Name () Class	Name	()	Class	



RIVER VALLEY HIGH SCHOOL 2012 Year 6 Preliminary Examination **Higher 2**

MATHEMATICS

Paper 2

9740/02 14 September 2012

3 hours

Additional Materials:

Answer Paper List of Formulae (MF15) Cover Page

READ THESE INSTRUCTIONS FIRST

Do not open this booklet until you are told to do so.

Write your name, class and index number in the space at the top of this page.

Write your name and class on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer all the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. You are expected to use a graphic calculator.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, place the cover page on top of your answer paper and fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

- 1 Use the substitution u = xy, where *u* is a function of *x*, to reduce the differential equation $x \frac{dy}{dx} + (1 - x^3)y - x^2 = 0$ to $\frac{du}{dx} = (u + 1)x^2$. Hence show that the general solution to the differential equation is of the form $y = \frac{Ae^{f(x)} - 1}{x}$, where f is a function in *x*. [5] Sketch on separate diagrams a member of the family of curves corresponding to (i) A > 1and (ii) A < -1. [2]
- 2 (a) Use the substitution $\tan y = e^x$ to find

$$\int \frac{1}{\mathrm{e}^x (1 + \mathrm{e}^{2x})} \mathrm{d}x \,. \tag{4}$$

- (b) The region bounded by the *x*-axis, the curve $y = e^{x^2+1}$, the lines x = 0 and x = 1 is rotated about the *y*-axis to form a solid of revolution of volume *V*. Find the exact value of *V*. [6]
- 3 (a) The diagram shows the graph of y = f(x). It has a maximum point at at (a, 2) where -4 < a < 0 and passes through the points (-6, 0) and (-2, 0). The curve has asymptotes at x = 0 and y = 4.



On separate diagrams, sketch the graphs of

- (i) y = f'(x), [3]
- (ii) $y = \frac{1}{f(x)}$, [3]
- (iii) y = f(|x| 4) [3]

(b) Describe a sequence of transformations that maps the graph of $x^2 - \frac{y^2}{4} = 1$ onto the graph of $x^2 - (y-1)^2 = 1$. [2]

- 4 (i) Solve the equation $z^3 8 = 0$, giving the roots in the form $re^{i\alpha}$, where r > 0 and $-\pi < \alpha \le \pi$. [3]
 - (ii) The roots denoted by z_1 , z_2 and z_3 are such that $-\pi < \arg(z_1) < \arg(z_2) < \arg(z_3) < \pi$. The complex numbers z_1 , z_2 and z_3 are represented by the points Z_1 , Z_2 and Z_3 in the Argand diagram respectively. Show and label the roots of the equation $z^3 8 = 0$ on the Argand diagram. [2]
 - (iii) The set of points P in the Argand diagram represents the complex number z that satisfies $|z z_2| = |z z_3|$. Explain why the locus of P passes through the point Z_1 . [1]
 - (iv) The set of points Q represents another complex number w that satisfies $|w-z_1| = 2\sqrt{3}$. Sketch the loci of P and Q on the same diagram, indicating clearly the positions of Z_1 , Z_2 and Z_3 . [3]
 - (v) The complex number v is represented by the point of intersection of the loci of P and Q in the first quadrant. Find v in the form x + iy, giving the exact values of x and y. [3]

Section B: Statistics [60 marks]

- 5 It is desired to interview residents of a district to find out whether there is a need to set up a nursing home in their district. In particular it is necessary to obtain a sample of 100 interviews representative of the various age groups.
 - (i) Explain how a quota sample might be obtained. [2]
 - (ii) Explain a disadvantage of quota sampling in the context of your answer to part (i).
 - (iii) Suggest an alternative method of sampling that would not have this disadvantage.

[1]

[1]

6 In a particular restaurant, set lunch menu is offered to customers. Under this menu, a customer can choose 1 soup, 1 main course, 1 dessert and 1 drink from among the following choices: 2 soups, 3 main courses, 3 desserts and 4 drinks.

Assuming every customer would choose 1 soup, 1 main course, 1 dessert and 1 drink from the menu, write down the number of combinations a customer can order for set lunch. [1]

Two friends place two sets of order under the set lunch menu. Suppose they do not mind sharing food and drinks with each other, how many ways can they place their order such that they get to consume the maximum variety of food and drinks? [4]

The restaurant also offers a special sandwich whereby customers can select the ingredients to be placed in a sandwich order. Suppose there are a total of 8 ingredients to choose from and a customer must select at least 1 ingredient, find the number of ways one can place a special sandwich order. [2]

- 7 (a) The equation of the estimated least squares regression line of y on x for a set of bivariate data is y = a + bx. Explain what do you understand by the least squares regression line of y on x. [2]
 - (b) The table below gives the values of seven observations of bivariate data, x and y.

x	1.2	2.0	2.7	3.8	4.8	5.6	6.9
у	2.2	4.5	5.8	7.3	7.6	9.0	9.9

- (i) Calculate the value of the product moment correlation coefficient, and explain why its value does not necessarily indicate that y = c + dx is a suitable model for the data. [2]
- (ii) Explain how the product moment correlation coefficient can be used to decide, for this data, whether $y = a + b \ln x$ or y = c + dx is a better model. Hence determine the equation of the suitable regression line for the above data. [2]
- (iii) It is desired to use the given data to estimate the value of x when y = 6.4. Use the regression line found in part (ii), to find the required estimate and comment on the reliability of your answer. [2]

8 A factory produces a particular brand of cooking oil in bottles of 2 sizes, namely Small and Regular. The amount of cooking oil in each type of bottle, in millilitres (ml), is assumed to be normally distributed with means and standard deviations as shown in the following table.

	Mean	Standard Deviation
Small	545	20
Regular	1020	50

- (i) A Regular bottle of cooking oil will be rejected if the amount of cooking oil is less than α ml. Find the greatest value of α , to the nearest integer such that at least 97% of the Regular bottles of cooking oil will not be rejected. [2]
- (ii) Find the probability that the mean volume of three Small bottles and two Regular bottles of cooking oil is less than 730 ml.
- (iii) Three Small bottles of cooking oil are randomly chosen. Find the probability that the bottle containing the least content holds more than 547 ml of cooking oil. [2]
- (iv) The factory found out that there was a change in the cooking oil dispensed for the Regular bottle. Assuming that the Regular bottle of cooking oil has a maximum capacity of 1080 ml, explain if the use of a normal distribution with the given mean and standard deviation is appropriate in modelling the amount of cooking oil dispensed.
 [1]
- 9 Singapore households are on the move to install fibre optic cables in their units. The continuous random variable X denotes the time, in minutes, spent by a technician of Open Network to install a fibre optic cable unit in a household unit. Open Network claims that a fibre optic cable unit can be installed within μ_0 minutes.

However, online critics refute this claim by Open Network. A sample of 12 independent observations of X was taken. The mean time and standard deviation from this sample were found to be 21.3 and 4.5 minutes respectively.

A test is carried out, at 2.5 % significance level, to determine the refutation of the online critics.

- (i) State the appropriate hypotheses for the test. [1]
- (ii) Explain, in the context of the question, the meaning of 'at the 2.5% significance level'. [1]
- (iii) Suppose the test yields a result that does not reject the null hypothesis, find the set of possible values of μ_0 . State any assumption that you made. [6]
- (iv) Given instead that the standard deviation of X is 5 minutes and $\mu_0 = 19$, find the least significance level at which the null hypothesis is rejected. [3]

- 10 (a) For events A and B, it is given that P(B) = 0.4 and P(A|B') = 0.15. Find $P(A \cup B)$. [3]
 - (b) In the last stage of a variety game show, the contestant aims to open a treasure chest containing the grand prize. The chest can be opened only when its two locks are unlocked. Each lock requires a unique key to be unlocked.

The contestant is given up to three rounds to select the two correct keys among six different keys on a tray. For each round, he can only select one key which will be tried on both locks. The selected key of a particular round will be removed from the tray after that round.

Show that the probability that a contestant wins the grand prize in the second round is $\frac{1}{15}$. [1]

Find the probability that

- (i) the grand prize is won and the contestant selects the correct key in the first round, [2]
- (ii) the contestant selects the correct key in the first round given that he wins the grand prize. [2]

The variety game show runs for five days a week, over a period of 60 weeks. Find the probability that the mean number of shows per week for which the grand prize is won in the second round is more than 0.4. [3]

- 11 The number of male and female customers entering a cafe in a randomly chosen 15-minute period, have independent Poisson distributions with means 1 and 2 respectively.
 - (i) Find the probability that in a randomly chosen 30-minute period, there are more than 5 customers entering the cafe. [2]
 - (ii) The cafe is open for business for 6 days each week. Using a suitable approximation, find the probability that in a period of 10 weeks there are no less than 30 days on which more than 5 customers entered the cafe between 9:00 am and 9:30 am.
 [4]
 - (iii) In a particular 15 minute period, there are less than 5 customers entering the cafe.
 Find the probability that at most 2 male customers entered the cafe during this 15-minute period. [4]
 - (iv) Explain why the Poisson distribution may not be a good model for the number of customers entering the cafe in a day. [1]

End of Paper –