BEATTY SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2022
SECONDARY FOUR NORMAL (ACADEMIC)

CANDIDATE NAME			
CLASS		REGISTER NUMBER	

ADDITIONAL MATHEMATICS

4051/01

Paper 1

3 August 2022
1 hour 45 minutes

Candidates answer on the Question Paper
Additional Materials: Nil

READ THESE INSTRUCTIONS FIRST

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Write in dark blue or black pen.
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Answer **all** the questions.
Given non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 70.

For Examiner's Use

This document consists of **16** printed pages and **0** blank page.

[Turn over

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 **Do not use a calculator in answering this question.**

It is given that $\sqrt{3}(x+3)=x+17$. Find x in the form $a+b\sqrt{3}$, where a and b are integers. [4]

2 **(a)** State, in terms of π, the principal value of

(i) $\tan^{-1}1$, [1]

(ii) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$. [1]

(b) Explain why the principal value of $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ cannot be $-\frac{\pi}{4}$. [1]

[Turn over

3 Given that these simultaneous equations

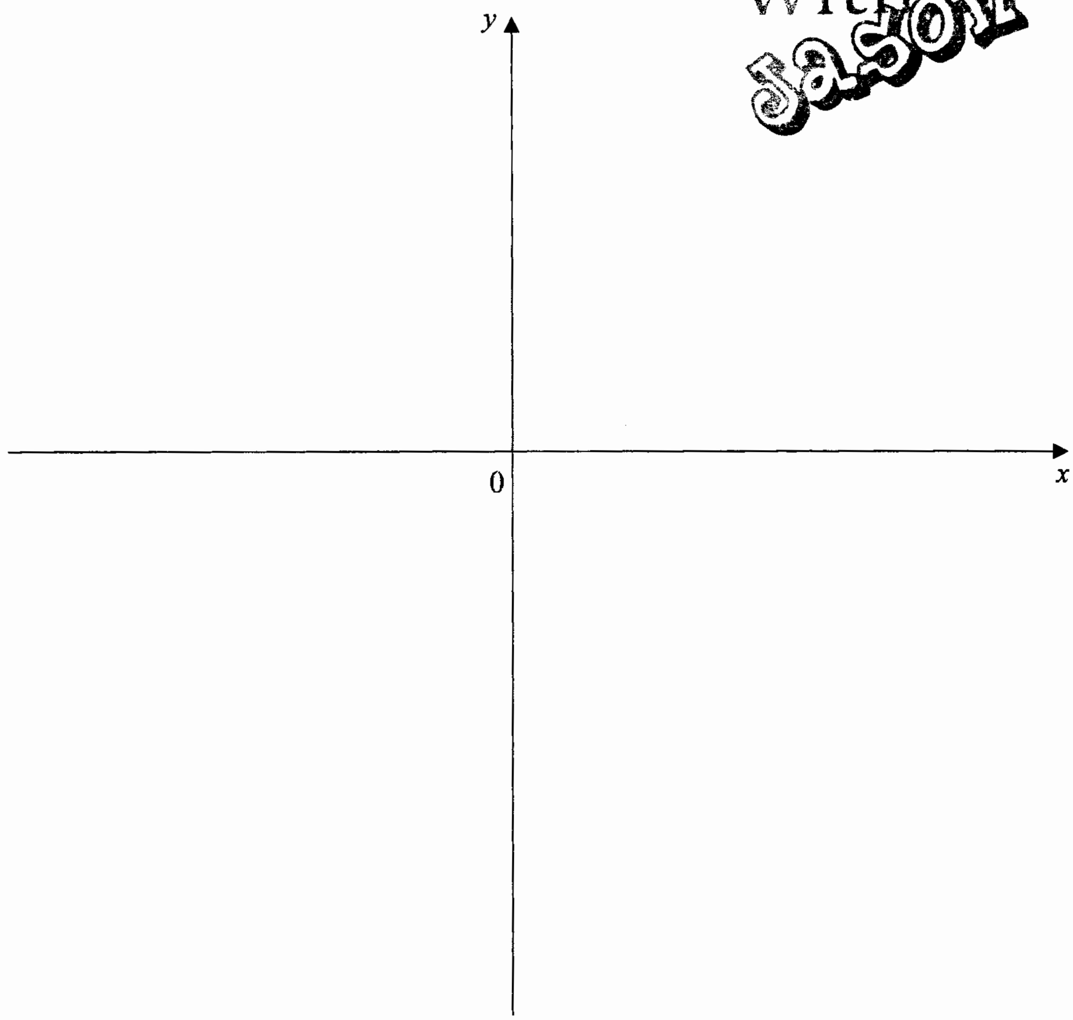
$$x^2 + y^2 - 16 = 0,$$
$$x - y = k$$

have exactly one pair of solutions, show that $k = \pm p\sqrt{2}$, where p is an integer. [6]

4 Given that the period of $y = 5\sin bx - 2$ is 120°.

(a) Find the value of b. [1]

(b) On the axis below, sketch the graph of $y = 5\sin bx - 2$ for $-90^\circ \le x \le 180^\circ$. [3]



[Turn over

5 **(a)** Show that $f(x) = 4x^2 - 10x + 16$ can be written as $f(x) = a(x - b)^2 + c$, where a, b and c are constants to be found. [3]

(b) Explain why the function $f(x)$ is always positive. [1]

6 (a) **Without using a calculator, and showing all your working,** find the exact value of $\sin\frac{\pi}{4}\cos\frac{5\pi}{12}+\cos\frac{\pi}{4}\sin\frac{5\pi}{12}$. [2]

(b) Given that $\tan\theta = k$ and that θ is acute, express in terms of k

(i) $\sec\theta$, [2]

(ii) $\sin 2\theta$. [2]

[Turn over

7 (a) Express $\dfrac{x^2+18x+50}{x(x+5)^2}$ in partial fractions. [5]

(b) Hence differentiate $\frac{x^2+18x+50}{x(x+5)^2}$ with respect to x. [2]

[Turn over

8 Solve the equation $5\cos 2x + 3\sin x = 4$ for $0^\circ < x < 360^\circ$. [5]

9 The equation of a curve is $y = \dfrac{\sqrt{2x^2+1}}{x-3}$.

(a) Find $\dfrac{dy}{dx}$. [4]

(b) Find the equation of the tangent to the curve at $x = 2$. [4]

[Turn over

10 **(a)** Differentiate $\dfrac{6}{(x^2+1)^2}$ with respect to x. [2]

(b) Hence evaluate $\displaystyle\int_{-1}^{2} \frac{12x}{(x^2+1)^3}\,\mathrm{d}x$. [3]

(c) Explain why the curve $y = \dfrac{6}{(x^2+1)^2}$ is a decreasing function for $x > 0$. [2]

[Turn over

11 A container, initially empty, is filled with water. After t seconds, the depth of water in the container is h cm and the volume of water, V cm^3, is given by

$$V = \frac{\pi}{8}h^2(3+h).$$

Given that the volume of water increases at a constant rate and that $h = 6$ when $t = 4$, find

(a) the constant rate of change of the volume of water, [3]

(b) the rate of change of the depth of water at the instant when $h = 2$. [4]

12 **(a)** Find the radius and coordinates of the centre of the circle

$$x^2+y^2-4x-3y-10=0.$$

[3]

(b) Determine whether the point (6,1) lies on the circle. [1]

(c) Find the equation of the tangent to the circle at the point $A(4,-2)$. [3]

[Turn over

(d) Given that AB is a diameter of this circle, find the coordinates of the point B. [2]

BEATTY SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2022
SECONDARY FOUR NORMAL (ACADEMIC)

CANDIDATE NAME	
CLASS	
REGISTER NUMBER	

ADDITIONAL MATHEMATICS

4051/02

Paper 2

5 August 2022
1 hour 45 minutes

Candidates answer on the Question Paper
Additional Materials: Nil

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[Turn over

Mathematical Formulae

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Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Express $\frac{2x^4-3x^3+5x^2+11x-15}{x^2-2x+5}$ in the form ax^2+bx+c, where a, b and c are integers. [3]

[Turn over

2 **(a)** Solve the inequality $2x^2+6x+13<3x(x-2)$. [3]

(b) A line has equation $y=x-7$ and a curve has equation $y=3x^2-5x+1$.
Determine whether the line intersects, is a tangent to, or does not intersect the curve. Give a reason for your answer. [3]

3 **(a)** Factorise $8a^3 + b^3$. [2]

(b) Hence express $8(\sqrt{3})^3 + (\sqrt{27})^3$ in the form $m\sqrt{3}$, where m is an integer. [3]

[Turn over

4 **(a)** Prove that $\frac{1}{\sin x+1}-\frac{1}{\sin x-1}=2\sec^2 x$. [3]

(b) Hence solve the equation $\frac{1}{\sin 2x+1}-\frac{1}{\sin 2x-1}=3$ for $0<x<\pi$. [5]

5 The point $(-3,0)$ is a stationary point on the graph of $y = x^3 + hx^2 + 3x + k$.

(a) Show that $h = 5$. [3]

(b) Find the value of k. [1]

[**Turn over**

(c) Find the x-coordinate of the other stationary point. [2]

(d) Determine the nature of each of these stationary points. [3]

6 **(a)** Express $6\sin t + 10\cos t$ in the form $R\sin(t+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$ radians. [3]

(b) The oscillation of a mass attached to a spring can be modelled by $x = 6\sin t + 10\cos t$, where x mm is the distance of the mass from its rest position A at time t seconds.

(i) Explain whether the mass can reach a distance of 18 mm from A. [1]

[Turn over

(ii) Find the time when the mass first reaches the maximum distance from A. [2]

(iii) After how many seconds does the mass first reach a height of 8 mm from A ? [3]

7 **Solutions to this question by accurate drawing will not be accepted.**

The coordinates of the points A, B and C are $(2,-4)$, $(2a+2, a)$ and $(-3,1)$ respectively. The area of the triangle ABC is 32.5 units^2.

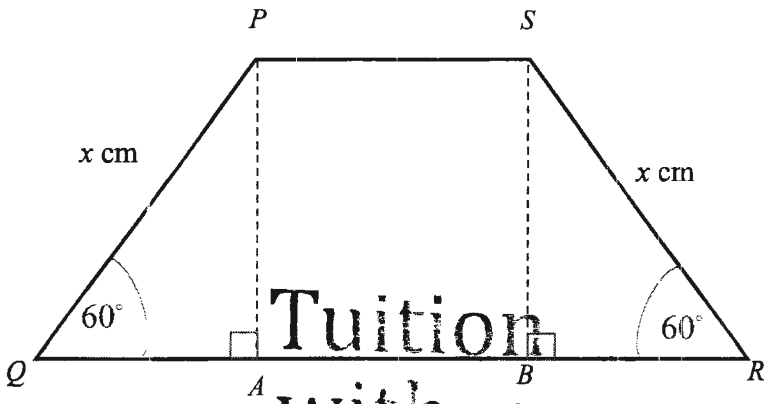
(a) Given that $a > 0$, show that $a = 3$. [4]

(b) Find the coordinates of the point when the line AC meets the x-axis. [3]

[Turn over

(c) Find the equation of the perpendicular bisector of *BC*. [4]

8 A piece of wire of length 100 cm is bent into the shape of a trapezium $PQRS$. Given that $PQ = SR = x$ cm, $\angle PQA = \angle SRB = 60^\circ$ and $PABS$ is a rectangle.



(a) Show that $PS = \dfrac{100-3x}{2}$. [3]

[Turn over

(b) Show that the area of the rectangle *ABSP*, *A* cm^2, is given by

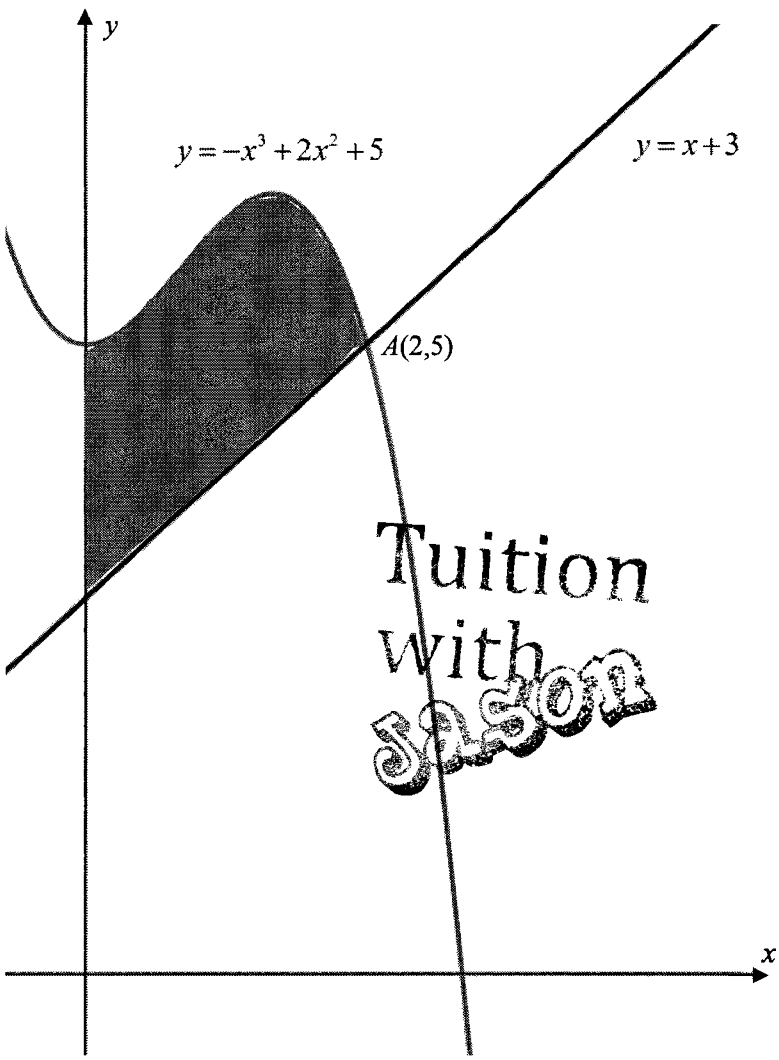
$$A = \frac{\sqrt{3}}{4}\left(100x - 3x^2\right).$$

[3]

(c) Given that x can vary, find the value of x for which A is maximum. [4]

[Turn over

9 The diagram shows the line $y = x + 3$ intersecting the curve $y = -x^3 + 2x^2 + 5$ at the point $A(2,5)$.



(a) Show that the curve $y = -x^3 + 2x^2 + 5$ intersects the line $y = x + 3$ at only one point, $A(2,5)$. [4]

[Turn over

(b) Find the area of the shaded region. [5]

Answer key

1. $2x^2 + x - 3$

2(a) $x < -1$ or $x > 13$

2(b) Since the discriminant is less than zero, the line and the curve does not meet.

3(a) $(2a+b)(4a^2 - 2ab + b^2)$ (b) $105\sqrt{3}$

4(b) 0.308, 1.26, 1.88, 2.83

5(b) -9 (c) $x = -\frac{1}{3}$

5(d)

When $x = -3, \frac{d^2y}{dx^2} = -8 < 0$. It is a maximum point.

When $x = -\frac{1}{3}, \frac{d^2y}{dx^2} = 8 > 0$. It is a minimum point .

6(a) $\sqrt{136}\sin(t + 1.03)$

6(b)(i) Since maximum value of $\sin(t+1.03) = 1$, maximum value of

$x = \sqrt{136} = 11.661$, so it is not possible for the mass to reach a distance of 18 mm.

6(b)(ii) 0.540 s

6(b)(iii) 1.36 s

7(b) $(-2,0)$ 7(c) $y = -\frac{11}{2}x + \frac{63}{4}$ or $y = -5.5x + 15.75$

8(c) $\frac{50}{3} = 16\frac{2}{3}$, A is a maximum

9(b) $3\frac{1}{3}$ units^2

BEATTY SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2022
SECONDARY FOUR NORMAL (ACADEMIC)

MARK SCHEME

CANDIDATE NAME	
CLASS	
REGISTER NUMBER	

ADDITIONAL MATHEMATICS — 4051/01

Paper 1
Setter:

3 August 2022
1 hour 45 minutes

Candidates answer on the Question Paper
Additional Materials: Nil

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1 **Do not use a calculator in answering this question.**

It is given that $\sqrt{3}(x+3)=x+17$. Find x in the form $a+b\sqrt{3}$, where a and b are integers. [4]

$\sqrt{3}(x+3)=x+17$

$x(\sqrt{3}-1)=17-3\sqrt{3}$ .... M1 (grouping of terms)

$x=\dfrac{17-3\sqrt{3}}{\sqrt{3}-1}$

$x=\dfrac{(17-3\sqrt{3})(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$ .... M1(rationalisation)

$x=\dfrac{17\sqrt{3}+17-9-3\sqrt{3}}{2}$ .... M1(simplification)

$x=\dfrac{14\sqrt{3}+8}{2}$

$x=4+7\sqrt{3}$ A1

2 (a) State, in terms of π, the principal value of

(i) $\tan^{-1}1$, [1]

$\frac{\pi}{4}$.............. B1

(ii) $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$. [1]

$-\frac{\pi}{3}$.............. B1

(b) Explain why the principal value of $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ cannot be $-\frac{\pi}{4}$. [1]

Since principal value of $\cos^{-1}\theta$ is between 0 and π, ... B1

it is not possible for the principal value of $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ to be $-\frac{\pi}{4}$.

Or principal value of $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ is $\frac{3\pi}{4}$ so it cannot be $-\frac{\pi}{4}$.

[Turn over

3 Given that these simultaneous equations

$$x^2 + y^2 - 16 = 0,$$
$$x - y = k$$

have exactly one pair of solutions, show that $k = \pm p\sqrt{2}$, where p is an integer. [6]

$x^2 + y^2 - 16 = 0$ .... (1)

$x - y = k$ .... (2)

From (2), $y = x - k$ .... (3)

Sub (3) into (1)

$x^2 + (x-k)^2 - 16 = 0$ .... M1(Substitution)

$x^2 + x^2 - 2kx + k^2 - 16 = 0$

$2x^2 - 2kx + k^2 - 16 = 0$........................M1(form of $ax^2 + bx + c = 0$)

Use $b^2 - 4ac = 0$

$(-2k)^2 - 4(2)(k^2 - 16) = 0$........... M1 √

$4k^2 - 8k^2 + 128 = 0$ .... M1 (simplification)

$-4k^2 = 128$

$k^2 = 32$ M1

$k = \pm\sqrt{32}$

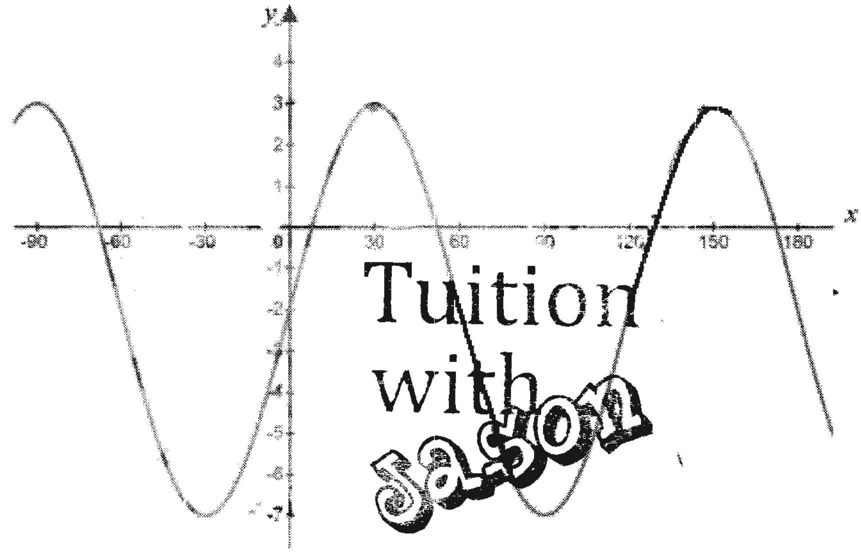
$k = \pm 4\sqrt{2}$ A1

4 Given that the period of $y = 5\sin bx - 2$ is 120°.

(a) Find the value of b. [1]

$b = 3$ B1

(b) On the axis below, sketch the graph of $y = 5\sin bx - 2$ for $-90^\circ \le x \le 180^\circ$. [3]



G1 – Shape of sine graph
G1 – Graph cuts y-axis at -2 and correct period
G1 – Correct maximum value of y = 3 and minimum value of y = –7

[Turn over

5 **(a)** Show that $f(x)=4x^2-10x+16$ can be written as $f(x)=a(x-b)^2+c$, where a, b and c are constants to be found. [3]

$$f(x)=4x^2-10x+16$$

$$=4\left(x^2-\frac{5}{2}x+4\right)\text{.................. M1 (factorisation)}$$

$$=4\left[\left(x-\frac{5}{4}\right)^2-\left(\frac{5}{4}\right)^2+4\right]\text{..............M1(completing of square)}$$

$$=4\left[\left(x-\frac{5}{4}\right)^2+\frac{39}{16}\right]$$

$$=4\left(x-\frac{5}{4}\right)^2+\frac{39}{4}\text{..........................A1}$$

(b) Explain why the function $f(x)$ is always positive. [1]

Since $a=4>0$ and minimum value of $f(x)=\frac{39}{4}>0$, then the function $f(x)$ is always positive.. ....B1

6 (a) **Without using a calculator, and showing all your working,** find the exact value of $\sin\frac{\pi}{4}\cos\frac{5\pi}{12}+\cos\frac{\pi}{4}\sin\frac{5\pi}{12}$. [2]

$\sin\frac{\pi}{4}\cos\frac{5\pi}{12}+\cos\frac{\pi}{4}\sin\frac{5\pi}{12}$

$=\sin\left(\frac{\pi}{4}+\frac{5\pi}{12}\right)$ .... M1 (use of addition formula)

$=\sin\left(\frac{2\pi}{3}\right)$

$=\sin\frac{\pi}{3}$

$=\frac{\sqrt{3}}{2}$ A1

(b) Given that $\tan\theta = k$ and that θ is acute, express in terms of k

(i) $\sec\theta$, [2]

$=\frac{1}{\cos\theta}$ B1

$=\sqrt{1+k^2}$ B1

(ii) $\sin 2\theta$. [2]

$=2\sin\theta\cos\theta$

$=2\left(\frac{k}{\sqrt{1+k^2}}\right)\left(\frac{1}{\sqrt{1+k^2}}\right)$.......... M1 (for either ratio)

$=\frac{2k}{1+k^2}$ A1

[Turn over

7 (a) Express $\dfrac{x^2+18x+50}{x(x+5)^2}$ in partial fractions. [5]

$$\frac{x^2+18x+50}{x(x+5)^2}=\frac{A}{x}+\frac{B}{x+5}+\frac{C}{(x+5)^2}$$ M1

$x^2+18x+50=A(x+5)^2+Bx(x+5)+Cx$

Sub $x=0$, $50=25A$

$A=2$ A1

Sub $x=-5$, $-15=-5C$

$C=3$ A1

Equating coefficient of x^2

$1=A+B$

$1=2+B$

$B=-1$ A1

$$\frac{x^2+18x+50}{x(x+5)^2}=\frac{2}{x}-\frac{1}{x+5}+\frac{3}{(x+5)^2}$$ A1

(b) Hence differentiate $\frac{x^2+18x+50}{x(x+5)^2}$ with respect to x. [2]

$$\frac{d}{dx}\left(\frac{2}{x}-\frac{1}{x+5}+\frac{3}{(x+5)^2}\right)$$

$$=\frac{d}{dx}\left[2x^{-1}-(x+5)^{-1}+3(x+5)^{-2}\right]$$

$= -2x^{-2}+(x+5)^{-2}-6(x+5)^{-3}$ ....M1 (Chain rule), A1

or $-\frac{2}{x^2}+\frac{1}{(x+5)^2}-\frac{6}{(x+5)^3}$

[Turn over

8 Solve the equation $5\cos 2x + 3\sin x = 4$ for $0^\circ < x < 360^\circ$. [5]

$5\cos 2x + 3\sin x = 4$

$5(1-2\sin^2 x) + 3\sin x - 4 = 0$ .... M1 (use of double angle formula)

$5 - 10\sin^2 x + 3\sin x - 4 = 0$

$10\sin^2 x - 3\sin x - 1 = 0$

$(5\sin x + 1)(2\sin x - 1) = 0$ ....M1 (factorisation)

$\sin x = -\frac{1}{5}$ or $\sin x = \frac{1}{2}$ M1

basic $\angle = 11.53^\circ$ basic $\angle = 30^\circ$

$x = 191.5^\circ, 348.5^\circ$ .... A1 $x = 30^\circ, 150^\circ$ A1

9 The equation of a curve is $y = \dfrac{\sqrt{2x^2+1}}{x-3}$.

(a) Find $\dfrac{dy}{dx}$. [4]

$y = \dfrac{\sqrt{2x^2+1}}{x-3}$

$u = \left(2x^2+1\right)^{\frac{1}{2}}$ $\qquad$ $v = x-3$

$\dfrac{du}{dx} = \dfrac{1}{2}(2x^2+1)^{-\frac{1}{2}}(4x)$ .... M1 $\qquad$ $\dfrac{dv}{dx} = 1$

$= \dfrac{2x}{\sqrt{2x^2+1}}$

$\dfrac{dy}{dx} = \dfrac{\dfrac{2x}{\sqrt{2x^2+1}}(x-3) - \sqrt{2x^2+1}}{(x-3)^2}$ .... M1(Quotient rule)

$= \dfrac{2x^2-6x-2x^2-1}{(x-3)^2\sqrt{2x^2+1}}$ .... M1 (simplification)

$= \dfrac{-6x-1}{(x-3)^2\sqrt{2x^2+1}}$ A1

(b) Find the equation of the tangent to the curve at $x = 2$. [4]

Sub $x = 2$

$\dfrac{dy}{dx} = -\dfrac{13}{3}$ and $y = -3$.................................... M1(substitution), A1

Sub into $y = -\dfrac{13}{3}x + c$

$-3 = -\dfrac{13}{3}(2) + c$ M1

$c = \dfrac{17}{3}$

equation of normal is $y = -\dfrac{13}{3}x + \dfrac{17}{3}$ A1

[Turn over

10 **(a)** Differentiate $\frac{6}{(x^2+1)^2}$ with respect to x. [2]

Let $y=\frac{6}{(x^2+1)^2}=6(x^2+1)^{-2}$

$\frac{dy}{dx}=-12(x^2+1)^{-3}(2x)$ ....M1(Chain rule)

$=-\frac{24x}{(x^2+1)^3}$ A1

(b) Hence evaluate $\int_{-1}^{2}\frac{12x}{(x^2+1)^3}\,dx$. [3]

$\int_{-1}^{2}\frac{12x}{(x^2+1)^3}\,dx$

$=-\frac{1}{2}\int_{-1}^{2}\frac{-24x}{(x^2+1)^3}\,dx$

$=-\frac{1}{2}\left[\frac{6}{(x^2+1)^2}\right]_{-1}^{2}$ M1

$=-\frac{1}{2}\left(\frac{6}{25}-\frac{6}{4}\right)$ .... M1(substitution)

$=0.63$ A1

(c) Explain why the curve $y = \dfrac{6}{(x^2+1)^2}$ is a decreasing function for $x > 0$. [2]

When $x > 0$, $-24x < 0$ and $(x^2+1)^3 > 0$, ... B1

so $\dfrac{dy}{dx} = -\dfrac{24x}{(x^2+1)^3} < 0$. This shows that the curve is a decreasing function for $x > 0$........B1

[Turn over

11 A container, initially empty, is filled with water. After t seconds, the depth of water in the container is h cm and the volume of water, V cm^3, is given by

$$V = \frac{\pi}{8}h^2(3+h).$$

Given that the volume of water increases at a constant rate and that $h = 6$ when $t = 4$, find

(a) the constant rate of change of the volume of water, [3]

Sub $h = 6$, $V = \frac{81}{2}\pi$ B1

$\frac{dV}{dt} = \frac{\frac{81}{2}\pi}{4}$ .... M1 v

constant rate $= \frac{81}{8}\pi$ cm^3/s A1

(b) the rate of change of the depth of water at the instant when $t = 2$. [4]

$V = \frac{\pi}{8}(3h^2 + h^3)$

$\frac{dV}{dh} = \frac{\pi}{8}(6h + 3h^2)$ B1

$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$

$\frac{81}{8}\pi = \frac{\pi}{8}(6h + 3h^2) \times \frac{dh}{dt}$ .... M1 (use of chain rule)

Sub $t = 2$,

$\frac{81\pi}{8} = \frac{\pi}{8}(24) \times \frac{dh}{dt}$ .... M1 (substitution)

$\frac{dh}{dt} = \frac{81}{24}$

$= 3\frac{3}{8}$ cm/s M1

12 **(a)** Find the radius and coordinates of the centre of the circle

$$x^2+y^2-4x-3y-10=0.$$

[3]

$x^2+y^2-4x-3y-10=0$

$(x-2)^2-2^2+(y-1.5)^2-1.5^2-10=0$ .... M1 (completing of square)

$(x-2)^2+(y-1.5)^2=16.25$

centre = (2,1.5) A1

radius = $\sqrt{16.25}$ units or 4.03 units................ A1

(b) Determine whether the point (6,1) lies on the circle. [1]

Sub (6,1)

$6^2+1^2-4(6)-3(1)-10$

$=0$

yes, (6,1) lies on the curcle. B1

(c) Find the equation of the tangent to the circle at the point $A(4,-2)$. [3]

(2,1.5) (4,−2)

gradient $=\frac{1.5+2}{2-4}=-\frac{7}{4}$.............. M1

Sub into $y=\frac{4}{7}x+c$

$-2=\frac{4}{7}(4)+c$ M1

$c=-\frac{30}{7}$

equation of tangent is $y=\frac{4}{7}x-\frac{30}{7}$ A1

[Turn over

(d) Given that AB is a diameter of this circle, find the coordinates of the point B. [2]

centre $= (2, 1.5)$ $A(4, -2)$

Let $B = (a, b)$

$\frac{a+4}{2} = 2$ and $\frac{b-2}{2} = 1.5$ .... M1(concept of midpoint)

$a + 4 = 4$ $b - 2 = 3$

$a = 0$ $b = 5$

$B = (0, 5)$ A1

BEATTY SECONDARY SCHOOL
PRELIMINARY EXAMINATION 2022
SECONDARY FOUR NORMAL (ACADEMIC)

MARK SCHEME

CANDIDATE NAME

CLASS

REGISTER NUMBER

ADDITIONAL MATHEMATICS

4051/02

Paper 2
Setter:

5 August 2022
1 hour 45 minutes

Candidates answer on the Question Paper
Additional Materials: Nil

READ THESE INSTRUCTIONS FIRST

Write your name, class and register number in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Given non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is 70.

For Examiner's Use

This document consists of **18** printed pages and **0** blank page.

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

1 Express $\dfrac{2x^4-3x^3+5x^2+11x-15}{x^2-2x+5}$ in the form ax^2+bx+c, where a, b and c are integers. [3]

$$\begin{array}{r} 2x^2+x-3 \\ x^2-2x+5 \overline{\big)\,2x^4-3x^3+5x^2+11x-15} \\ \underline{2x^4-4x^3+10x^2} \qquad\qquad \\ x^3-5x^2+11x \qquad \\ \underline{x^3-2x^2+5x} \qquad \\ -3x^2+6x-15 \\ \underline{-3x^2+6x-15} \end{array}$$

M1 – Attempt long division, A1 – all steps are correct.

$\dfrac{2x^4-3x^3+5x^2+11x-15}{x^2-2x+5}=2x^2+x-3$ A1

[Turn over

2 **(a)** Solve the inequality $2x^2+6x+13<3x(x-2)$. [3]

$2x^2+6x+13<3x(x-2)$

$2x^2+6x+13<3x^2-6x$ .... M1 (expansion)

$-x^2+12x+13<0$

$x^2-12x-13>0$

$(x-13)(x+1)>0$ .... M1 (factorisation)

$x<-1$ or $x>13$ A1

(b) A line has equation $y=x-7$ and a curve has equation $y=3x^2-5x+1$. Determine whether the line intersects, is a tangent to, or does not intersect the curve. Give a reason for your answer. [3]

$3x^2-5x+1=x-7$ M1

$3x^2-6x+8=0$

b^2-4ac

$=(-6)^2-4(3)(8)$ M1

$=-60<0$

Since the discriminant is less than zero, the line and the curve does not meet. A1

3 (a) Factorise $8a^3+b^3$. [2]

$8a^3+b^3$

$=(2a)^3+b^3$

$=(2a+b)(4a^2-2ab+b^2)$ M1, A1

(b) Hence express $8(\sqrt{3})^3+(\sqrt{27})^3$ in the form $m\sqrt{3}$, where m is an integer. [3]

Sub $a=\sqrt{3}$ and $b=\sqrt{27}$

$8(\sqrt{3})^3+(\sqrt{27})^3$

$=\left[2\sqrt{3}+\sqrt{27}\right]\left[4(\sqrt{3})^2-2(\sqrt{3})(\sqrt{27})+(\sqrt{27})^2\right]$ .... M1(substitution)

$=\left(2\sqrt{3}+3\sqrt{3}\right)(12-18+27)$ .... M1(simplification of surds)

$=5\sqrt{3}(21)$

$=105\sqrt{3}$ A1

[Turn over

4 **(a)** Prove that $\dfrac{1}{\sin x+1}-\dfrac{1}{\sin x-1}=2\sec^2 x$. [3]

$$\frac{1}{\sin x+1}-\frac{1}{\sin x-1}$$
$$=\frac{\sin x-1-(\sin x+1)}{(\sin x+1)(\sin x-1)} \quad \text{............... M1}$$
$$=\frac{-2}{\sin^2 x-1}$$
$$=\frac{-2}{1-\cos^2 x-1} \quad \text{.................... M1}$$
$$=\frac{-2}{-\cos^2 x}$$
$$=2\sec^2 x \quad \text{............................ A1}$$

(b) Hence solve the equation $\dfrac{1}{\sin 2x+1}-\dfrac{1}{\sin 2x-1}=3$ for $0<x<\pi$. [5]

$$\frac{1}{\sin 2x+1}-\frac{1}{\sin 2x-1}=3$$
$$2\sec^2 2x=3 \quad \text{...............}$$
$$\sec^2 2x=\frac{3}{2}$$
$$\cos^2 2x=\frac{2}{3} \quad \text{.............................. M1}$$
$$\cos 2x=\pm\sqrt{\frac{2}{3}} \quad \text{.......................... M1}$$

basic angle = 0.615479

$2x = 0.61547,\ 2.52611,\ 3.75707,\ 5.66771$ M1

$x = 0.308,\ 1.26,\ 1.88,\ 2.83$ (to 3sf)................ A1

5 The point $(-3,0)$ is a stationary point on the graph of $y = x^3 + hx^2 + 3x + k$.

(a) Show that $h = 5$. [3]

$\frac{dy}{dx} = 3x^2 + 2hx + 3$ B1

sub $x = -3$ and $\frac{dy}{dx} = 0$

$3(-3)^2 + 2h(-3) + 3 = 0$ M1

$27 - 6h + 3 = 0$

$-6h = -30$

$h = 5$ A1

(b) Find the value of k. [1]

$y = x^3 + 5x^2 + 3x + k$

Sub $(-3,0)$

$(-3)^3 + 5(-3)^2 + 3(-3) + k = 0$

$-27 + 45 - 9 + k = 0$

$k = -9$ B1

[Turn over

(c) Find the x-coordinate of the other stationary point. [2]

Let $\frac{dy}{dx} = 0$

$3x^2 + 10x + 3 = 0$ M1

$(x+3)(3x+1) = 0$

$x = -3$(given) or $x = -\frac{1}{3}$ A1

(d) Determine the nature of each of these stationary points. [3]

$\frac{dy}{dx} = 3x^2 + 10x + 3$

$\frac{d^2y}{dx^2} = 6x + 10$ .... B1 √

When $x = -3$, $\frac{d^2y}{dx^2} = -8 < 0$. It is a maximum point. B1

When $x = -\frac{1}{3}$, $\frac{d^2y}{dx^2} = 8 > 0$. It is a minimum point. B1

Accept the first derivative test using change in the gradients.

6 **(a)** Express $6\sin t + 10\cos t$ in the form $R\sin(t+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$ radians. [3]

$6\sin t + 10\cos t = R\sin(t+\alpha)$

$= R\sin t\cos\alpha + R\cos t\sin\alpha$

$R\cos\alpha = 6$ and $R\sin\alpha = 10$

$R = \sqrt{6^2 + 10^2}$ M1

$R = \sqrt{136}$ or $2\sqrt{34}$

$\tan\alpha = \frac{10}{6}$ M1

$\alpha = 1.0303$ radians

$6\sin t + 10\cos t = \sqrt{136}\sin(t + 1.03)$ A1

(b) The oscillation of a mass attached to a spring can be modelled by $x = 6\sin t + 10\cos t$, where x mm is the distance of the mass from its rest position A at time t seconds.

(i) Explain whether the mass can reach a distance of 18 mm from A. [1]

Since maximum value of $\sin(t + 1.03) = 1$, maximum value of $x = \sqrt{136} = 11.661$, so it is not possible for the mass to reach a distance of 18 mm. .... B1√

[Turn over

(ii) Find the time when the mass first reaches the maximum distance from A. [2]

Let $\sin(t + 1.03) = 1$ M1

$t + 1.03 = 1.5707$

$t = 0.540$ s (to 3sf) A1

(iii) After how many seconds does the mass first reach a height of 8 mm from A? [3]

Let $\sqrt{136}\sin(t + 1.03) = 8$ .... M1 √

$\sin(t + 1.03) = 0.68599$

$t + 1.03 = 0.75596,\ 2.38563$ M1

(angles in 1st and 2nd quadrants)

$t = -0.274$(rejected) , 1.36 s (to 3sf) A1

7 **Solutions to this question by accurate drawing will not be accepted.**

The coordinates of the points A, B and C are $(2,-4)$, $(2a+2, a)$ and $(-3,1)$ respectively. The area of the triangle ABC is 32.5 units².

(a) Given that $a > 0$, show that $a = 3$. [4]

$$\frac{1}{2}\begin{vmatrix} 2 & 2a+2 & -3 & 2 \\ -4 & a & 1 & -4 \end{vmatrix} = 32.5 \text{ .... M1}$$

$$2a+2a+2+12-(-8a-8-3a+2)=65 \text{ .... M1}$$

$$15a+20=65$$

$$15a=45 \text{ .... M1}$$

$$a=3 \text{ .... A1}$$

(b) Find the coordinates of the point when the line AC meets the x-axis. [3]

$A(2,-4)$ $C(-3,1)$

$m = \dfrac{1+4}{-3-2}$ M1

$m = -1$

Let point be $(x,0)$

$\dfrac{0-1}{x+3} = -1$ M1

$x+3=1$

$x=-2$

Point is $(-2,0)$ A1

[Turn over

(c) Find the equation of the perpendicular bisector of BC. [4]

$B(8,3)$ $C(-3,1)$

$m = \frac{3-1}{8+3}$ .... M1 √

$m = \frac{2}{11}$

Midpoint $= \left(\frac{8-3}{2}, \frac{3+1}{2}\right)$ .... M1 √

$= \left(\frac{5}{2}, 2\right)$

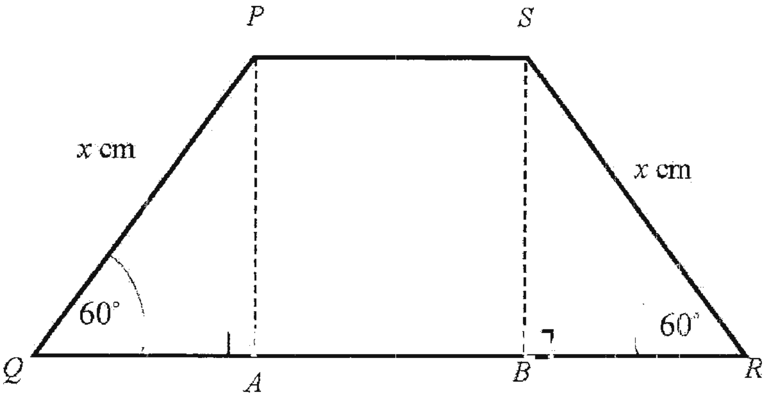
Sub $\left(\frac{5}{2}, 2\right)$ into $y = -\frac{11}{2}x + c$

$2 = -\frac{11}{2}\left(\frac{5}{2}\right) + c$ .... M1 (equation of line)

$c = \frac{63}{4}$

equation is $y = -\frac{11}{2}x + \frac{63}{4}$ or $y = -5.5x + 15.75$ A1

8 A piece of wire of length 100 cm is bent into the shape of a trapezium $PQRS$. Given that $PQ = SR = x$ cm, $\angle PQA = \angle SRB = 60^\circ$ and $PABS$ is a rectangle.



(a) Show that $PS = \dfrac{100 - 3x}{2}$. [3]

$\cos 60^\circ = \dfrac{QA}{x}$ M1

$\dfrac{1}{2} = \dfrac{QA}{x}$

$QA = \dfrac{x}{2} = RB$

$2x + 2\left(\dfrac{x}{2}\right) + 2PS = 100$ M1

$3x + 2PS = 100$

$PS = \dfrac{100 - 3x}{2}$ A1

[Turn over

(b) Show that the area of the rectangle *ABSP*, *A* cm^2, is given by [3]

$$A=\frac{\sqrt{3}}{4}\left(100x-3x^2\right).$$

$\sin 60 = \frac{PA}{x}$ M1

$\frac{\sqrt{3}}{2}=\frac{PA}{x}$ .... M1 (value of sin 60)

$PA=\frac{\sqrt{3}}{2}x$

$A=\frac{\sqrt{3}}{2}x\left(\frac{100-3x}{2}\right)$

$=\frac{\sqrt{3}}{4}\left(100x-3x^2\right)$ A1

(c) Given that x can vary, find the value of x for which A is maximum. [4]

$A=\frac{\sqrt{3}}{4}\left(100x-3x^2\right)$

$\frac{dA}{dx}=\frac{\sqrt{3}}{4}(100-6x)$ B1

Let $\frac{dA}{dx}=\frac{\sqrt{3}}{4}(100-6x)=0$

$100-6x=0$ M1

$6x=100$

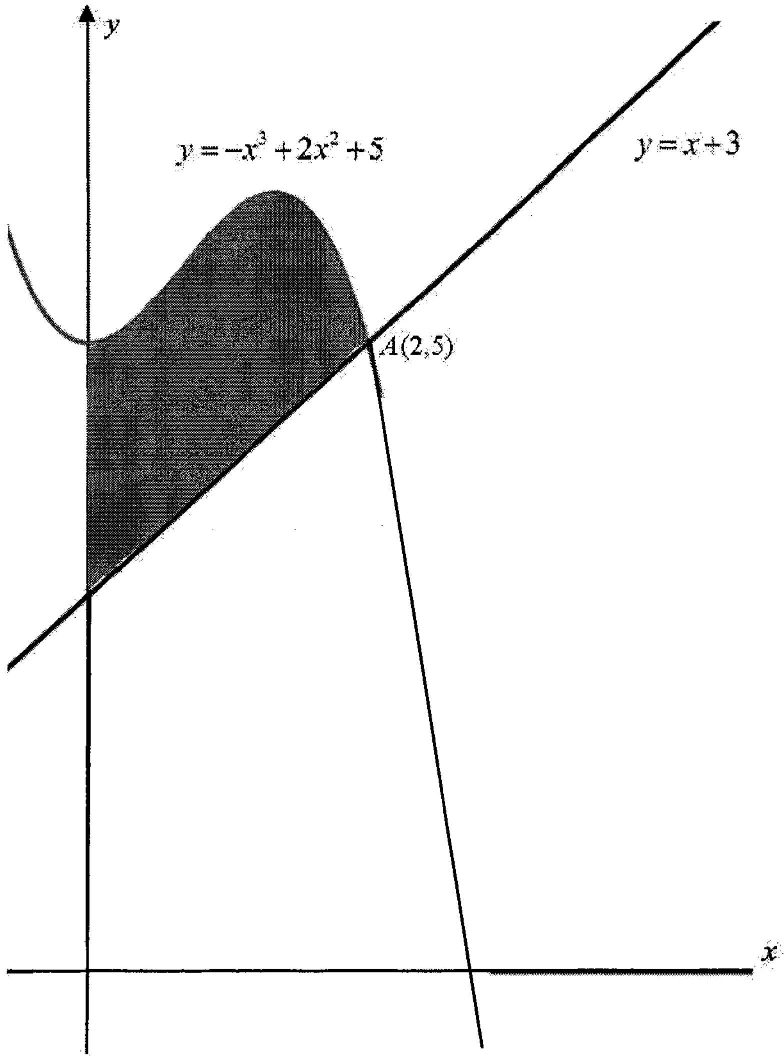
$x=\frac{50}{3}=16\frac{2}{3}$ A1

$\frac{d^2A}{dx^2}=-\frac{3\sqrt{3}}{2}<0$ B1

Therefore A is a maximum value.

[Turn over

9 The diagram shows the line $y = x + 3$ intersecting the curve $y = -x^3 + 2x^2 + 5$ at the point $A(2,5)$.



(a) Show that the curve $y = -x^3 + 2x^2 + 5$ intersects the line $y = x + 3$ at only one point, $A(2,5)$. [4]

$-x^3 + 2x^2 + 5 = x + 3$

$x^3 - 2x^2 + x - 2 = 0$ M1

Since the line and curve cuts at A, it shows that $(x-2)$ is a factor.

$$\begin{array}{r} x^2 + 1 \\ x-2 \overline{\big) x^3 - 2x^2 + x - 2} \\ \underline{x^3 - 2x^2} \\ x - 2 \\ \underline{x - 2} \\ 0 \end{array}$$

.... M1 (o e)

$x^3 - 2x^2 + x - 2 = 0$

$(x-2)(x^2+1) = 0$ A1

$x = 2$ or $x^2 = -1$

Since x^2 cannot be negative, there is no solution for $x^2 = -1$ B1

and the line and curve intersects only at one point A.

[Turn over

(b) Find the area of the shaded region. [5]

Area under curve $= \int_0^2 (-x^3 + 2x^2 + 5)\,dx$

$= \left[\frac{-x^4}{4} + \frac{2x^3}{3} + 5x\right]_0^2$ .... M1 (integration)

$= \frac{34}{3}$ A1

Area under line $= \frac{1}{2}(3+5)(2)$ M1

$= 8$

Area of shaded region $= \frac{34}{3} - 8$ M1

$= \frac{10}{3} = 3\frac{1}{3}$ units^2 ... A1