and gravitational field

(d) recall and use Coulomb's law in the form $F = \dfrac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$ for the electric force between two point charges in free space or air

(e) recall and use $E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$ for the electric field strength of a point charge in free space or air

(f) calculate the electric field strength of the uniform field between charged parallel plates in terms of the potential difference and plate separation

(g) calculate the forces on charges in uniform electric fields

(h) describe the effect of a uniform electric field on the motion of charged particles

(i) define the electric potential at a point as the work done per unit positive charge in bringing a small test charge from infinity to that point

(j) state that the field strength of the electric field at a point is numerically equal to the potential gradient at that point

(k) use the equation $V = \dfrac{Q}{4\pi\varepsilon_0 r}$ for the electric potential in the field of a point charge, in free space or air.

13.1 Force Fields

A "field" is a *region in space* where a force is experienced by an "entity" without contact. A *mass* experiences a force when placed in a *gravitational field*. Similarly, a *charged particle* experiences force when placed in an *electric field.*

> An **electric field** is
>
> a region of space where an electric charge experiences an electric force

Forces are vector quantities so the direction has to be well-defined. The direction of an electric field is the direction of force on a *positive* charge, by convention and by definition.

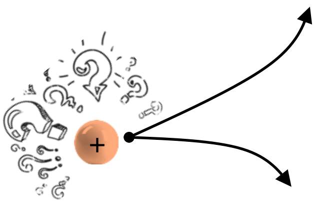
An isolated mass generates its gravitational field in the region surrounding the mass and it permeates all of space ("to infinity"). Similarly, a charged particle or a collection of charged bodies, generate electric field in the surrounding region, all the way to infinity.

When an *additional* charge particle comes into this region, the *additional* charged particle *interacts* with the existing electric field and experiences an electric force.

13.2 Field Lines Represent a Field Visually

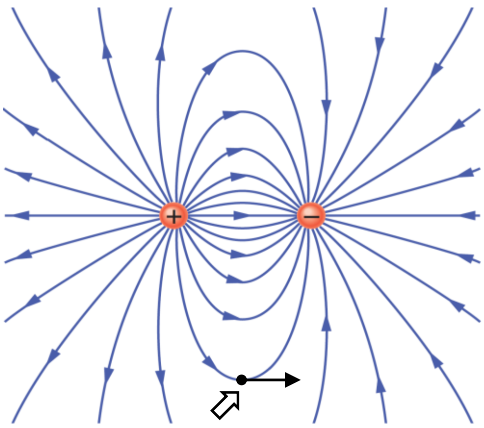
Specifically, the *tangent* at a point on the electric field line shows the direction of electric force that acts on a small stationary positive test charge if the charge is placed at that point and is free to move.

> electric field lines
> - start on positive charges
> - end on negative charges



An electric field line must clearly define the direction of force experienced by a positive charge at that location; therefore ***field lines must never intersect.***

The **arrows** on electric field lines point **away** from positive charges and **towards** negative charges.



The figure on the left represents the electric field between 2 particles of equal but opposite charges.

A small stationary positive test charge at the position marked by ⇗ will experience an electric force directed to the right.

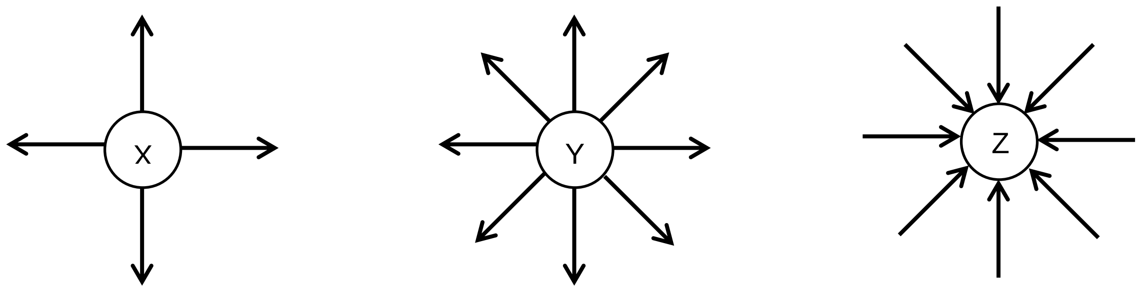
What do you think is the direction of the electric force acting on a negative charge placed at the same position?

direction of electric field lines
is **from** region of **higher electric potential**
to region of **lower electric potential**

A positive charge placed in an electric field tends to move from a region of "higher (potential) energy" to a region of "lower (potential) energy".

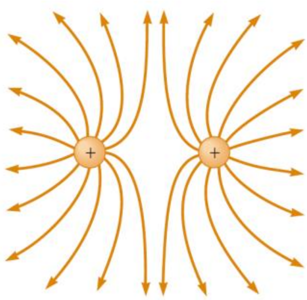
(within the same diagram) electric field strength is

- stronger where electric field lines are closer together
- weaker where electric field lines are further away from each other

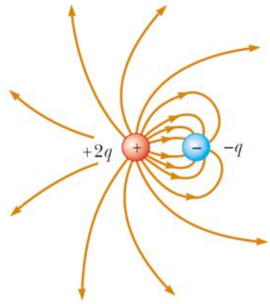


3 isolated point charges are shown above. X and Y are both positively charged. X has less charge than Y. Y and Z have equal magnitude of electric charges of opposite polarities. Individually, the electric fields weaken with increasing distance from the charges.

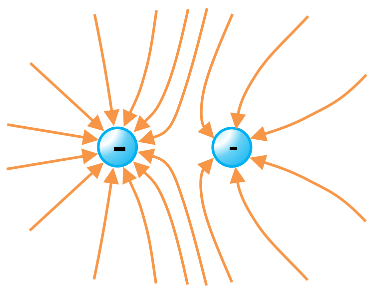
electric field lines are perpendicular to conducting surfaces



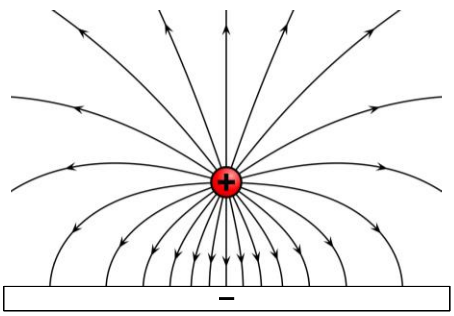
Two positive charges of same magnitude.



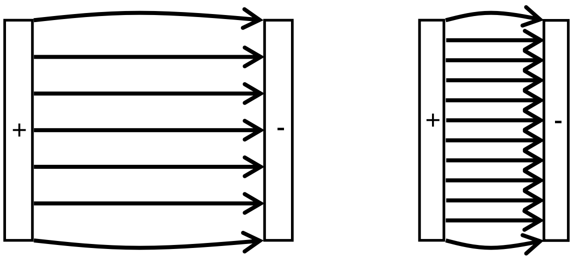
Unlike charges of different magnitude.



Two negative charges of different magnitudes.



A point positive charge and a negatively-charged flat plate. The electric field lines reach the flat plate at right-angles.



Two identical parallel plates of opposite charge. The strength of the uniform electric field in-between is stronger when the plates are closer to each other.

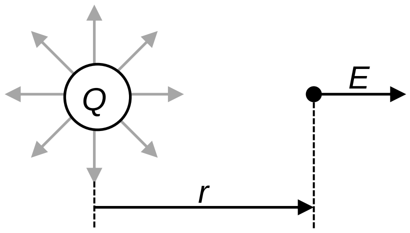
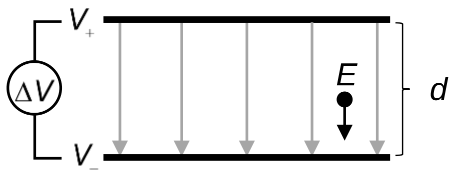
13.3 Electric Field Strength

Electric field strength *E* at a point in the field is	
[type of force]	electric force
[ratio]	*per* unit positive charge

The units of E is: [N C^{-1}] or [V m^{-1}].

Electric field strength is a vector quantity and therefore undergoes vector addition.

The two basic geometries for electric field are (i) radial electric field due to an isolated point charge and (ii) a uniform electric field in-between oppositely-charged parallel plates.

 electric field strength of a **point charge** in free space or air: $$E = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{r^2}\right)$$	 electric field strength of uniform electric field between **charged parallel plates**: $$E = \frac{\Delta V}{d}$$ *(ΔV : potential difference, $V_+ - V_-$)*

13.4 Electric Force

The electric force F_E experienced by an electric charge q at a point in an electric field E where E is the resultant electric field strength at that point.

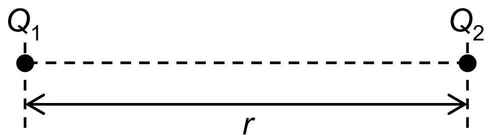
This is analogous to placing a mass m inside a gravitational field of field strength g, $F_g = mg$.

$$F_E = qE$$

13.4.1 Coulomb's Law

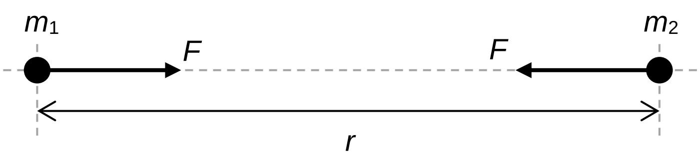
Coulomb's Law states that the	
[type of force]	electric force between two point charges is
[magnitude]	directly proportional to product of the two charges and inversely proportional to the square of separation between the two charges

$$F = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{Q_1Q_2}{r^2}$$



(Compare the above with the gravitational force between two point masses, m_1 and m_2 separated by a distance r)

$$F = G\frac{m_1m_2}{r^2}$$



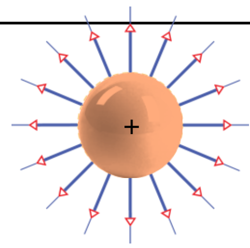
The direction of the electric force acts along the line joining the two point charges.

The electric force can be attractive between unlike charges, or repulsive between like charges (in comparison to gravitational force which can only be attractive).

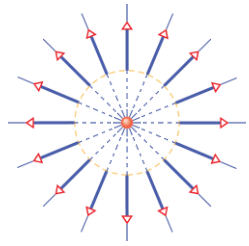
By Newton's 3rd Law, the electric force that Q_1 exerts on Q_2 is equal in magnitude and opposite in direction to the electric force that Q_2 exerts on Q_1 i.e. $\vec{F}_{Q_1 \text{ on } Q_2} = -\vec{F}_{Q_2 \text{ on } Q_1}$

ε_0 is a constant known as the permittivity of free space (vacuum) or air at 8.85×10^{-12} C^2 N^{-1} m^{-2}.

By reference to electric field fields, for a point outside a spherical conductor, the charge on the sphere seems to act as a point charge at its centre because the



electric field lines are perpendicular to the surface of sphere, radial pattern make field lines appear to originate from centre of sphere



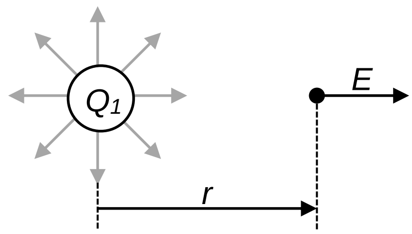
Example 1

By considering electric field strength and Coulomb's Law, provide the expression describing the electric field strength generated by a single point charge.

Solution:

By Coulomb's Law:

$$F_e = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}$$



Recall that electric field strength is the electric force per unit positive charge experienced by a small stationary positive test charge at that point.

The field strength due to Q_1 is:

$$\begin{aligned} E &= \frac{F}{Q_2} \\ &= \frac{1}{\cancel{Q_2}}\left(\frac{Q_1 \cancel{Q_2}}{4\pi\varepsilon_0 r^2}\right) \\ &= \frac{Q_1}{4\pi\varepsilon_0 r^2} \end{aligned}$$

Note: similar to gravitational field strength, electric field strength due to a point charge follows an inverse square law, $E \propto \frac{1}{r^2}$

(F is the electric force experienced by Q_2 due to the electric field set up by Q_1)

Example 2

A hydrogen atom consists of an electron (m_e = 9.11 × 10^{-31} kg) orbiting around a nucleus of 1 proton (m_p = 1.67 × 10^{-27} kg). 1 electronic charge of a proton or electron is ±1.6 × 10^{-19} C. Find the ratio of the electrical force to the gravitational force between the proton and electron.

Solution:

By Newton's Law of Gravitation:

$$F_g = \frac{G m_e m_p}{r^2}$$

By Coulomb's Law:

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{q_e^2}{r^2}$$

Ratio:

$$\frac{F_e}{F_g} = \frac{1}{4\pi\varepsilon_0}\frac{q_e^2}{r^2} \times \frac{r^2}{Gm_em_p}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_e^{\,2}}{Gm_em_p}$$

$$= \frac{(1.6\times10^{-19})^2}{4\pi(8.85\times10^{-12})(6.67\times10^{-11})(9.11\times10^{-31})(1.67\times10^{-27})}$$

$$= 2.27\times10^{39}$$

$$F_e \gg F_g$$

Note: Therefore, we typically ignore the negligible gravitational attraction in such systems.

Example 3

The radius of a hydrogen atom can be estimated by the radius of orbit of the lone electron. Calculate the velocity of the electron (of mass m_e = 9.11 × 10^{-31} kg) if the radius is about 25 pm.

Solution:

Electric force of attraction provides the centripetal force that keeps electron in circular motion:
By Coulomb's Law:

$$F_e = \frac{1}{4\pi\varepsilon_0}\frac{q_e^2}{r^2} = \frac{m_ev^2}{r}$$

$$v = q_e\sqrt{\frac{1}{4\pi\varepsilon_0 m_e r}}$$

$$= (1.6\times10^{-19})\sqrt{\frac{1}{4\pi(8.85\times10^{-12})(9.11\times10^{-31})(25\times10^{-12})}}$$

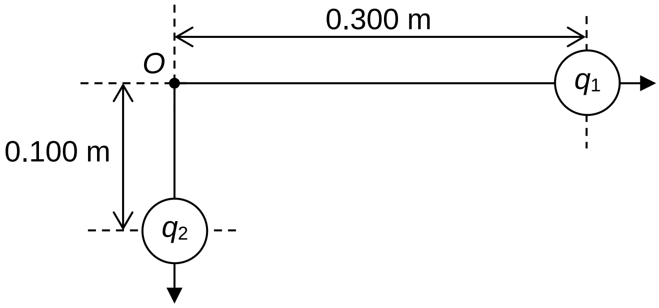
$$= 3.2\times10^{6}\ \text{m s}^{-1}$$

Example 4

Two charges q_1 and q_2 , of −6.0 nC and 3.0 nC respectively, are arranged as shown.

(i) Calculate the magnitude and direction of the electric field strength at *O*.

(ii) Hence calculate the force experienced by a −5.0 nC charge placed at *O* the origin and state its direction.



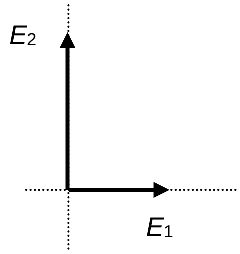
Solution:

(i) Let E_1 be the electric field due to the −6.0 nC charge.
Let E_2 be the electric field due to the 3.0 nC charge.

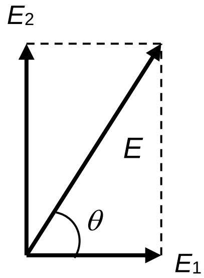
By Coulomb's Law:

$$E_1 = \frac{q_1}{4\pi\varepsilon_0 r^2} = \frac{6.0\times10^{-9}}{4\pi\left(8.85\times10^{-12}\right)\left(0.300^2\right)} = 599 \text{ N C}^{-1} \text{ to right}$$

$$E_2 = \frac{q_2}{4\pi\varepsilon_0 r^2} = \frac{3.0\times10^{-9}}{4\pi\left(8.85\times10^{-12}\right)\left(0.100^2\right)} = 2698 \text{ N C}^{-1} \text{ to top}$$



$$\left|E_{\text{resultant}}\right| = \sqrt{E_1^2 + E_2^2} = \sqrt{599^2 + 2698^2} = 2764 \approx 2760 \text{ N C}^{-1}$$



$$\tan\theta = \frac{E_2}{E_1}$$

$$\theta = \tan^{-1}\left(\frac{2698}{599}\right) = 77.5°$$

anticlockwise from positive x-axis

$$F = qE_{\text{resultant}} = \left(-5.0\times10^{-9}\right)\left(2764\right) = 1.34\times10^{-5} \text{ N}$$

directed 180° – 77.5° = 102.5° clockwise from positive *x*-axis (i.e. opposite direction from electric field line)

13.4.2 Electric Field Due to 2 or more Charged Particles

We perform vector addition to find the resultant electric field $E_{\text{resultant}}$ at a point if the field is set up by multiple charged particles. The force acting on an *additional* charged particle that is later placed inside that field at a point is $F = q_{\text{additional}}E_{\text{resultant}}$.

13.4.3 Point Charges vs Conducting Shells

Electric charges are redistributed in a conductor to achieve electrostatic equilibrium i.e. the charges do not move at equilibrium. If a conductor has a net electric charge, then the net excess charges will be positioned furthest away from each other / spread out as far as possible:

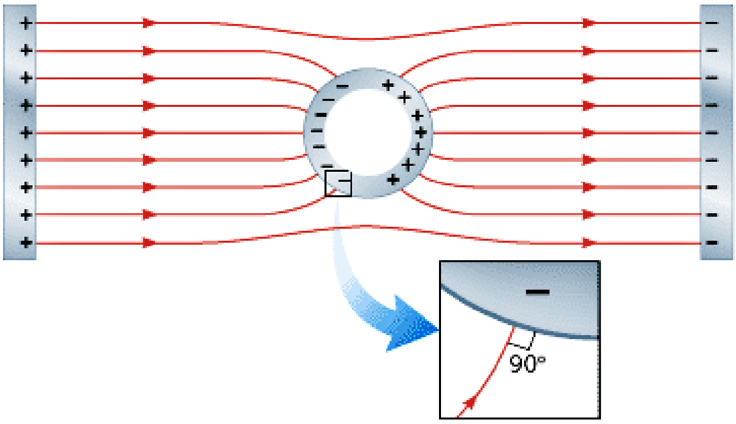
electric charges in a conductor always reside entirely on the conductor's surface

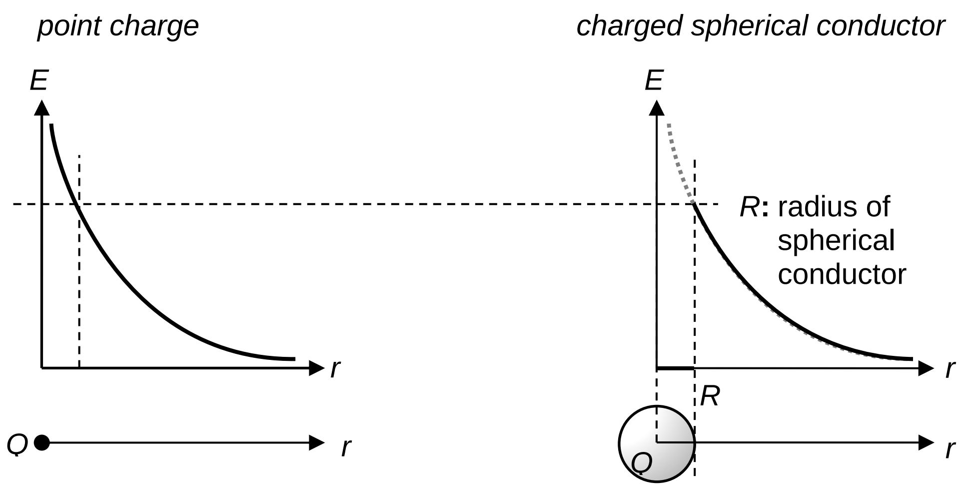
Explain why the electric field strength inside a thin conducting sphere is zero.

Electric charges in a conductor will move in an external electric field.
Electric charges in a conductor move under the influence of one another's electric field. There is a redistribution of charges until no resultant electric force acts on the charges. This is because The redistributed charges set up an electric field that is equal and opposite to the external electric field. Net field strength is now zero. Thus, no electric field.

Since the electric field inside a conducting surface must be zero, a thin metallic shell effectively shields its inside from external fields.

A Faraday's Cage (cover photo) shields its contents from external fields. Cables carrying digital data are often shielded against stray EM waves from corrupting the signals. This is why our mobile connection may be cut when entering a lift.





$$E = \frac{1}{4\pi\epsilon_0}\left(\frac{Q}{r^2}\right)$$

for $r < R$, $E = 0$

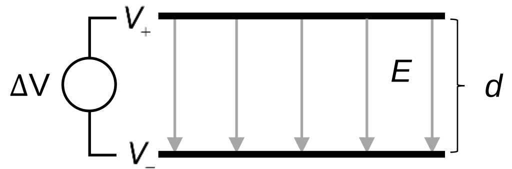
for $r \geq R$, $E = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r^2}\right)$

note: for $r \geq R$, regard all of Q to be at the centre

13.4.4 Uniform Electric Field Between Charged Parallel Plates

between charged parallel plates, the electric field strength is uniform at every point in the region:

$$E = \frac{\Delta V}{d}$$



We assume the separation between the plates, *d*, to be small compared to the dimensions of the plate. More accurate diagrams may choose to include "fringing of the electric field", where the field lines bend outward at the edge of the charged parallel plates.

As the electric field strength is uniform, the force experienced by a charged particle inside the uniform electric field is uniform, and therefore the charged particle exhibits constant acceleration. We can use the kinematics equations to analyse motion of charged particles inside a uniform electric field.

13.4.4.1 Motion Parallel to Uniform Electric Field

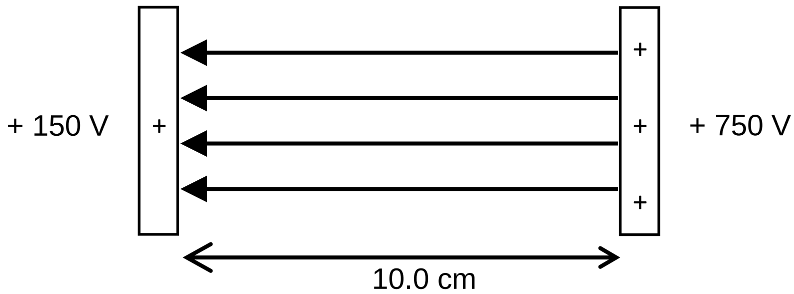
Example 5a

2 plates placed 10.0 cm apart have potential values of +750 V and +150 V. Calculate

a) the electric field between the plates, and

b) the acceleration experienced by a Ca^{2+} ion (40 g mol^{-1}) placed in between the plates.

Solution:



a) electric field strength:

$$E = \frac{\Delta V}{d} = \frac{750-150}{10\times10^{-2}} = 6000 \text{ V m}^{-1}$$

b) electric force: $F = qE$

By Newton's 2nd Law: $F = ma$

$$qE = ma \rightarrow a = \frac{(2q_e)E}{m_{Ca}}$$

$$a = \frac{(2(1.6\times10^{-19}))(6000)}{(40\times10^{-3})\dfrac{1}{N_{Avogadro}}}$$

$$= 2.9\times10^{10} \text{ m s}^{-2}$$

13.4.4.2 Parabolic Path In Uniform Electric Field

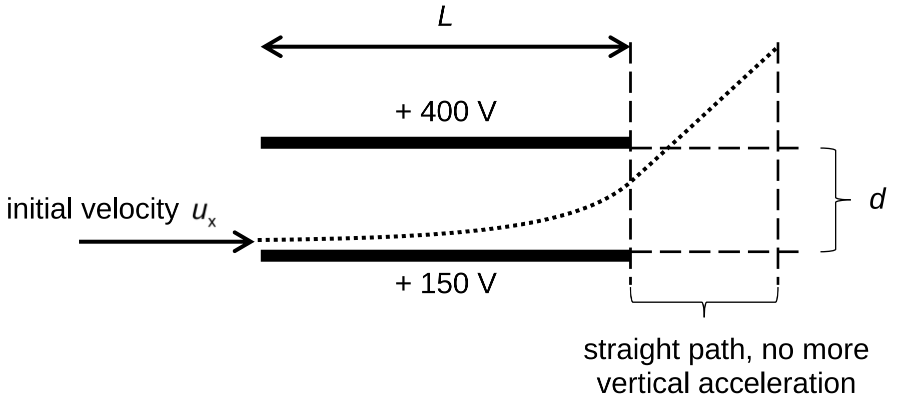
Example 5b

An electron beam enters a uniform electric field between a pair of parallel charged plates near the plate of lower potential as shown.

(i) Show that the magnitude of acceleration experienced by an electron is given by $\dfrac{4.4\times10^{13}}{d}$.

(ii) Find the expression, in terms of u_x and d, for the maximum length L such that the electrons do not collide with the upper plate.

Solution:



$$\text{electric force } F_e = qE$$
$$\text{By Newton's 2nd Law } F = ma$$
$$qE = ma$$

$$\text{vertical acceleration } a_y = \frac{qE}{m} = \frac{q_e}{m_e}\frac{\Delta V}{d}$$
$$= \frac{1.6\times10^{-19}}{9.11\times10^{-31}}\frac{(400-150)}{d}$$
$$= \frac{4.4\times10^{13}}{d}$$
$$s_y = u_y t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}a_y t^2$$
$$\text{time of flight inside field } t = \sqrt{\frac{2s_y}{a_y}} = \sqrt{\frac{2d}{a_y}}$$

$$L_{\max} = u_x t = u_x\sqrt{\frac{2d}{a_y}}$$
$$= u_x\sqrt{2d\left(\frac{d}{4.4\times10^{13}}\right)}$$
$$= \left(2.13\times10^{-7}\right)u_x d$$

13.5 Electric Potential and Electric Potential Energy

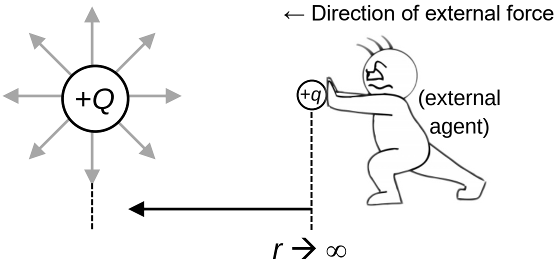
Electric potential is the

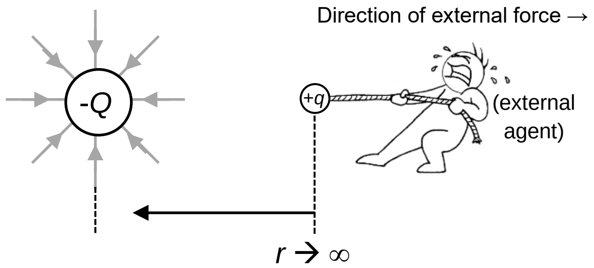
work done per unit positive charge
in moving a small positive test charge
from infinity to that point

$$V = \frac{U}{Q}$$

The units of V is: [J C^{-1}] or the volt, [V]. Electric potential is a scalar quantity.

The electric potential at infinity is defined as zero. For e.g., a valence electron of an atom "reaches infinity" when the atom is ionised, as the electron no longer interacts with the electric field of the positively charged nucleus.





In many other definitions of potential, the concept of "an external agent" causing the change in location of the test charge "without a change in the kinetic energy" is included for further clarification – that the work done by the external agent represents the change in potential energy of the system. These phrases are not required formally for exam answers.

The electric potential *energy* of a charge, U, is the

electric potential energy at a point

$$U = QV$$

work done in moving *that particular charge* from infinity to that point in the electric field.

change in electric potential energy

$$\Delta U = Q\Delta V$$
$$= Q\left(V_{\text{final}} - V_{\text{initial}}\right)$$

Unlike gravity, which is always attractive and so always results in a negative value for potential energy, the electric potential energy of a charge will depend on whether the interaction is attraction or repulsion.

sign of ΔU	interpretation
positive	• work done <u>on</u> electric field • gain in electric potential energy of system
negative	• work is done <u>by</u> electric field • loss of electric potential energy from system

The units for electric potential energy U can be in Joules [J] or the electron-volt [eV].

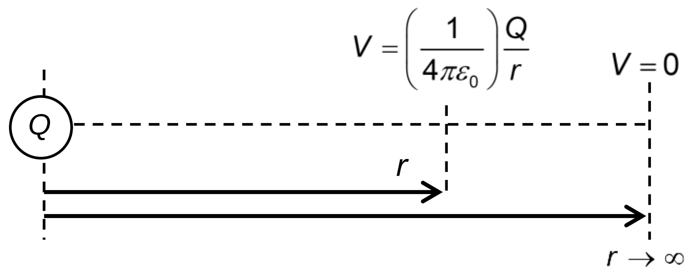
1 eV is the (kinetic) energy gained by an electron when it is accelerated through a potential difference of 1 V.

$$U = QV$$
$$1\text{ eV} = q_e V$$
$$= \left(1.6\times10^{-19}\text{ C}\right)\left(1\text{ V}\right)$$
$$= 1.6\times10^{-19}\text{ J}$$

13.5.1 Electric Potential due to a Point Charge or Spherical Charged Conductor

electric potential
in the field of a point charge
in free space or air is

$$V = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{Q}{r}$$

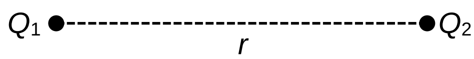


The potential at a point outside a spherical charged conductor (of radius R, carrying charge Q) has the same form as that due to a point charge of the same charge.

13.5.2 Electric Potential Energy of a System of 2 Point Charges

For an electric field generated by point charge Q_1, the electric potential energy U of the system when another charge Q_2 is brought from infinity to a distance r away from the centre of Q_1:

$$U = Q_2 V_{\text{due to } Q_1} = Q_2\left(\frac{1}{4\pi\varepsilon_0}\frac{Q_1}{r}\right)$$



electric potential energy stored between two isolated point charges $U = \dfrac{Q_1 Q_2}{4\pi\varepsilon_0 r}$

Charges Q_1 and Q_2 should be substituted together with their electrical polarities (+ or –).

Example 6

A spherical conductor of radius 25 cm carries a charge of +6.0 μC. Calculate the

(a) electric potential at a point X, 25 cm from the surface of the sphere,

$$V_x = \frac{1}{4\pi\epsilon_0}\frac{Q}{r} = \frac{+6.0\times10^{-6}}{4\pi(8.85\times10^{-12})[(25+25)\times10^{-2}]} = 1.08\times10^{5}\text{ V}$$

(b) electric potential at a point Y, 50 cm from the surface of the sphere,

$$V_Y = \frac{1}{4\pi\epsilon_0}\frac{Q}{r} = \frac{+6.0\times10^{-6}}{4\pi(8.85\times10^{-12})[(25+50)\times10^{-2}]} = 7.19\times10^{4}\text{ V}$$

(c) change in potential energy of a +10 μC in moving from point X to point Y,

$$\Delta U = q\Delta V = q\left(V_{\text{final}} - V_{\text{initial}}\right) = q\left(V_Y - V_X\right) = \left(+10\times10^{-6}\right)(7.19\times10^{4} - 1.08\times10^{5}) = -0.361\text{ J}$$

(d) the change in potential energy of a −10 μC in moving from point X to point Y, and

$$\Delta U = q\Delta V = q\left(V_{\text{final}} - V_{\text{initial}}\right) = q\left(V_Y - V_X\right) = \left(-10\times10^{-6}\right)(7.19\times10^{4} - 1.08\times10^{5}) = +0.361\text{ J}$$

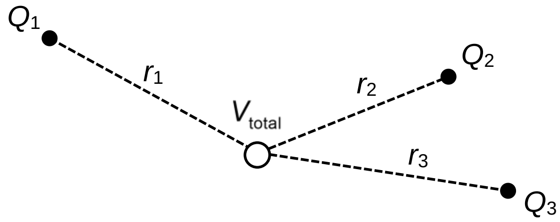
(e) the change in potential energy of a −10 μC in moving from point Y to point X.

$$\Delta U = q\Delta V = q\left(V_{\text{final}} - V_{\text{initial}}\right) = q\left(V_X - V_Y\right) = \left(-10\times10^{-6}\right)(1.08\times10^{5} - 7.19\times10^{4}) = -0.361\text{ J}$$

13.5.3 Electric Potential of a System of Multiple Point Charges

Electric potential is a scalar quantity. The total electric potential at a point V_{total} due to 3 point charges Q_1, Q_2 and Q_3 is the scalar sum of electric potential V_1, V_2 *and* V_3 at that point, due to the individual charges Q_1, Q_2 *and* Q_3.

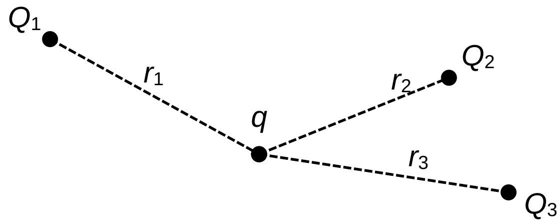
$$V_{\text{total}} = V_1 + V_2 + V_3$$
$$= \left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \frac{Q_3}{r_3}\right)$$



13.5.4 Electric Potential Energy of a System of Multiple Point Charges

The electric potential energy of a charge q placed at a point due to multiple point charges can be found by the product of q and the scalar sum of electric potential V_{total} at that point.

$$U_{\text{total}} = qV_{\text{total}}$$
$$= q\left(V_1 + V_2 + V_3\right)$$
$$= q\left(\frac{1}{4\pi\varepsilon_0}\right)\left(\frac{Q_1}{r_1} + \frac{Q_2}{r_2} + \frac{Q_3}{r_3}\right)$$



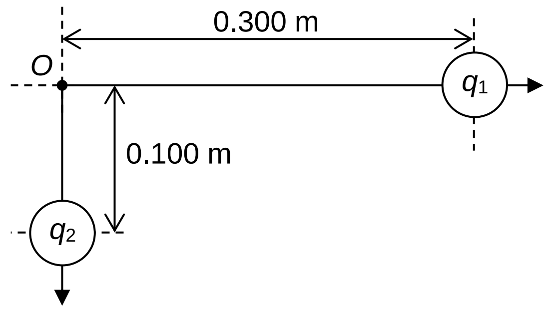
Example 7

Consider the set up in Example 4 where there are two charges q_1 and q_2, of −6.0 nC and 3.0 nC respectively. Determine the

(i) electric potential at *O*,

(ii) electric potential energy that a −5.0 nC charge has when placed at *O*,

(iii) work done in bringing a charge of −5.0 nC from *O* to infinity.



Solution:

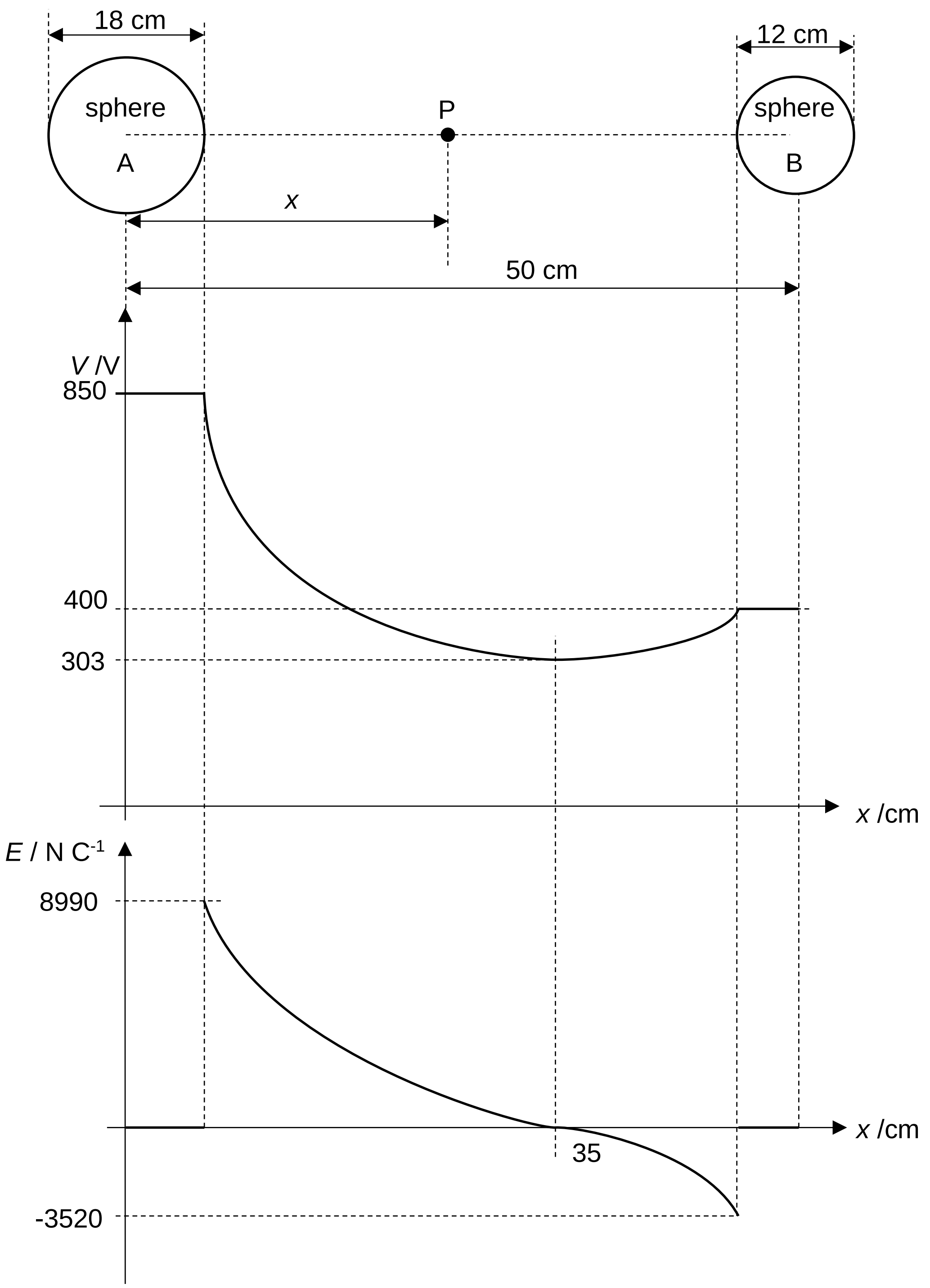
$$\text{(i) } V_{\text{total}} = \frac{q_1}{4\pi\varepsilon_0 r_1} + \frac{q_2}{4\pi\varepsilon_0 r_2}$$
$$= \frac{-6.0\times10^{-9}}{4\pi\left(8.85\times10^{-12}\right)(0.300)} + \frac{3.0\times10^{-9}}{4\pi\left(8.85\times10^{-12}\right)(0.100)}$$
$$= 89.9 \text{ V}$$

$$\text{(ii) } U = qV$$
$$= \left(-5.0\times10^{-9}\right)(89.9)$$
$$= -4.50\times10^{-7} \text{ J}$$

$$\text{(iii) work done to bring to infinity } = -\left(\text{work done to bring FROM infinity}\right)$$
$$= -U$$
$$= 4.50\times10^{-7} \text{ J}$$

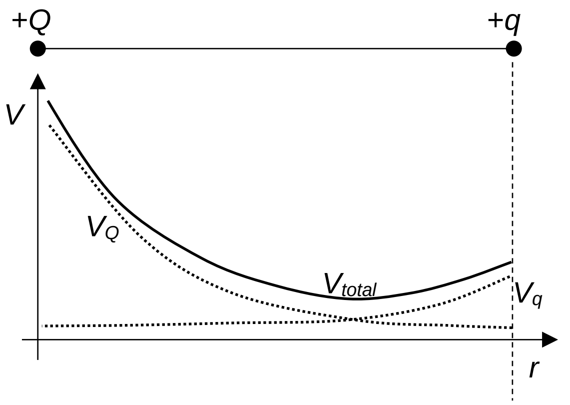
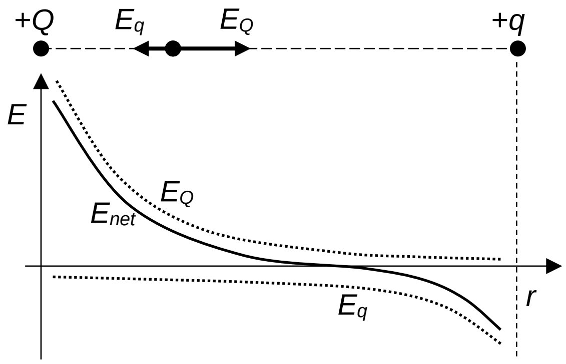
Example 8

Two charged metal spheres A and B, of diameters 18 cm and 12 cm respectively, are isolated in space and separated by a distance of 50 cm. The potential at the surface of A is 850 V and the potential at the surface of B is 400 V. Sketch the variation with *x*, of the electric potential *V* and the electric field strength *E*.



13.5.5 explains in detail how the shape and critical values of the above curves are obtained.

13.5.5 Curve Sketching

Resultant electric *potential* graph is a *scalar addition* of individual potentials of charged bodies. For two positive point charges carrying unequal charges:	Resultant electric *field strength* graph is a *vector addition*. Therefore, need to define direction. For the same system below, we defined positive to be pointing to the right:
	

It is **not trivial** to sketch *and label* the curve in Example 8 above. The following is needed:

<u>find magnitude of charges:</u>

$$850 = \frac{Q_A}{4\pi\varepsilon_0 r_A} + \frac{Q_B}{4\pi\varepsilon_0 (0.50 - r_A)}$$

$$400 = \frac{Q_A}{4\pi\varepsilon_0 (0.50 - r_B)} + \frac{Q_B}{4\pi\varepsilon_0 r_B}$$

$$\begin{cases} 850(4\pi(8.85\times10^{-12})) = \dfrac{Q_A}{(0.09)} + \dfrac{Q_B}{(0.50-0.09)} \\ 400(4\pi(8.85\times10^{-12})) = \dfrac{Q_A}{(0.50-0.06)} + \dfrac{Q_B}{(0.06)} \end{cases}$$

solve simultaneously

$Q_A = 8.17$ nC, $Q_B = 1.56$ nC

<u>find location of zero field strength , where E = 0</u>

$$|E_A| = |E_B| \rightarrow \frac{Q_A}{4\pi\varepsilon_0 r^2} = \frac{Q_B}{4\pi\varepsilon_0 (0.5 - r)^2}$$

$r = 35$ cm

<u>find potential at point of zero field strength:</u>

$$V_{r=35cm} = V_A + V_B = \frac{Q_A}{4\pi\varepsilon_0 r_{35}} + \frac{Q_B}{4\pi\varepsilon_0 (0.50 - r_{35})}$$

$$= \frac{1}{4\pi(8.85\times10^{-12})}\left(\frac{8.17\times10^{-9}}{0.35} + \frac{1.56\times10^{-9}}{(0.50-0.35)}\right)$$

$$= 303 \text{ V}$$

<u>find field strengths at surfaces of spheres:</u>

$$E_A = \frac{Q_A}{4\pi\varepsilon_0 {r_A}^2} - \frac{Q_B}{4\pi\varepsilon_0 (0.50 - r_A)^2}$$

$$= \frac{\dfrac{8.17\times10^{-9}}{(0.09)^2} - \dfrac{1.56\times10^{-9}}{(0.50-0.09)^2}}{4\pi(8.85\times10^{-12})}$$

$$= 8990 \text{ N C}^{-1} \ (\text{to the right})$$

$$E_B = \frac{Q_A}{4\pi\varepsilon_0 (0.50 - r_B)^2} - \frac{Q_B}{4\pi\varepsilon_0 {r_B}^2}$$

$$= \frac{\dfrac{8.17\times10^{-9}}{(0.50-0.06)^2} - \dfrac{1.56\times10^{-9}}{(0.06)^2}}{4\pi(8.85\times10^{-12})}$$

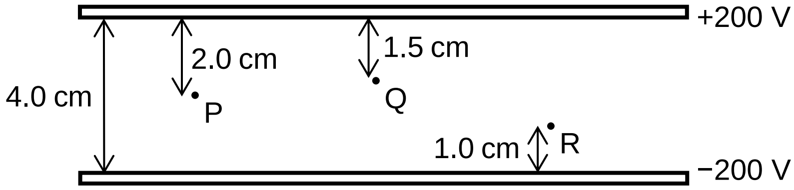
$$= -3520 \text{ N C}^{-1} \text{ (to the left)}$$

13.5.6 Electric Potential due to Charged Parallel Plates

The electric potential changes uniformly with the perpendicular distance from the plates since E is uniform and given by $E = \dfrac{\Delta V}{d}$.

Example 9

Two charged parallel plates of potential +200 V and –200 V are 4.0 cm apart as shown.

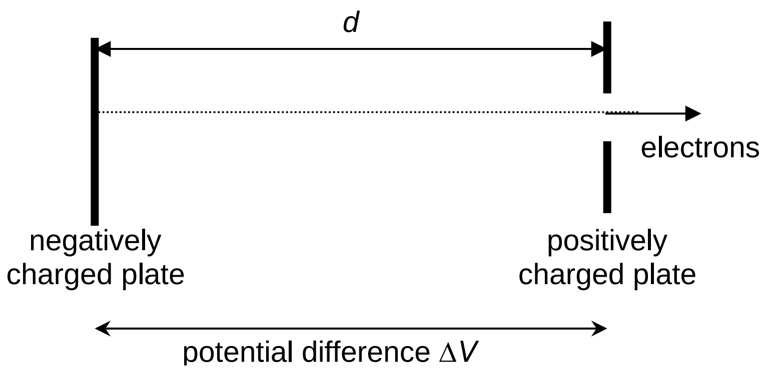


The table below shows the electric potentials, as well as the electric potential energy possessed by an electron at points P, Q and R:

	P	Q	R
electric potential	0 V	+ 50 V	-100 V
electric potential energy of an electron	0 eV	- 50 eV	+ 100 eV
	0 J	-8×10^{-18} J	$+1.6 \times 10^{-17}$ J

Example 10

Electrons are accelerated from rest at the negatively charged plate. Obtain an expression for the velocity of electrons as they exit the electric field from the positively charged plate.



Method 1

By Principle of Conservation of Energy

loss of E_P by electron = gain in E_K by electron

$$\Delta U = E_{K,\text{ final}} - E_{K,\text{ initial}}$$

$$q_e \Delta V = \frac{1}{2} m_e v^2 - 0$$

$$v = \sqrt{\frac{2 q_e \Delta V}{m_e}}$$

Method 2

Electric force: $F = qE$

By Newton's 2nd Law: $F = ma$

$$qE = ma$$

$$\text{acceleration } a = \frac{qE}{m} = \frac{q_e}{m_e}\frac{\Delta V}{d}$$

$$v^2 = u^2 + 2as$$

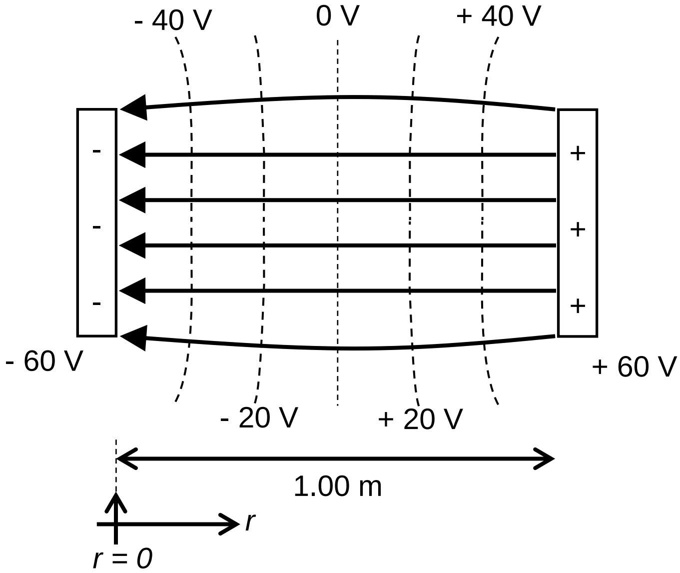
$$= 0 + 2\left(\frac{q_e \Delta V}{m_e d}\right) d$$

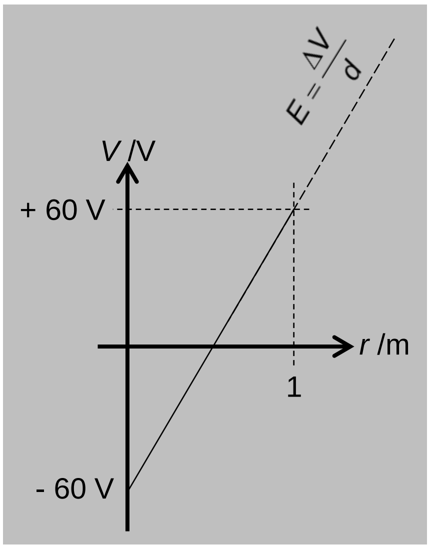
$$v = \sqrt{\frac{2 q_e \Delta V}{m_e}}$$

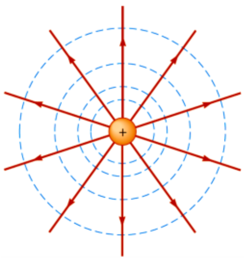
13.6 Equipotential Lines

equipotential lines are lines joining points that have the same potential in a field	equipotential lines always meet electric field lines at right angles

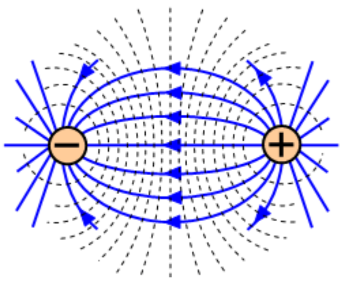
For 3D geometries, there would be equipotential surfaces as opposed to equipotential lines for 2D geometries. No work is done when a (external) charge is moved along an equipotential line or surface. In the following diagrams, equipotential lines are shown as dotted lines. In particular for the pair of charged parallel plates, the (non-ideal case) field fringing is shown here.



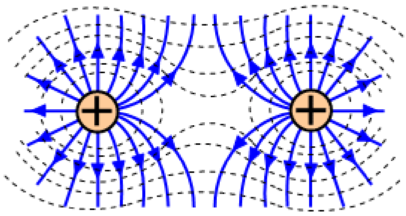




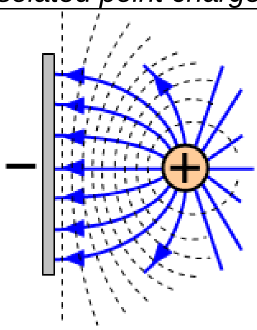
equipotential lines surrounding a single isolated point charge



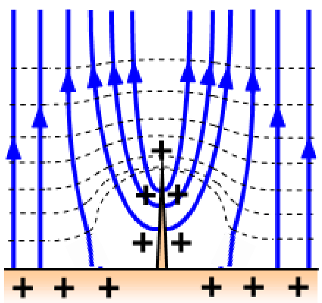
equipotential lines around charges of equal magnitude and opposite charges



equipotential lines around equal charges



equipotential lines around a point charge and charged surface



electric field lines leave a conducting surface at right angles. The electric field at pointed tips are stronger (field lines closer to each other), increasing the chance of lightning striking a lightning rod when the surge of electrons seeks out the path of least resistance.

13.7 Relationship between Electric Field Strength and Electric Potential

[magnitude]	electric field strength E is numerically equal to the electric potential gradient at a point in the field
[direction]	negative sign shows that the direction of field strength points towards direction of lower potential

$$E = -\frac{dV}{dr}$$

$\frac{dV}{dr}$ is the electric potential gradient.

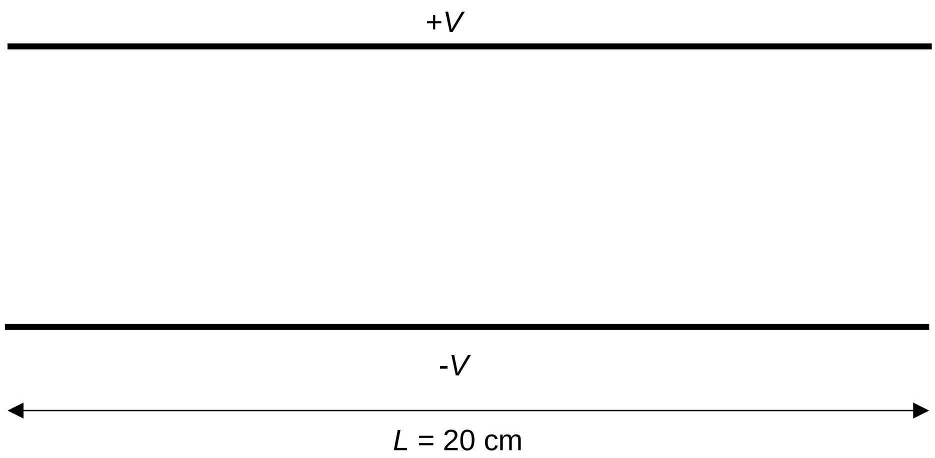
Similarly, $qE = -\frac{d}{dr}(qV)$

$$\boxed{F = -\frac{dU}{dr}}$$

This means that the electric *force* experienced by a charged particle q is numerically equal to the rate of change of *potential energy* $\frac{dU}{dr}$ and is directed towards decreasing potential energy.

Example 11

The region of uniform electric field in between two parallel charged plates of length L = 20 cm has an electric field strength of 50 N C^{-1}.



(a) In the diagram above, draw lines to represent the
 (i) electric field in between the charged plates,
 (ii) equipotential lines using dotted lines.

(b) For a free electron in the region, calculate the
 (i) electric force experienced,
 (ii) acceleration experienced.

A horizontal electron beam enters with speed of 2.0×10^6 m s^{-1} and emerges from this region.

(c) Draw lines to show the beam trajectory inside and after emerging from this region.

(d) Show that the beam deviates 4.4 cm vertically from its original line of motion of the electrons at the point it emerges.

Solution:

(b)(i)

$$F_e = qE$$
$$= (1.6\times10^{-19})(50)$$
$$= 8.00\times10^{-18}\text{ N}$$

(b)(ii)

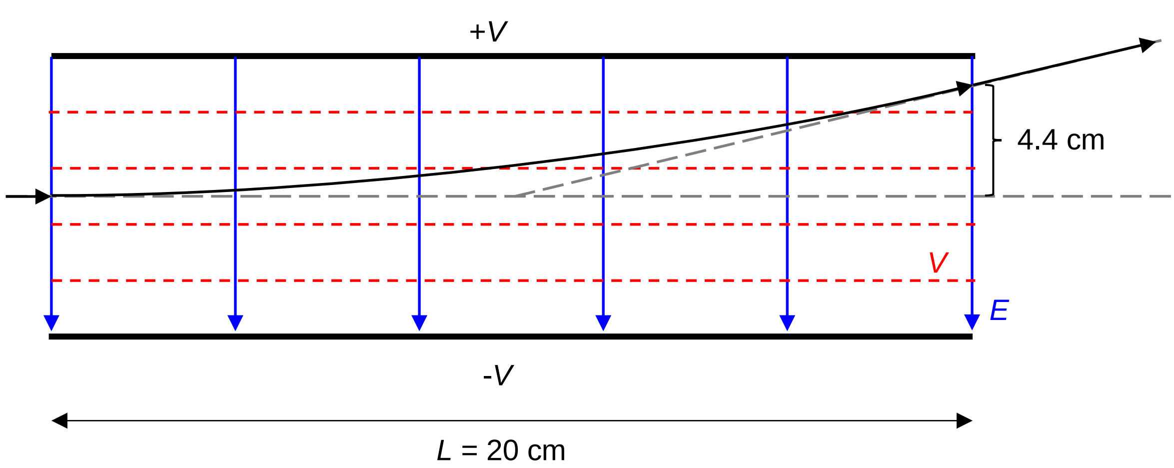
$$F_e = ma \qquad \text{by Newton's 2nd Law}$$
$$a = \frac{F_e}{m}$$
$$= \frac{8.00\times10^{-18}}{9.11\times10^{-31}}$$
$$= 8.78\times10^{12}\text{ m s}^{-2}$$

(d)

$$L = v_x t$$

$$\text{time of flight } t = \frac{L}{v_x}$$
$$= \frac{20\times10^{-2}}{2.0\times10^{-6}}$$
$$= 1.00\times10^{-7}\text{ s}$$

$$\text{vertically } s_y = u_y t + \frac{1}{2}a_y t^2$$
$$= 0 + \frac{1}{2}(8.78\times10^{12})(1.00\times10^{-7})^2$$
$$= 0.0439\text{ m}$$
$$= 4.4\text{ cm}$$



13.8 Comparisons with Gravitational Field

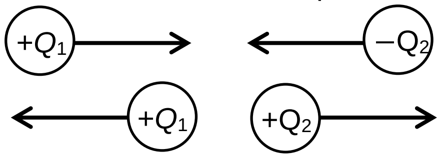
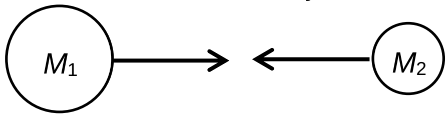
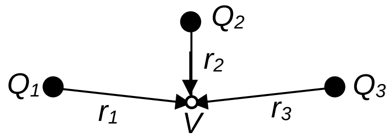
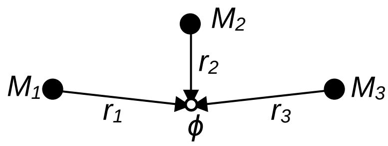
13.8.1 Similarities

(for isolated point mass/charge) variation of field strength with distance	field strength inversely proportional to square of distance from centre of mass/charge (field strength) $\propto \dfrac{1}{r^2}$
(for isolated point mass/charge) field lines	• greater separation of field lines with increasing distance from point mass/charge • field lines are normal to surface of mass/charge
energy considerations	Both electric force and gravitational force are conservative forces. (i.e. the work done is **independent of the path taken**, and depends only on the initial and final positions.)
relationship between field strength and potential / force and potential energy	**[magnitude]** field strength is numerically equal to the potential gradient at that point **[direction]** field points towards direction of lower potential $E = -\dfrac{dV}{dr}$ and $g = -\dfrac{d\phi}{dr}$ **[magnitude]** force is numerically equal to the rate of change of potential energy at that point **[direction]** force is directed towards lower potential energy electric force, $F = -\dfrac{dU}{dr}$ gravitational force, $F = -\dfrac{dE_P}{dr}$

13.8.2 Differences

	electric Field	gravitational Field
direction of field lines	towards negative charge and away from positive charge	towards point mass
nature of force interactions	can be attractive or repulsive	always attractive

13.8.3 Analogies

	electric Field	gravitational Field
origin	charge interaction	mass interaction
nature	can be attractive or repulsive 	attractive only 
force law	Coulomb's Law $F=\left(\frac{1}{4\pi\varepsilon_0}\right)\frac{Q_1Q_2}{r^2}$	Newton's Law of Gravitation $F=G\frac{M_1M_2}{r^2}$
field strength	force per unit positive charge on a small stationary test charge at that point $E=\frac{F}{Q}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$ [N C^{-1}]	force per unit mass on a small test mass at that point $g=\frac{F}{m}=\frac{GM}{r^2}$ [N kg^{-1}]
potential	work done per unit positive charge in moving a small test charge from infinity to that point $V=\frac{U}{q}=\frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$ [J C^{-1}]	work done per unit mass in bringing small test mass from infinity to that point $\phi=\frac{W}{m}=-\frac{GM}{r}$ [J kg^{-1}]
potential due to multiple masses or charges	$V=\frac{Q_1}{4\pi\varepsilon_0 r_1}+\frac{Q_2}{4\pi\varepsilon_0 r_2}+\ldots$ 	$\phi=\frac{-GM_1}{r_1}+\frac{-GM_2}{r_2}+\ldots$ 
potential energy	$U=QV$	$E_P=m\phi$
relationship between field and potential	$E=-\frac{dV}{dr}$ field strength numerically equal to potential gradient at that point negative sign indicates that the force acts in the direction of decreasing potential	$g=-\frac{d\phi}{dr}$
relationship between force and potential energy	$F=-\frac{dU}{dr}$	$F=-\frac{dE_p}{dr}$

13.9 Concept Map