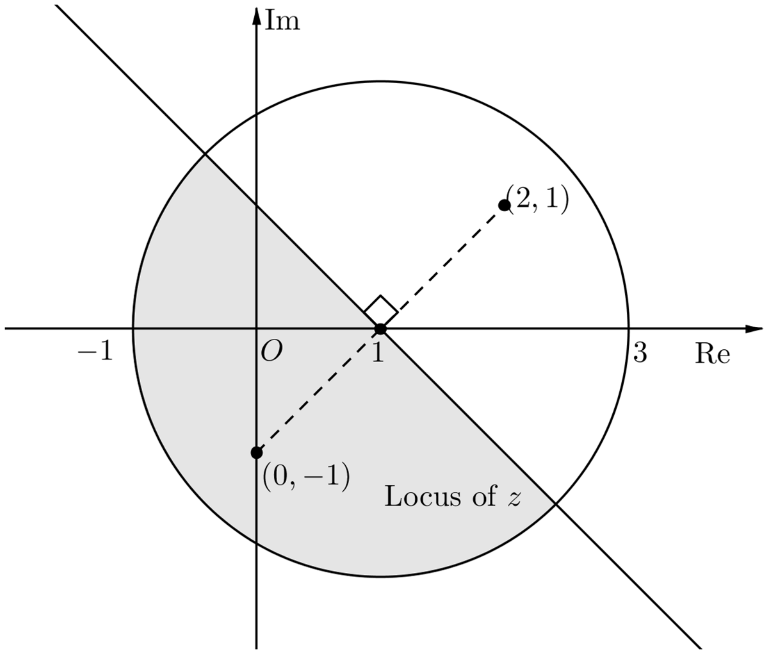
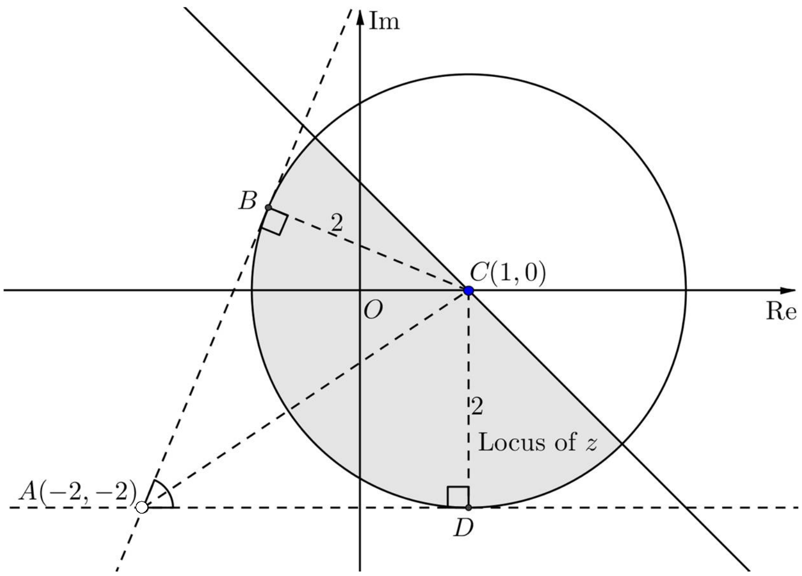
National Junior College
2016 – 2017 H2 Further Mathematics
Topic F7: Further Complex Numbers (Assignment 1 Solutions)

Qn No.	Solution	Marking Scheme
1		B1: circle with centre at (1,0) and radius 2; B1: perpendicular bisector that passes through the centre of circle B1: shading the correct region with the correct boundaries
1	 Consider triangle ABC or ACD where AB and AD are tangents to the circle. Then $AB = AD = 3$ and $AC = \sqrt{AB^2 + BC^2} = \sqrt{2^2 + 3^2} = \sqrt{13}$ $\sin \angle BAC = \left(\frac{2}{\sqrt{13}}\right) \Rightarrow \angle BAD = 2\sin^{-1}\left(\frac{2}{\sqrt{13}}\right)$	M1: construct appropriate right angled triangle B1: correct length of AC A1: Working to get angle BAD

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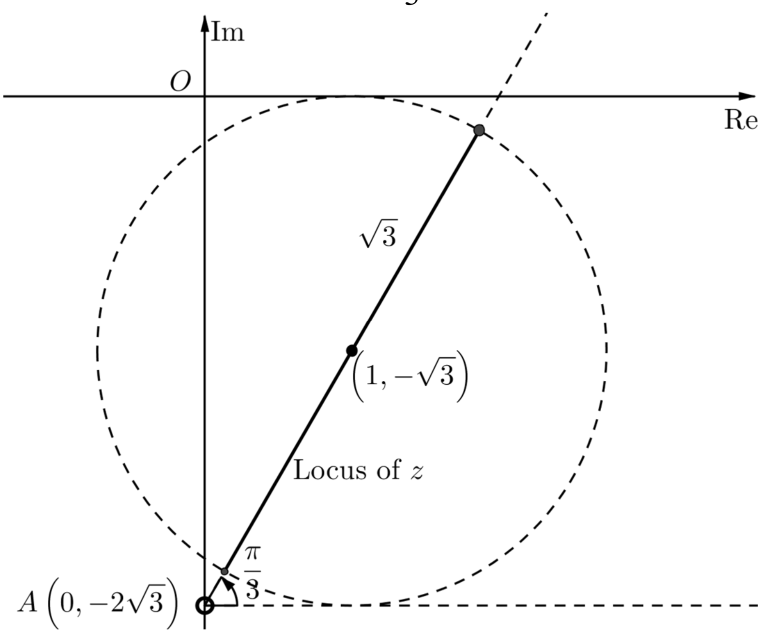
2(a) (i)	$\lvert z+w\rvert = \lvert z+(\mathrm{i}\sqrt{3})z\rvert$ $= \lvert (1+\mathrm{i}\sqrt{3})z\rvert$ $= \lvert 1+\mathrm{i}\sqrt{3}\rvert\lvert z\rvert = 2\lambda$	M1 : Use of $\lvert z_1z_2\rvert = \lvert z_1\rvert\lvert z_2\rvert$ A1 : 2λ
(ii)	$\arg(z+w) = \arg\left((1+\mathrm{i}\sqrt{3})z\right)$ $= \arg(1+\mathrm{i}\sqrt{3}) + \arg(z)$ $= \dfrac{\pi}{3}+\theta$	M1 : Use of $\arg(z_1z_2) = \arg(z_1)+\arg(z_2)$ A1 : $\dfrac{\pi}{3}+\theta$
2 (b) (i)	$\lvert z-1+\mathrm{i}\sqrt{3}\rvert = \sqrt{3}$ is a circle with centre $(1, \sqrt{3})$ and radius $\sqrt{3}$ units. $\arg(2z+\mathrm{i}4\sqrt{3}) = \dfrac{\pi}{3}$ $\Rightarrow \arg(2)+\arg(z+\mathrm{i}2\sqrt{3}) = \dfrac{\pi}{3}$ $\Rightarrow \arg(z+\mathrm{i}2\sqrt{3}) = \dfrac{\pi}{3}$ $\arg(z+\mathrm{i}2\sqrt{3}) = \dfrac{\pi}{3}$ is a half-line with starting point at $(0, -2\sqrt{3})$ that makes angle of $\dfrac{\pi}{3}$ with the positive real axis.  Note that 1. the half line passes through the centre of circle. 2. The real axis is a tangent to the circle.	B1: correct sketch of circle (or the line segment must appear to be confined in the circle) B1: correct sketch of half line (hollow circle is not required at end point) B1: Required locus is a diameter of the circle (must be clearly and correctly labeled)
2b		

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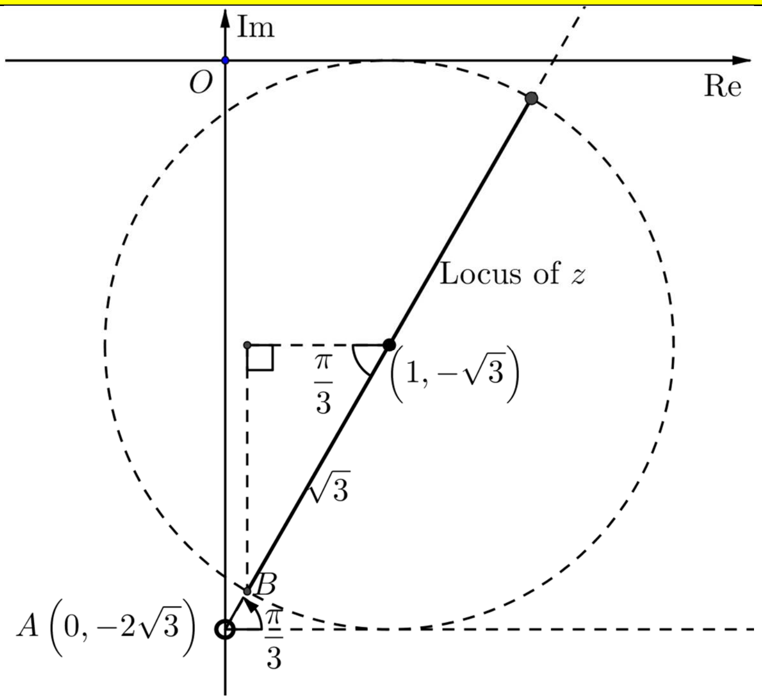
<tr><th>Qn No.</th><th>Solution</th><th>Marking Scheme</th></tr>
<tr><td>(ii)</td><td>

B is the point that represents the complex number $z = x + \mathrm{i}y$ that gives the greatest possible value of $\left|\arg(z)\right|$.

From the diagram,
$x = 1 - \left(\sqrt{3}\right)\cos\frac{\pi}{3} = 1 - \frac{\sqrt{3}}{2}$,
$y = -\sqrt{3} - \left(\sqrt{3}\right)\sin\frac{\pi}{3} = -\sqrt{3} - \frac{3}{2}$
$\therefore z = \left(1 - \frac{\sqrt{3}}{2}\right) + \mathrm{i}\left(-\sqrt{3} - \frac{3}{2}\right)$</td><td>B1: choose the correct point

M1: use of trigonometry

A1:
$z = \left(1 - \frac{\sqrt{3}}{2}\right) + \mathrm{i}\left(-\sqrt{3} - \frac{3}{2}\right)$</td></tr>
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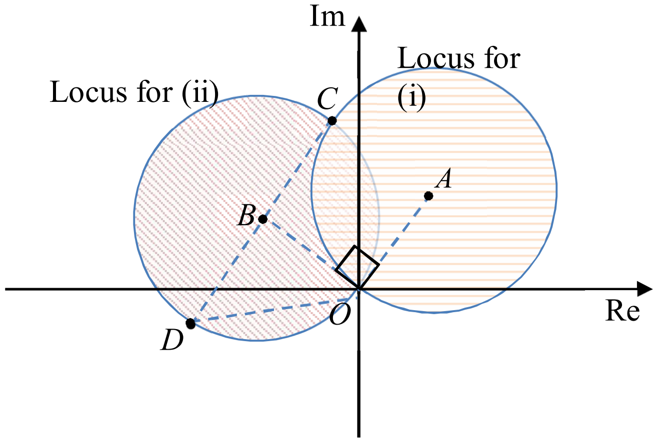
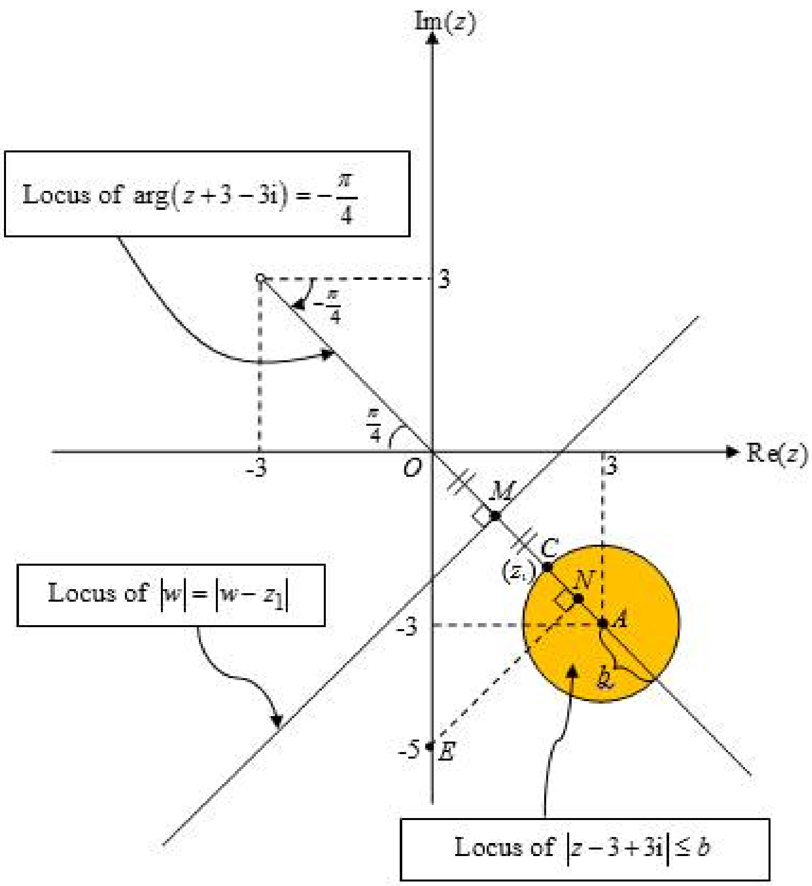
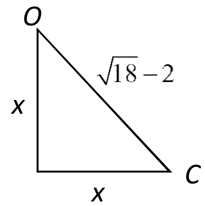
Qn No.	Solution	Marking Scheme
3 (i), (ii)		**(i)** B1: Circle centered at A with radius 5 (must pass through origin) B1: Correct shaded region **(ii)** B1: Circle centered at B with radius 5 (must pass through origin) B1: Correct shaded region and relative position of A
3 (iii)	A 90° anti-clockwise rotation about the origin	B1
3 (iv)	Since $\|(a+\mathrm{i}a)-a\|=\|(a+\mathrm{i}a)-\mathrm{i}a\|=5$, thus $z=a+\mathrm{i}a$ satisfies the equations $\|z-a\|=5$ and $\|z-\mathrm{i}a\|=5$. Thus C lies on the loci of $\|z-a\|=5$ and $\|z-\mathrm{i}a\|=5$.	B1: Or any other appropriate explanations
3(v)	Since triangle OBC is an isosceles right angled triangle, Required area $=2\left(\frac{1}{4}\pi(5)^2-\frac{1}{2}(5)^2\right)$ $=\frac{25}{2}(\pi-2)$	M1 A1
3 (vi)	Let D represents $\mathrm{i}a-a$ on the Argand diagram. By circle properties, $\angle ODC=\frac{\pi}{4}$. Since $\mathrm{i}a-a=-7-\mathrm{i}$, thus $\max\arg(z-\mathrm{i}a+a)=\frac{\pi}{4}+\tan^{-1}\left(\frac{1}{7}\right)$. **OR** Since DC is parallel to OA, $\max\arg(z-\mathrm{i}a+a)=\arg(a)=\tan^{-1}\left(\frac{4}{3}\right)$	M1: Identify point D correctly A1

Qn No.	Solution	Marking Scheme
4 (i)		B1: correct sketch of half-line. B1: correct sketch of circle with its interior shaded.
4(ii)	Let d be the least possible value of $\|z+5\mathrm{i}\|$. Using ΔONE , $\sin\frac{\pi}{4}=\frac{d}{5}$ $d=\frac{5}{\sqrt{2}}=\frac{5\sqrt{2}}{2}$	B1
4 (iii) (a)	$OA=\sqrt{3^2+3^2}=\sqrt{18}$ $b=OA-\left(\sqrt{18}-2\right)$ $b=2$	B1

4 (iii) (b)	Method 1: $z=\left(\sqrt{18}-2\right)\left[\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right]$ $=\left(3\sqrt{2}-2\right)\left[\frac{1}{\sqrt{2}}-\mathrm{i}\frac{1}{\sqrt{2}}\right]$ $=3-\frac{2}{\sqrt{2}}-\mathrm{i}\frac{\left(3\sqrt{2}-2\right)}{\sqrt{2}}$ $=3-\frac{2}{\sqrt{2}}-3\mathrm{i}+\mathrm{i}\frac{2}{\sqrt{2}}$ $=\left(3-\frac{2}{\sqrt{2}}\right)+\left(-3+\frac{2}{\sqrt{2}}\right)\mathrm{i}$ $=\left(3-\sqrt{2}\right)+\left(-3+\sqrt{2}\right)\mathrm{i}$ Method 2: $x^2+x^2=\left(\sqrt{18}-2\right)^2$ $x^2=\frac{\left(\sqrt{18}-2\right)^2}{2}$ $x=\sqrt{\frac{\left(\sqrt{18}-2\right)^2}{2}}$ $x=\frac{\sqrt{18}-2}{\sqrt{2}}$ $x=3-\sqrt{2}$ $z=\left(3-\sqrt{2}\right)+\left(-3+\sqrt{2}\right)\mathrm{i}$  	M1 A1
4 (iii) (c)	The locus of $\|w\|=\|w-z_1\|$ is a perpendicular bisector of the line segment joining the points O and C. Since gradient of line segment OC is -1, Hence, gradient of perpendicular bisector = 1. Let point M be the midpoint of OC. Hence, point M is $\left(\frac{\left(3-\sqrt{2}\right)}{2}, \frac{\left(-3+\sqrt{2}\right)}{2}\right)$. Equation of locus: $y-\frac{\left(-3+\sqrt{2}\right)}{2}=(1)\left(x-\frac{\left(3-\sqrt{2}\right)}{2}\right)$ $y=x-\frac{\left(3-\sqrt{2}\right)}{2}+\frac{\left(-3+\sqrt{2}\right)}{2}$ $y=x-3+\sqrt{2}$	B1