YISHUN TOWN SECONDARY SCHOOL
Mathematics Department

MARKING SCHEME

Examination : 4NA Preliminary Examination **Date**: 14 Aug 2023

Subject : Additional Maths **Paper No.** : 1

Q^n	Key Steps / Solution	Scheme

1	$\frac{2+x^2}{(x-2)^2(x+4)} = \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{C}{x+4}$
	$2+x^2 = A(x-2)(x+4) + B(x+4) + C(x-2)^2$
	Sub $x=2$, $B=1$
	Sub $x=-4$, $C=\frac{1}{2}$
	Sub $x=0$, $A=\frac{1}{2}$
	$\frac{2+x^2}{(x-2)^2(x+4)} = \frac{1}{2(x-2)} + \frac{1}{(x-2)^2} + \frac{1}{2(x+4)}$
		For decomposition of partial fractions
		
		A1 for each fraction

2	$\frac{1}{2}\left(8-\sqrt{48}+12-\sqrt{12}\right) height = 12+11\sqrt{3}$ $\frac{1}{2}\left(8-2\sqrt{12}+12-\sqrt{12}\right) height = 12+11\sqrt{3}$ $\left(20-3\sqrt{12}\right) height = 24+22\sqrt{3}$ $\left(20-6\sqrt{3}\right) height = 24+22\sqrt{3}$ $height = \frac{12+11\sqrt{3}}{\left(10-3\sqrt{3}\right)} \times \frac{\left(10+3\sqrt{3}\right)}{\left(10+3\sqrt{3}\right)}$ $= \frac{120+36\sqrt{3}+110\sqrt{3}+99}{100-27}$ $= \frac{219+146\sqrt{3}}{73}$ $= 3+2\sqrt{3}$	 Rationalise $\sqrt{48}$ to simplify expression For conjugate surd Numerator Denominator
3	$x^2 - xy + y^2 - 7 = 0$ ----------------(1) $y - 3x + 7 = 0$ $y = 3x - 7$ --------------------------(2) Sub (2) to (1) $x^2 - x(3x-7) + (3x-7)^2 - 7 = 0$ $x^2 - 3x^2 + 7x + 9x^2 - 42x + 49 - 7 = 0$	 Substitution from (2) Expansion and

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	$7x^2 - 35x + 42 = 0$ $x^2 - 5x + 6 = 0$ $x = 3$ or $x = 2$ $y = 2$ or $y = -1$	simplification Solving quadratic equation Both sets of answers
4	Let $f(x) = 3x^3 + hx^2 + kx - 4$ $(3x-1)$ is a factor of $f(x)$ $f\left(\frac{1}{3}\right) = 0$ $3\times\left(\frac{1}{3}\right)^3 + h\left(\frac{1}{3}\right)^2 + k\left(\frac{1}{3}\right) - 4 = 0$ $\frac{1}{9} + \frac{h}{9} + \frac{k}{3} - 4 = 0$ $h + 3k = 35$ ------------(1) Divisor = $x+1$, reminder = -4 $f(-1) = -4$ $-3 + h - k - 4 = -4$ $h - k = 3$ -----------(2) $(1)-(2)$, $4k = 32$ $k = 8$ $h = 3 + k = 3 + 8 = 11$	Use of Factor Theorem Use of Remainder Theorem Solving For values of *k* and *h*
5	$x^2 + 12x - 4k + 41 = kx + \frac{9}{4}k$ $x^2 + (12-k)x - \frac{25}{4}k + 41 = 0$ $(12-k)^2 - 4(1)(-\frac{25}{4}k + 41) < 0$ $144 - 24k + k^2 + 25k - 164 < 0$ $k^2 + k - 20 < 0$ $(k-4)(k+5) < 0$ $-5 < k < 4$	For equating the 2 expressions and equating to 0 For discriminant<0 For factorisation
6a	$27x^3 + 125$ $= (3x)^3 + 5^3$ $= (3x+5)(9x^2 - 15x + 25)$	
6b	Consider $9x^2 - 15x + 25$ $b^2 - 4ac$ $= (-15)^2 - 4(9)(25)$ $= -675$ Since $b^2 - 4ac < 0$, therefore $9x^2 - 15x + 25$ cannot be factorised.	Finding value of discriminant

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	Thus, $x=-\frac{5}{3}$	Explanation
7a	$\frac{\sin(A+B)+\sin(A-B)}{\cos(A+B)+\cos(A-B)}$ $=\frac{\sin A\cos B+\cos A\sin B+\sin A\cos B-\cos A\sin B}{\cos A\cos B-\sin A\sin B+\cos A\cos B+\sin A\sin B}$ $=\frac{2\sin A\cos B}{2\cos A\cos B}$ $=\tan A$	
7b	$\frac{\sin 105^o+\sin 15^o}{\cos 105^o+\cos 15^o}$ $A+B=105^o$ $A-B=15^o$ $2A=120^o$ $A=60^o$ $\frac{\sin 105^o+\sin 15^o}{\cos 105^o+\cos 15^o}=\tan 60^o=\sqrt{3}$	
8a	$\cos ecA=\frac{p}{2}$ $\sin A=\frac{2}{p}$ $\cot A=\frac{1}{\tan A}$ $=-\frac{\sqrt{p^2-4}}{2}$	For sqrt
8b	$\cos 2A=2\cos^2 A-1$ $=2\left(-\frac{\sqrt{p^2-4}}{p}\right)^2-1$ $=\frac{2p^2-8}{p^2}-1$ $=\frac{p^2-8}{p^2}$	Or equivalent
9ai	$-\frac{\pi}{6}$	
9aii	$\frac{3\pi}{4}$	

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9b	$-1 \le -\frac{1}{2}x \le 1$ $-2 \le x \le 2$	
9c	$\sin(45^o - 2x) = 0.75$ Ref angle = 48.590^o $0^o \le x \le 180^o$ $-315^o \le 45^o - 2x \le 45^o$ $45^o - 2x = 48.590^o$ (rej), $-(180^o + 48.590^o), -(360^o - 48.590^o)$ $= -273.590^o, -311.410^o$ $x = 136.8^o, 178.2^o$	Ref angle
10	$(\sin 45^o + \cos 30^o)(\sin 60^o - \cos 45^o)$ $= \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\right)$ $= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{2}}{2}\right)^2$ $= \frac{3}{4} - \frac{2}{4}$ $= \frac{1}{4}$	Special angles
11a	$y = (x-2)\sqrt{2x+1}$ $\frac{dP}{dx} = \frac{x-2}{\sqrt{2x+1}} + \sqrt{2x+1}$ $= \frac{x-2+2x+1}{\sqrt{2x+1}}$ $= \frac{3x-1}{\sqrt{2x+1}}$	Product rule Same base
11b	$\int_0^4 \frac{6x-2}{\sqrt{2x+1}}\,dx = 2\int_0^4 \frac{3x-1}{\sqrt{2x+1}}\,dx$ $= 2\left[(x-2)\sqrt{2x+1}\right]_0^4$ $= 2\left[2(3) - (-2)(1)\right]_0^2$ $= 16$	

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12	$V=\frac{2}{3}\pi h^3$ $\frac{dV}{dh}=2\pi h^2$ $\frac{dV}{dt}=\frac{dV}{dh}\times\frac{dh}{dt}$ $30=2\pi(2.5)^2\times\frac{dh}{dt}$ $\frac{dh}{dt}=\frac{30}{2\pi(2.5)^2}$ $=\frac{12}{5\pi}$	Differentiation
13a	Let $\sqrt{3}\cos\theta+\sin\theta=R\cos(\theta-\alpha)$ $R=\sqrt{\left(\sqrt{3}\right)^2+1^2}$ $R=2$ $\alpha=\tan^{-1}\frac{1}{\sqrt{3}}$ $\alpha=\frac{\pi}{6}$ $2\sin\theta+2\cos\left(\theta+\frac{\pi}{6}\right)=2\cos\left(\theta-\frac{\pi}{6}\right)$	Finding R
13b	Minimum value $=-2$ Occurs when $\theta=\pi+\frac{\pi}{6}=\frac{7\pi}{6}$	
13c	$2\cos\left(\theta-\frac{\pi}{6}\right)=-1$ $\cos\left(\theta-\frac{\pi}{6}\right)=-\frac{1}{2}$ (2^{nd} / 3^{rd} quads) Domain: $0<\theta<6$ $-0.52360<\theta-\frac{\pi}{6}<5.4764$ Reference angle of $\theta-\frac{\pi}{6}=\frac{\pi}{3}$ $\theta-\frac{\pi}{6}=\pi-\frac{\pi}{3},\pi+\frac{\pi}{3}$ $\theta=\frac{5\pi}{6},\frac{3\pi}{2}$	Finding reference angle

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14a	gradient of AB = $\frac{3}{-9} = -\frac{1}{3}$ gradient of the perpendicular bisector = $-\frac{1}{-\frac{1}{3}}$ $= 3$ midpoint of $AB = \left(\frac{13}{2}, -\frac{1}{2}\right)$ Equation of perpendicular bisector is $y + \frac{1}{2} = 3\left(x - \frac{13}{2}\right)$ $y = 3x - 20$	Gradient Midpoint Form equation
14b	For centre of circle, $\frac{4}{3}x - 10 = 3x - 20$ $\frac{5}{3}x = 10$ $x = 6$ $y = 18 - 20 = -2$ centre $= (6, -2)$ radius, $r = \sqrt{(6-2)^2 + (-2-1)^2}$ or $r = 11 - 6$ $= 5$ $= 5$ Equation of circle is $(x-6)^2 + (y+2)^2 = 25$	Equating Centre Radius
14c	point $= (6, -7)$	