



RAFFLES INSTITUTION
MATHEMATICS DEPARTMENT
2022 YEAR 4 RP MATHEMATICS
TOPIC 3B: DIFFERENTIATION (MATHS 2)

SUPPLEMENTARY WORKSHEET

Name: _____

Class: Sec 4 () Date: _____

1 **2020/Y4RP/M2/T1/Q1**

Differentiate $\ln(\cos^5 3x)$ with respect to x , leaving your answer as a single trigonometric term. [2]

[Ans: $-15 \tan 3x$]

2 **2020/Y4RP/M2/T1/Q3**

Given that $y = \tan 5x$, show that $\frac{d^2 y}{dx^2}$ can be written in the form $ky \frac{dy}{dx}$, where k is a constant to be determined. [4]

[Ans: $k = 10$]

3 **2019/Y4RP/M2/T1/Q1**

Differentiate the following with respect to x , simplifying your answer:

(i) $\frac{\sin 3x}{x^2}$, [2]

(ii) $\ln\left(\frac{e^{3x^2}}{\cos 3x}\right)$. [2]

[Ans: (i) $\frac{3x \cos 3x - 2 \sin 3x}{x^3}$ (ii) $6x + 3 \tan 3x$]

4 **2019/Y4RP/M2/T1/Q4**

(i) By writing $\sec x$ as $(\cos x)^{-1}$, show that $\frac{d}{dx}(\sec x) = \sec x \tan x$. [1]

(ii) Hence find the following in terms of $\sec x$,

(a) $\frac{d}{dx}(\sec x \tan x)$, [2]

(b) $\frac{d}{dx}[\ln(\sec x + \tan x)]$. [2]

[Ans: (ii)(a) $\sec x(2 \sec^2 x - 1)$ (b) $\sec x$]

5 **2018/Y4RP/CT/Q1**

Differentiate each of the following with respect to x , leaving your answer in the simplest form.

(b) $5 \tan^5(2x)$. [2]

[Ans: (b) $50 \tan^4(2x) \sec^2(2x)$]

6 **2018/Y4RP/CT/Q6**

It is given that $y = \sqrt{1 - \sin x}$.

(i) Find an expression for $\frac{dy}{dx}$, leaving your answer in the simplest form. [2]

(ii) Show that $\frac{d^2y}{dx^2} = ky$, where k is a constant to be determined. [4]

$$[Ans: (i) -\frac{\cos x}{2\sqrt{1-\sin x}} \quad (ii) -\frac{1}{4}\sqrt{1-\sin x}]$$

7 2017/Y4RP/CT/Q1

A curve has the equation $y = x \tan(1 - 3x)$. Find the gradient of the curve at the point where $x = 1$. [3]

$$[Ans: -15.1]$$

8 2017/Y4RP/CT/Q3

Given that $y = \cos^2 2x$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$, simplifying your answers. Hence show that

$$\left(\frac{d^2y}{dx^2}\right)^2 + 16\left(\frac{dy}{dx}\right)^2 - 128y = k \cos 4x, \text{ where } k \text{ is a constant to be determined.} \quad [5]$$

$$[Ans: \frac{dy}{dx} = -2 \sin 4x, \frac{d^2y}{dx^2} = -8 \cos 4x]$$

9 2016/Y4RP/CT/Q1

Differentiate each of the following with respect to x , expressing your answers as a single fraction.

(ii) $\frac{(3x+1)^4}{\tan^2 5x}$. [3]

$$[Ans: (ii) \frac{2(3x+1)^3 [6 \tan 5x - 5(3x+1) \sec^2 5x]}{\tan^3 5x}]$$

10 2016/Y4RP/CT/Q4

It is given that $y = \sqrt{1 + \cos x}$.

(i) Find $\frac{dy}{dx}$, leaving your answer in its simplest form. [1]

(ii) Show that $\frac{d^2y}{dx^2} = \frac{1}{4}y$. [4]

$$[Ans: (i) -\frac{\sin x}{2\sqrt{1+\cos x}}]$$

11 2015/Y4RP/CT/Q2

Given that $y = \ln(1 + \sin x)$,

(a) write down an expression for $\frac{dy}{dx}$ in terms of x , [1]

(b) show that $\frac{d^2y}{dx^2} = \sec x \left(\frac{dy}{dx}\right)$. [3]

$$[Ans: (a) \frac{dy}{dx} = \frac{\cos x}{1 + \sin x}]$$

12 2015/Y4RP/CT/Q4

For each of the following functions, find $\frac{dy}{dx}$, expressing your answers **as a single fraction**

(a) $y = \frac{e^{2x+1}}{\cos^3 4x},$ [4]

[Ans: (a) $\frac{2e^{2x+1}(\cos 4x + 6 \sin 4x)}{\cos^4 4x}$]

13 2014/Y4RP/CT/Q1

Find $\frac{dy}{dx}$ for each of the following functions, expressing your answers as a single fraction.

(a) $y = x \operatorname{cosec}^2 x,$ [3]

[Ans: (a) $\frac{\sin x - 2x \cos x}{\sin^3 x}$]

14 2013/Y4RP/MYCT/Q2

If $y = \frac{\sin x + \cos x}{\cos x}$, show that $\frac{d^2 y}{dx^2} - 2(y-1)\frac{dy}{dx} = 0.$ [4]

15 2013/Y4RP/MYCT/Q3

Differentiate $2e^{2x-1} \sin^4 4x$ with respect to x , simplifying your answer. [3]
[Ans: $4e^{2x-1} \sin^3 4x(8 \cos 4x + \sin 4x)$]

16 2012/Y4RP/T2/Q1

Differentiate the following with respect to x , simplifying your answers.

(b) $x \sin^3 x,$ [2]

[Ans: (b) $\sin^2 x(3x \cos x + \sin x)$]

17 2009/Y4RP/T3/Q2

Differentiate the following with respect to x , simplifying your answers.

(b) $3x^2 \cos^3 \left(2x + \frac{\pi}{3}\right),$ [3]

[Ans: (b) $6x \cos^2 \left(2x + \frac{\pi}{3}\right) \left[\cos \left(2x + \frac{\pi}{3}\right) - 3x \sin \left(2x + \frac{\pi}{3}\right) \right]$]

18 2008/Y4RP/T3/Q1

Differentiate the following expressions with respect to x , giving your answers in the simplest form possible:

(c) $e^{3x} \tan 2x,$ [2] (d) $\frac{\sin 3x}{3x}.$ [2]

[Ans: (c) $e^{3x}(3 \tan 2x + 2 \sec^2 2x)$ (d) $\frac{3x \cos 3x - \sin 3x}{3x^2}$]

19 2007/Y4RP/T3/Q1

Differentiate the following with respect to x , simplifying your answers.

(c) $\left(3x + \frac{\pi}{4}\right) \tan \left(3x - \frac{\pi}{4}\right)$ [2]

$$[Ans: (c) \ 3 \left[\tan \left(3x - \frac{\pi}{4} \right) + \left(3x + \frac{\pi}{4} \right) \sec^2 \left(3x - \frac{\pi}{4} \right) \right]]$$

20 2007/Y4RP/T3/Q2

Find $\frac{d}{dx}(\sin^4 x + \cos^4 x)$, giving your answer as a single trigonometric function. [3]

$$[Ans: -\sin 4x]$$

21 2007/Y4RP/T3/Q3

Find the value of t , where $0 < t < 1$, for which $\frac{d^2}{dt^2}(\cos 2t) = 1$. [3]

$$[Ans: 0.912]$$

22 2006/Y4RP/T3/Q1

Differentiate each of the following with respect to x , simplifying your answer.

(a) $(\ln \sqrt{x}) \tan^2(1-3x)$, [3]

$$[Ans: (a) \ \frac{\tan(1-3x) [\tan(1-3x) - 6x \ln x \sec^2(1-3x)]}{2x}]$$

23 2005/Y4RP/T3/Q1

Differentiate the following with respect to x , simplifying your answers.

(a) $\sin x^2$, [1]

$$[Ans: (a) 2x \cos x^2]$$

24 2004/S4AM/T2/Q1

Differentiate each of the following with respect to x , simplifying your answers as far as possible.

(b) $\sec^3(2x^2+1)$ [3]

$$[Ans: (b) 12x \tan(2x^2+1) \sec^3(2x^2+1)]$$

25 2004/S4AM/T2/Q2

Find the value of a and b for which $\frac{d}{dx} \left(\frac{\sin x}{3 \tan x + \cos x} \right) = \frac{a + b \sin x \tan^2 x}{(3 \tan x + \cos x)^2}$. [4]

$$[Ans: a = 1 \text{ and } b = -3]$$

26 2004/S4AM/T2/Q2 (V2)

Given $y = \sec^3(2x^2+1)$, find $\frac{dy}{dx}$ and simplify your answers as far as possible. Calculate the gradient of the curve at the point where $x=2$, giving your answer correct to 2 decimal places. [4]

$$[Ans: \frac{12x \sin(2x^2+1)}{\cos^4(2x^2+1)}, 14.35]$$

27 2003/S4AM/T2/Q1

Differentiate each of the following with respect to x , simplifying your answers as far as possible.

(b) $\sqrt{2-3\cos^2 4x}$, [2] (d) $2x^3 \tan 6x$, [2]

$$[Ans: (b) \frac{6 \sin 8x}{\sqrt{2-3\cos^2 4x}} \quad (d) \ 6x^2(\tan 6x + 2x \sec^2 6x)]$$