<u>MARKING SCHEME FOR ADDITIONAL MATHEMATICS</u>
<u>SECONDARY 4 EXPRESS</u>
<u>PRELIMINARY EXAMINATION 2021</u>

Qn		**Solution**	**Marking Point**	**Remarks**
1		$y = mx^2 + 9x + k$ $y = kx^2 + 9x + k$ For curve to lie completely above the *x*-axis, $k > 0$ and $9^2 - 4(k)(k) < 0$	**M1**	Discriminant < 0
		$-4k^2 + 81 < 0$ $4k^2 - 81 > 0$		
		$(2k+9)(2k-9) > 0$ $k < -\frac{9}{2}$ or $k > \frac{9}{2}$	**M1**	Factorizing quad. inequality
		$\therefore k > \frac{9}{2}$	**A1**	
2		$\sqrt{343^x} = \frac{7^{1-x}}{49}$		
		$(7^{3x})^{\frac{1}{2}} = \frac{7^{1-x}}{7^2}$ $7^{\frac{3}{2}x} = 7^{1-x-2}$	**M1**	Expressing in base 7
		$\frac{3}{2}x = -x - 1$ $\frac{5}{2}x = -1$	**M1**	Equating powers
		$x = -\frac{2}{5}$	**M1**	
		$\therefore \sqrt{343^{-\frac{2}{5}}} = 0.311$ (to 3s.f)	**A1**	
3		$\frac{dy}{dx} = \frac{(x-1)(2)e^{2x} - (1)e^{2x}}{(x-1)^2}$	**M1**	
		$= \frac{e^{2x}(2x-2-1)}{(x-1)^2}$ $= \frac{e^{2x}(2x-3)}{(x-1)^2}$		

		For y to be decreasing, $\frac{dy}{dx} < 0$ $\frac{e^{2x}(2x-3)}{(x-1)^2} < 0$ Since $e^{2x} > 0$ and $(x-1)^2 > 0$ for $x > 1$, $2x - 3 < 0$ $x < \frac{3}{2}$ $\therefore 1 < x < \frac{3}{2}$	**M1** **M1** **A1**	$\frac{dy}{dx} < 0$
4		Height $\frac{44+4\sqrt{5}}{(\sqrt{10}+\sqrt{2})^2}$ $= \frac{44+4\sqrt{5}}{10+2\sqrt{20}+2}$ $= \frac{44+4\sqrt{5}}{12+2\sqrt{20}}$ $= \frac{22+2\sqrt{5}}{6+\sqrt{20}}$ $= \frac{22+2\sqrt{5}}{6+2\sqrt{5}} \times \frac{6-2\sqrt{5}}{6-2\sqrt{5}}$ $= \frac{(22+2\sqrt{5})(6-2\sqrt{5})}{36-4(5)}$ $= \frac{132-44\sqrt{5}+12\sqrt{5}-4(5)}{16}$ $= \frac{112-32\sqrt{5}}{16}$ $= 7-2\sqrt{5}$	**M1** **M1** **M1** **M1** **A1**	For obtaining h volume/base area For area of base For conjugate surd For correct expansion of numerator
5	(i)	f(x) $= 2(x+2)(x-3)(2x^2+3x+2)$ $= 2(x^2-x-6)(2x^2+3x+2)$ $= 2(2x^4+3x^3+2x^2-2x^3-3x^2-2x-12x^2-18x-12)$ $= 2(2x^4+x^3-13x^2-20x-12)$ $= 4x^4+2x^3-26x^2-40x-24$	**M1, M1** **A1**	For coefficient 2, at least 1 correct ( )
5	(ii)	f(x) $= 0$		

		$2(x+2)(x-3)(2x^2+3x+2)=0$ Discriminant $=3^2-4(2)(2)$ $=-7<0$ Since discriminant < 0, **the quadratic equation** $2x^2+3x+2=0$ **has no real roots** and hence $2(x+2)(x-3)(2x^2+3x+2)$ has exactly two real roots, –2 and 3.	**M1** **A1**	For computing discriminant
6	(a)	For $a^2x^3+7x+b^2$: Remainder = a^2+7+b^2 For $-18+2abx^3$: Remainder = $-18+2ab$ $a^2+7+b^2=-18+2ab$ $a^2-2ab+b^2=-18-7$ $(a-b)^2=-25$ Since $(a-b)^2 \geq 0$ for all real values of a and b, the equation $(a-b)^2=-25$ has no solutions.	**M1** **M1** **M1** **A1**	For any correct expression for remainder Equating remainders
6	(b)	$g(x)=-18+2abx^3$ By Factor Theorem, $-18+2ab\left(\frac{2}{3}\right)^3=0$ $a=\frac{243}{8b}$	**M1** **A1**	
7	(i)	1 cycle occurs every 12 hours. $\frac{2\pi}{k}=12$ $k=\frac{2\pi}{12}$ $=\frac{\pi}{6}$ (shown)	**M1** **A1**	
7	(ii)	$a=4$ $b=-2$	**B1** **B1**	

7	(iii)	$d = 4 - 2\cos\frac{\pi}{6}t$ When $d = 3$, $3 = 4 - 2\cos\frac{\pi}{6}t$ $\cos\frac{\pi}{6}t = \frac{3-4}{-2}$ $\cos\frac{\pi}{6}t = \frac{1}{2}$ basic angle, $\alpha = \cos^{-1}(\frac{1}{2}) = \frac{\pi}{3}$ $\frac{\pi}{6}t = \frac{\pi}{3}$ $t = 2$ Earliest time = 0830 + 0200 = 1030	**M1** **M1** **A1**	For finding basic angle
8	(i)	At stationary points, $\frac{dy}{dx} = 0.$ $y = 2x + \frac{8x}{x-1}$ $\frac{dy}{dx} = 2 + \frac{(x-1)(8) - 8x}{(x-1)^2}$ $= 2 - \frac{8}{(x-1)^2}$ $2 - \frac{8}{(x-1)^2} = 0$ $(x-1)^2 = 4$ $x = 1 \pm \sqrt{4}$ $x = -1$ or 3 When $x = -1$, $y = 2(-1) + \frac{8(-1)}{(-1-1)}$ $y = 2$	**M1** **M1** **A1**	$\frac{dy}{dx} = 0$

		When $x = 3$, $y = 2(3) + \frac{8(3)}{(3-1)}$ $y = 18$ Coordinates of stationary points = (–1, 2) and (3, 18)	**A1**	
8	(ii)	$\frac{dy}{dx} = 2 - 8(x-1)^{-2}$ $\frac{d^2y}{dx^2} = \frac{16}{(x-1)^3}$ When $x = -1$, $\frac{d^2y}{dx^2} = \frac{16}{(-2)^3} = -2 < 0$ When $x = 3$, $\frac{d^2y}{dx^2} = \frac{16}{(2)^3} = 2 > 0$ (–1, 2) is a maximum point and (3, 18) is a minimum point.	**M1** **M1** **A1**	For substitution into $\frac{d^2y}{dx^2}$
9	(i)	At instantaneous rest, $v = 0$. $2\cos\frac{t}{2} + 1 = 0$ $\cos\frac{t}{2} = -\frac{1}{2}$ basic angle, $\alpha = \cos^{-1}(\frac{1}{2}) = \frac{\pi}{3}$ Since $\cos\frac{t}{2} < 0$, $\frac{t}{2}$ lies in the 2nd or 3rd quadrant. Since $0 \le t \le 2\pi$, $0 \le \frac{t}{2} \le \pi$.	**M1** **M1**	For finding basic angle

		$\frac{t}{2} = \pi - \frac{\pi}{3}$ or $\pi + \frac{\pi}{3}$ (N.A) $\frac{t}{2} = \frac{2\pi}{3}$ $t = \frac{4\pi}{3}$	**A1**	
9	(ii)	$v = 2\cos\frac{t}{2} + 1$ $s = \int 2\cos\frac{t}{2} + 1 \, \mathrm{d}t$ $= \frac{2}{\frac{1}{2}}(\sin\frac{t}{2}) + t + c$ $= 4\sin\frac{t}{2} + t + c$ When $t = 0,\ s = 0.$ $0 = 4\sin 0 + 0 + c$ $c = 0$ When $t = 0,\ s = 0.$ When $t = \frac{4\pi}{3},\ s = 4\sin\frac{2\pi}{3} + \frac{4\pi}{3} = 7.6529$ When $t = 2\pi,\ s = 4\sin\pi + 2\pi = 2\pi = 6.2832$ Total distance $= 7.6529 + (7.6529 - 6.2832)$ $= 9.02$ m (to 3s.f)	**M1** **M1** **M1** **A1**	
9	(iii)	$v = 2\cos\frac{t}{2} + 1$ $a = 2\left(\frac{1}{2}\right)(-\sin\frac{t}{2})$ $a = -\sin\frac{t}{2}$ When $t = \pi$,	**M1**	

		$a=-\sin\frac{\pi}{2}$ $=-1\text{ m/s}^2$	**A1**	
10	(i)	$\frac{52x^2-20x-6}{(x-2)(4x+1)^2}=\frac{A}{x-2}+\frac{B}{4x+1}+\frac{C}{(4x+1)^2}$ $52x^2-20x-6=A(4x+1)^2+B(x-2)(4x+1)+C(x-2)$ Sub $x=2$, $52(2)^2-20(2)-6=A[4(2)+1]^2$ $162=81A$ $A=2$ Sub $x=-\frac{1}{4}$, $52\left(-\frac{1}{4}\right)^2-20\left(-\frac{1}{4}\right)-6=C\left(-\frac{1}{4}-2\right)$ $2.25=-2.25C$ $C=-1$ Sub $x=3, A=2, C=-1$, $52(3)^2-20(3)-6=2[4(3)+1]^2+13B+(-1)(1)$ $65=13B$ $B=5$ $\therefore \frac{52x^2-20x-6}{(x-2)(4x+1)^2}=\frac{2}{x-2}+\frac{5}{4x+1}-\frac{1}{(4x+1)^2}$	**M1** **M1** **M1** **M1** **M1** **A1**	Realizing form of the partial fraction Eliminating denominator

10	(ii)	$\int_3^4 \frac{52x^2-20x-6}{(x-2)(4x+1)^2}\ \mathrm{d}x$ $=\int_3^4 \frac{2}{x-2}+\frac{5}{4x+1}-\frac{1}{(4x+1)^2}\ \mathrm{d}x$ $=\left[2\ln(x-2)+\frac{5}{4}\ln(4x+1)-\frac{(4x+1)^{-2+1}}{(-2+1)(4)}\right]_3^4$ $=\left[2\ln(x-2)+\frac{5}{4}\ln(4x+1)+\frac{1}{4(4x+1)}\right]_3^4$ $=(2\ln 2+\frac{5}{4}\ln 17+\frac{1}{68})-(2\ln 1+\frac{5}{4}\ln 13+\frac{1}{52})$ $=1.72$ (to 3s.f)	**M1** **M1, M1** **A1**	
11	(i)	$y=\ln x$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{x}$ At B, $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{e^2}$ $\frac{1}{x}=\frac{1}{e^2}$ $\therefore x = e^2$ When $x=e^2$, $y=\ln e^2$ $=2$ $\therefore B(e^2, 2)$	**M1** **M1** **A1**	
11	(ii)	$y-2=\frac{1}{e^2}(x-e^2)$ $y=\frac{1}{e^2}x-1+2$ $\therefore$ eqn of tangent: $y=\frac{1}{e^2}x+1$	**M1**	

		When $y=0$, $0=\frac{1}{e^2}x+1$ $x=-e^2$ $\therefore C(-e^2,\ 0)$	**M1** **A1**	
11	(iii)	$\frac{d}{dx}(x\ln x-x)=\ln x$ $\int \ln x\ dx=x\ln x-x$ $\therefore$ area of shaded region $=\frac{1}{2}[e^2-(-e^2)](2)-\int_1^{e^2}\ln x\ dx$ $=2e^2-\left[x\ln x-x\right]_1^{e^2}$ $=2e^2-[(e^2\ln e^2-e^2)-(\ln 1-1)]$ $=2e^2-(2e^2-e^2+1)$ $=e^2-1$ units^2	 **M1** **M1, M1** **A1**	
12	(i)	$x^2+y^2-2x-8y-33=0$ $(x-1)^2-1+(y-4)^2-16-33=0$ $(x-1)^2+(y-4)^2=50$ Coordinates of centre = (1, 4) Radius = $\sqrt{50}\ /\ 5\sqrt{2}$ units	 **M1** **A1** **A1**	
12	(ii)	When $y=0$, $(x-1)^2+(0-4)^2=50$ $(x-1)^2=50-16$ $x-1=\pm\sqrt{34}$ $x=1+\sqrt{34}$ or $1-\sqrt{34}$ Midpoint of *PQ* $=\left(\frac{1+\sqrt{34}+1-\sqrt{34}}{2},0\right)$ $=\left(\frac{2}{2},0\right)$ $=(1,0)$	 **M1** **M1**	

			A1	
12	(iii)	Gradient of normal through (6, –1) $= \frac{-1-4}{6-1}$ $= -1$ $\therefore$ gradient of tangent $= 1$ $y+1 = 1(x-6)$ $y = x-6-1$ $y = x-7$ (shown)	 **M1** **M1** **A1**	
12	(iv)	Radius of C_2 = 9 units $\therefore (x-1)^2 + (y-4)^2 = 9^2$	**B1**	
13	(a) (i)	$\cos 75°$ $= \cos(30° + 45°)$ $= \cos 30° \cos 45° - \sin 30° \sin 45°$ $= \frac{\sqrt{3}}{2} \times \frac{\sqrt{2}}{2} - \frac{1}{2} \times \frac{\sqrt{2}}{2}$ $= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$ $= \frac{\sqrt{6}-\sqrt{2}}{4}$ **OR** $\cos 75°$ $= \cos(30° + 45°)$ $= \cos 30° \cos 45° - \sin 30° \sin 45°$ $= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}$ $= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$ $= \frac{\sqrt{3}-1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ $= \frac{\sqrt{6}-\sqrt{2}}{2(2)}$ $= \frac{\sqrt{6}-\sqrt{2}}{4}$	 **M1** **M1** **A1** **M1**	 For Addition Angle Formula For Addition Angle Formula

			M1 **A1**	
13	(a) (ii)	$\cos 150°$ $= \cos[2(75°)]$ $= 2\cos^2 75° - 1$ $= 2\left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)^2 - 1$ $= 2\left(\frac{6-2\sqrt{12}+2}{16}\right) - 1$ $= \frac{8-2\sqrt{12}}{8} - 1$ $= 1 - \frac{4\sqrt{3}}{8} - 1$ $= -\frac{\sqrt{3}}{2}$	 **M1** **M1** **A1**	 For Double Angle Formula For expansion

13	(b)	L.H.S $=\left(\dfrac{1}{\tan x}+\dfrac{1}{\sin x}\right)^2$ $=\left(\dfrac{\cos x}{\tan x\cos x}+\dfrac{1}{\sin x}\right)^2$ or $\left(\dfrac{\cos x}{\sin x}+\dfrac{1}{\sin x}\right)^2$ $=\left(\dfrac{\cos x+1}{\sin x}\right)^2$ $=\dfrac{(\cos x+1)^2}{\sin^2 x}$ $=\dfrac{(\cos x+1)^2}{1-\cos^2 x}$ $=\dfrac{(\cos x+1)^2}{(1+\cos x)(1-\cos x)}$ $=\dfrac{1+\cos x}{1-\cos x}$ = R.H.S (proven)	**M1** **M1** **M1** **A1**	Re-express in terms of $\sin\theta$ or $\cos\theta$