2022 JC1 H2 MA Common Test Markers Comments

1	Solution [4] MOD	
	Let $\frac{2}{(2r-1)(2r+1)}=\frac{A}{(2r-1)}+\frac{B}{(2r+1)}$ $A=\left.\frac{2}{(2r+1)}\right\|_{x=\frac{1}{2}}=1$ $B=\left.\frac{2}{(2r-1)}\right\|_{x=-\frac{1}{2}}=-1$ $\frac{2}{(2r-1)(2r+1)}=\frac{1}{(2r-1)}-\frac{1}{(2r+1)}$	This part is well-done by almost every student.
	$\sum_{r=1}^{n}\left[\frac{1}{(2r-1)(2r+1)}\right]=\frac{1}{2}\sum_{r=1}^{n}\left[\frac{1}{2r-1}-\frac{1}{2r+1}\right]$ $=\frac{1}{2}\left\{\begin{array}{l}\frac{1}{1}-\cancel{\frac{1}{3}}\\+\cancel{\frac{1}{3}}-\frac{1}{5}\\+\ldots\\+\cancel{\frac{1}{2n-3}}-\cancel{\frac{1}{2n-1}}\\+\cancel{\frac{1}{2n-1}}-\frac{1}{2n+1}\end{array}\right\}$ $=\frac{1}{2}\left(1-\frac{1}{2n+1}\right)$	The common mistake is missing out the $\frac{1}{2}$ but most students recognize it is MOD.

	$\sum_{r=4}^{n}\left[\frac{1}{(2r-3)(2r-1)}\right]$ $=\sum_{k+1=4}^{k+1=n}\left[\frac{1}{(2(k+1)-3)(2(k+1)-1)}\right]$ ***(let r = k+1)*** $=\sum_{k=3}^{n-1}\left[\frac{1}{(2k-1)(2k+1)}\right]$ $=\sum_{k=1}^{n-1}\left[\frac{1}{(2k-1)(2k+1)}\right]-\sum_{k=1}^{2}\left[\frac{1}{(2k-1)(2k+1)}\right]$ $=\frac{1}{2}\left(1-\frac{1}{2(n-1)+1}\right)-\frac{1}{2}\left(1-\frac{1}{2(2)+1}\right)$ $=\frac{1}{2}\left(1-\frac{1}{2(n-1)+1}-1+\frac{1}{2(2)+1}\right)$ $=\frac{1}{2}\left(\frac{1}{5}-\frac{1}{2n-1}\right)$	The challenging part of this question is letting $r=k+1$ and more importantly considering the starting and ending values of k.

2	Solution [4] SLEs	
	Let b, s and f be the price per cup of butter, sugar and flour respectively. $200b + 50s + 200f = 129$ $40b + 40s + 40f = 32.4$ $100b + 50s + 300f = 88$ From GC, $b = 0.5$, $s = 0.22$, $f = 0.09$ Therefore, the price of a cup of butter, sugar, and flour is \$0.50, \$0.22, and \$0.09 respectively. Hence, the cost to produce the pastries is \$81.25	Very well-done by almost all students

3	Solution [5] Inequality	
(a)	$\sqrt{\frac{x+2}{x}} = \left(1+\frac{2}{x}\right)^{\frac{1}{2}}$ $\Rightarrow \left(1+\frac{2}{x}\right)^{\frac{1}{2}}$ $= \left(1+\frac{1}{2}\left(\frac{2}{x}\right)+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)}{2!}\left(\frac{2}{x}\right)^2+\frac{\frac{1}{2}\left(\frac{1}{2}-1\right)\left(\frac{1}{2}-2\right)}{3!}\left(\frac{2}{x}\right)^3+\ldots\right)$ (*) $=1+\frac{1}{x}-\frac{1}{2x^2}+\frac{1}{2x^3}+\ldots$ Expansion is valid for $\left\|\frac{2}{x}\right\|<1 \Rightarrow \frac{2}{\|x\|}<1 \Rightarrow \|x\|>2$ $x>2$ or $x<-2$ Since $\|-6\|=6>2$, $x=-6$ is valid for getting an estimate for $\sqrt{6}$. $\sqrt{\frac{x+2}{x}} \approx 1+\frac{1}{x}-\frac{1}{2x^2}+\frac{1}{2x^3}$ Let $x=-6$ $\sqrt{\frac{-4}{-6}} \approx 1+\frac{1}{(-6)}-\frac{1}{2(-6)^2}+\frac{1}{2(-6)^3}$ $\frac{2}{\sqrt{6}} \approx \frac{353}{432}$ $\sqrt{6} \approx \frac{864}{353}$	Most students are able to come out with the series; however some did not read the question clearly and attempt to expand in increasing order instead. Not many students can write down the range of values of x correctly and subsequently give a reason why -6 is a good estimate. There is another approximate value of $\sqrt{6} \approx \frac{353}{144}$. $\sqrt{\frac{2}{3}} \approx \frac{353}{432}$ $\sqrt{\frac{2}{3}.\frac{3}{3}} \approx \frac{353}{432}$ $\frac{\sqrt{6}}{3} \approx \frac{353}{432}$ $\sqrt{6} \approx \frac{353}{144}$

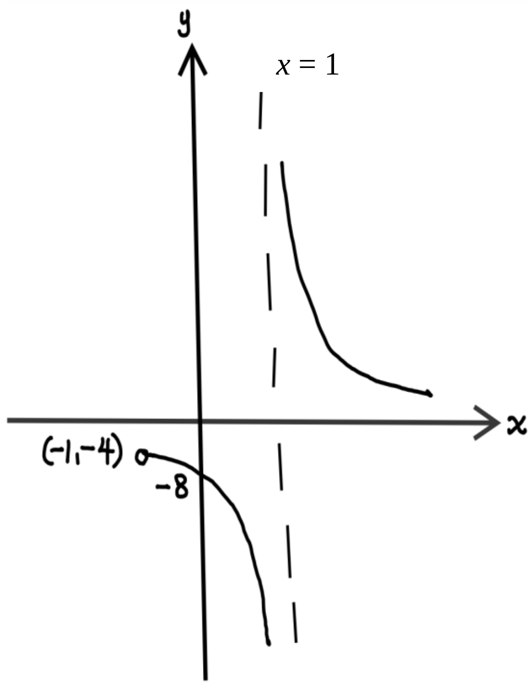
4	Solution [5] Summation	
(a)	$\mathrm{u}(r)-\mathrm{u}(r-1)=2^{-r}+4r-2$ $\sum_{r=1}^{N}\mathrm{u}(r)-\mathrm{u}(r-1)=\sum_{r=1}^{N}2^{-r}+4r-2$ $\left\{\begin{array}{l}\mathrm{u}(1)-\mathrm{u}(0)\\+\mathrm{u}(2)-\mathrm{u}(1)\\\ldots\\+\mathrm{u}(N)-\mathrm{u}(N-1)\end{array}\right\}=\sum_{r=1}^{N}2^{-r}+4\sum_{r=1}^{N}r-\sum_{r=1}^{N}2$ $\mathrm{u}(N)-\mathrm{u}(0)=\frac{\frac{1}{2}\left(1-\frac{1}{2^N}\right)}{1-\frac{1}{2}}+4\left(\frac{1}{2}N(N+1)\right)-2N$ $\mathrm{u}(N)-5=1-\frac{1}{2^N}+2N(N+1)-2N$ $\mathrm{u}(N)=6-\frac{1}{2^N}+2N^2$ Therefore $\mathrm{u(r)}=6-\frac{1}{2^r}+2r^2$	Question was challenging for most students. For those who attempted, many used the original form $\mathrm{u}(r)=\mathrm{u}(r-1)2^{-r}+4r-2$ and kept expanding the RHS with no avail. For some, MOD was used for $\mathrm{u}(r)-\mathrm{u}(r-1)$ but did not apply the summation to $2^{-r}+4r-2$. To apply MOD, students need to recognize that they need the form $\sum \mathrm{u}(r)-\mathrm{u}(r-1)$. Thus, the first step is to move u(r – 1) to the LHS. Then take $\sum_{r=1}^{N}$ on both sides. Then realise that the RHS can be split into sum of a GP, AP and constant respectively.

5	Solution [7] Inequality	
(a)	$\frac{x^3+4x^2+5x-2}{2-x} \geq -1$ $\frac{x^3+4x^2+5x-2}{2-x}+1 \geq 0$ $\frac{x^3+4x^2+5x-2+2-x}{2-x} \geq 0$ $\frac{x^3+4x^2+4x}{2-x} \geq 0$ $\frac{x\left(x^2+4x+4\right)}{2-x} \geq 0$ $\frac{x(x+2)^2}{2-x} \geq 0$ $-\quad -\quad +\quad -$ $-2\quad 0\quad 2$ $0 \leq x < 2$ but $x=-2$ also satisfy the inequality Therefore $0 \leq x < 2$ or $x=-2$ Alternatively $\frac{x(x+2)^2}{2-x} \geq 0$ Since $(x+2)^2 \geq 0$, with $x=-2$ satisfy the inequality $\frac{x}{2-x} > 0$ $-\quad +\quad -$ $0\quad 2$ Therefore $0 \leq x < 2$ or $x=-2$	Generally ok except for some students missing out on the answer $x=-2$. Also, there were mistakes with the combining of fraction expressions as well as in the operation with inequality signs.

(b)	$\dfrac{x^3-4x^2+5x+2}{2+x}\le 1$ Replace x with $-y$ $\dfrac{-y^3-4y^2-5y+2}{2-y}\le 1$ $\dfrac{y^3+4y^2+5y-2}{2-y}\ge -1$ Using earlier result $0\le y<2$ or $\mathrm{y}=-2$. Thus $0\le -x<2$ or $-x=-2$ $-2<x\le 0$ or $x=2$	Most students were able to identify the replacement for x and proceed to solve the inequality for this part. Some missed the answer $x=2$ due to the first part.

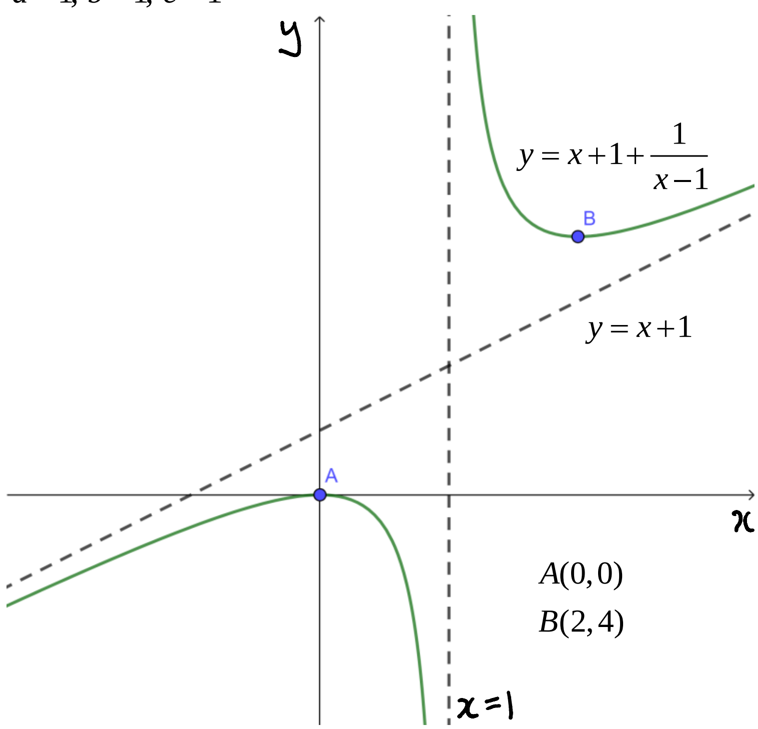
6	Solution [7] Implicit Differentiation	
(a)	$x^3 + y^3 = 4xy$ $3x^2 + 3y^2 \frac{dy}{dx} = 4y + 4x\frac{dy}{dx}$ $\frac{dy}{dx}\left[3y^2 - 4x\right] = 4y - 3x^2$ $\frac{dy}{dx} = \frac{4y - 3x^2}{3y^2 - 4x}$	Generally, well done.
(b)	$\frac{dy}{dx} \to \infty$ Let $3y^2 - 4x = 0$ $x = \frac{3}{4}y^2$ $\left(\frac{3}{4}y^2\right)^3 + y^3 = 4y\left(\frac{3}{4}y^2\right)$ $\left(\frac{3}{4}\right)^3 y^6 + y^3 = 3y^3$ $y^3\left(\frac{27}{64}y^3 - 2\right) = 0 \quad ----(*)$ $y = 0$ or $\mathrm{y} = \left(\frac{128}{27}\right)^{\frac{1}{3}} = \frac{(128)^{\frac{1}{3}}}{3}$ *or* 1.6798947 $x = 0$ or $\mathrm{x} = \frac{1}{12}(128)^{\frac{2}{3}}$ *or* 2.1165347 The coordinates where the tangent to curve is parallel to the y-axis are $(0,0)$ and $\left[\left(\frac{128^{\frac{1}{3}}}{3}, \frac{128^{\frac{2}{3}}}{12}\right) \text{ or } (1.68, 2.12)\right]$	Many candidates miss out the solution $y = 0$ from equation (*). The question did not ask for exact values of the coordinates, candidates could have used GC function to evaluate the roots to (*).

7	Solution [10] Tangent normal Parametric Equation	
(i)	$x=2t+1,\quad y=\frac{4}{t}$ where $t>-1$ and $t\neq 0$. When $t\to 0$, $x\to 1$ and $y\to\infty$, therefore the curve has a vertical asymptote at $x=1$. When $t\to\infty$, $x\to\infty$ and $y\to 0$, therefore the curve has a horizontal asymptote at $y=0$.	To show that a line is an asymptote, we need to describe the behaviour of <u>both</u> *x* and *y* variables near the line (eg $x\to 1$ and $y\to\infty$). In this question, since t is the parameter where both *x* and *y* are dependent on, we <u>must</u> start with the behaviour of *t* first ($t\to 0$ and $t\to\infty$), then talk about the behaviour of *x* and *y*.
	<u>Alternative Method</u> $x=2t+1,\quad y=\frac{4}{t}$ where $t>-1$ and $t\neq 0$. Sub $t=\frac{x-1}{2}$ into $y=\frac{4}{t}$, $y=4\div\frac{x-1}{2}$ $y=\frac{8}{x-1}$ where $x>-1$ and $t\neq 1$ since ($t>-1$ and $t\neq 0$) As $x\to 1$, $y\to\infty$, therefore the curve has a vertical asymptote at $x=1$. As $x\to\infty$ and $y\to 0$, therefore the curve has a horizontal asymptote at $y=0$.	

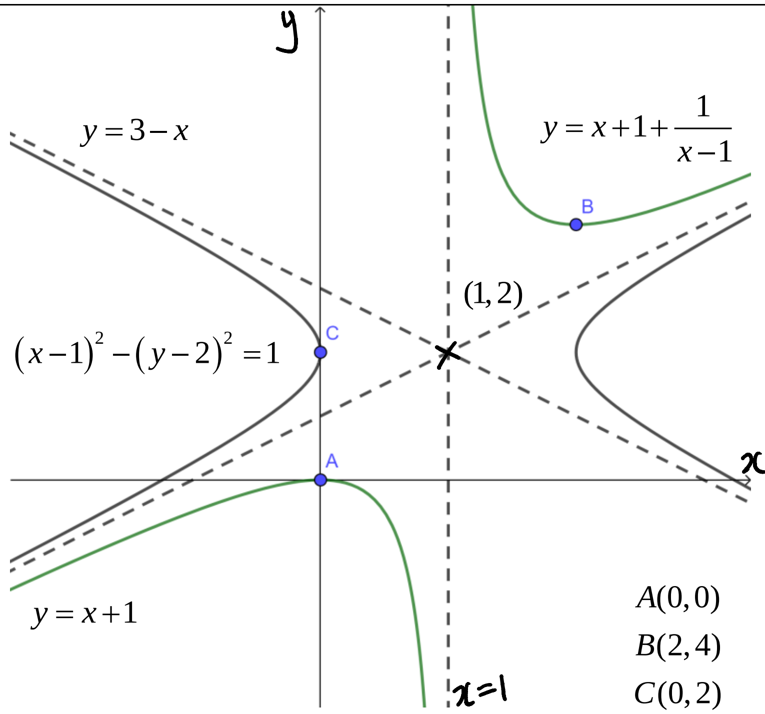
(ii)	 $x = 2t + 1, \quad y = \dfrac{4}{t}$ where $t > -1$ and $t \neq 0$.	Most students were unable to get the graph right. Most only had the portion for $x > 1$ because the default Tmin = 0 in the GC was used. To sketch the other portion using the GC, you will need to sketch separately for Tmin = –1 and Tmax = 0. Another note is that the point (–1, –4) corresponds to t = –1 which not in the range of t, thus this point has to be excluded.

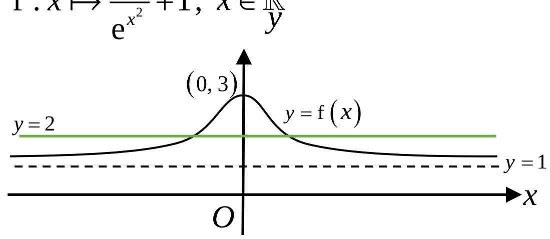
(iii)	$\dfrac{dx}{dt} = 2, \qquad \dfrac{dy}{dt} = -\dfrac{4}{t^2}$ $\dfrac{dy}{dx} = \dfrac{dy}{dt} \times \dfrac{dt}{dx} = -\dfrac{2}{t^2}$ **Alternatively** $y = \dfrac{8}{x-1}$ $\dfrac{dy}{dx} = -\dfrac{8}{(x-1)^2}$ When $x = 2t+1$, $\dfrac{dy}{dx} = -\dfrac{2}{t^2}$ Equation of tangent : $y - \left(\dfrac{4}{r}\right) = \dfrac{-2}{r^2}\left(x - 2r - 1\right)$ At point S : $y = 0 \Rightarrow \quad 0 - \left(\dfrac{4}{r}\right) = \dfrac{-2}{r^2}\left(x - 2r - 1\right)$ $x = 4r + 1$	Manageable for most students, except for few careless mistakes in $x - 2r - 1$. If $\dfrac{dy}{dx} = -\dfrac{8}{(x-1)^2}$ is used, then x must be replaced by $2r + 1$ to obtain the gradient at the point ($2r + 1$, $4/r$).

	At point T: $x=0 \Rightarrow$ $y-\left(\frac{4}{r}\right)=\frac{-2}{r^2}\left(-2r-1\right)$ $y=\frac{2\left(4r+1\right)}{r^2}$ Area of triangle OST = $\frac{1}{2}\left(\frac{2(4r+1)}{r^2}\right)\left(4r+1\right)$ $=\left(\frac{4r+1}{r}\right)^2$ units^2	
(iv)	C: $x=2t+1,\quad y=\frac{4}{t}$ Replace x with $x-1$, replace y with -y $x-1=2t+1$, $-y=\frac{4}{t}$ $x=2t+2$, $y=-\frac{4}{t}$	Manageable for most students, except a few were confused about what to replace x and y. Few students used cartesian equations instead of parametric though the question asked for parametric equation.

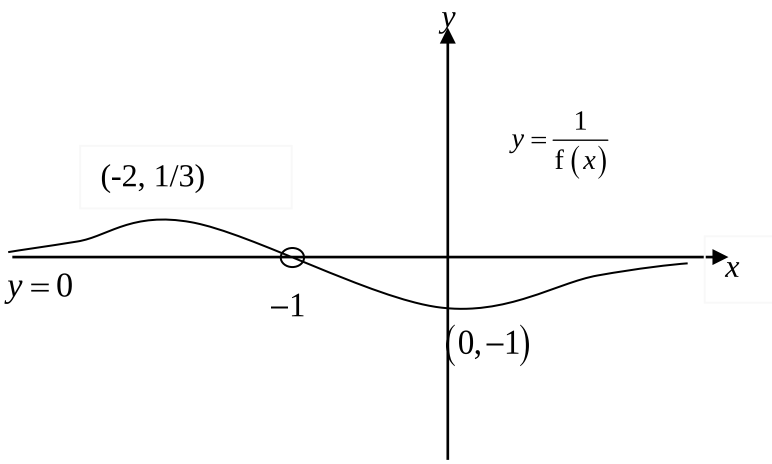
8	Solution [10] Graphing techniques	
(i)	Since $x=1$ is a vertical asymptote, therefore $d=1$. Since the asymptotes intersect at (1, 2), and the oblique asymptote has equation $y = ax + b$: When $x = 1$, $a + b = 2$, $b = 2 - a$. Since there is a stationary point when $x = 0$: $\frac{\mathrm{d}y}{\mathrm{d}x}=a-\frac{c}{(x-1)^2}$ $\left.\frac{\mathrm{d}y}{\mathrm{d}x}\right\|_{x=0}=a-c=0$ Therefore, $c = a$.	Generally very well done for this part except for some careless algebraic manipulation errors. Some students did not have the correct expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.
(ii) (iii)	Since the graph passes through (0, 0): When $x = 0$, $y = 0$, $0=a(0)+2-a+\frac{a}{(0-1)}$ $a=1,\ b=1,\ c=1$ 	Mistakes for this part include not labelling the max/min points or the oblique asymptote $y=x+1$

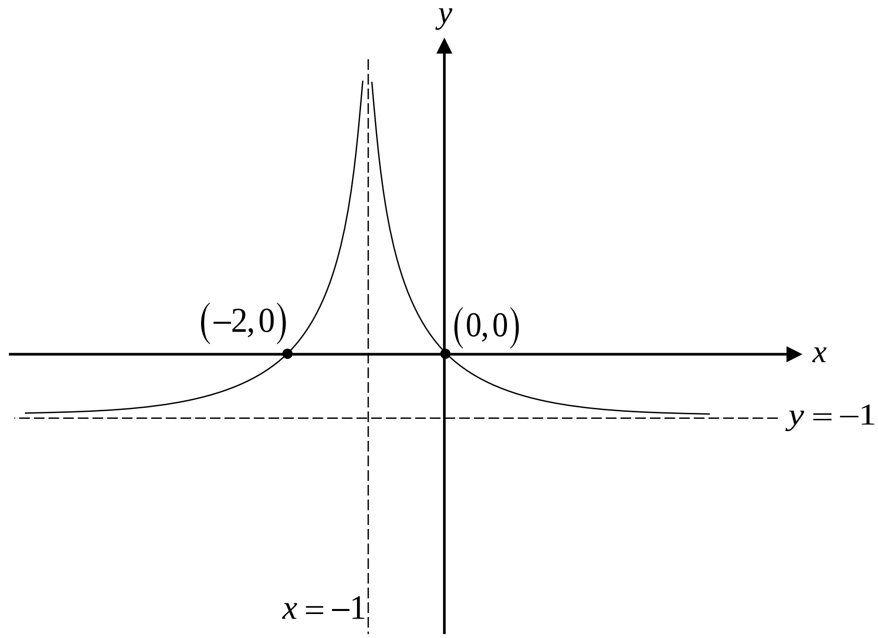
$y=x+1+\frac{1}{x-1}$

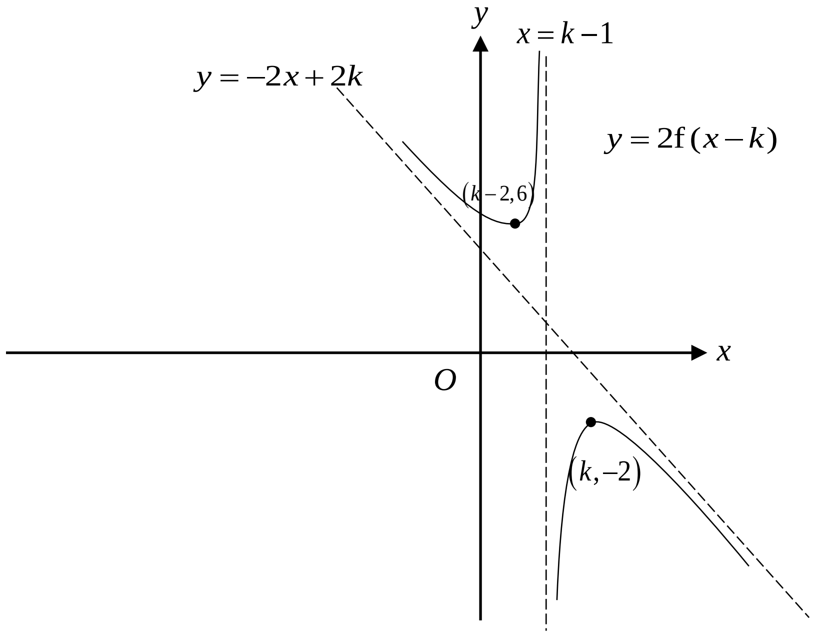
(c) (iii)		This part is generally not well done. Some students drew ellipses instead of the correct hyperbola. Students did not label all necessary features of the hyperbola: the oblique asymptotes, centre and turning points (0,2) and (2,2) etc.
	Equations of asymptotes of hyerbola are $y=kx-k+2$ and $y=-kx+k+2$. From the graph, the curves will not intersect, so long as the asymptotes do not intersect. Therefore, $0<k\le 1$ Note: when $k=1$, equations of asymptotes of hyperbola are $y=x+1$ and $y=3-x$	Many students were not able to link the value of k to the value of the gradient of the oblique asymptotes of the hyperbola and hence the explanation for the answer $0<k\le 1$.

9	Solution [10] Function	
(i)	$\mathrm{f}: x \mapsto \frac{2}{\mathrm{e}^{x^2}} + 1,\ x \in \mathbb{R}$  The horizontal line $y = k$, for $1 < k < 3$ intersects the graph of $y = \mathrm{f}(x)$ more than once and thus f is not one-one, f does not have an inverse.	Very badly done. Majority of the cohort missed out $y = 1$ as the asymptote and was not reflected on their graph. Justification was badly explained. Many students wrote the line $y = k$ without specifying the values of k.
(ii)	Largest value of a is 0.	Very well done. Except a bunch of students who have mistaken $a = 3$ when the values we are looking from are from the domain.
(iii)	Let $y = 2\mathrm{e}^{-x^2} + 1$ $y - 1 = 2\mathrm{e}^{-x^2}$ $\mathrm{e}^{-x^2} = \frac{y-1}{2}$ $-x^2 = \ln\left(\frac{y-1}{2}\right)$....(*) $x^2 = -\ln\left(\frac{y-1}{2}\right)$ $x = \pm\sqrt{-\ln\left(\frac{y-1}{2}\right)}$ (**) Since $x \le 0$ (found previously in **(ii)**), $x = -\sqrt{-\ln\left(\frac{y-1}{2}\right)}$ $\mathrm{f}^{-1}(x) = -\sqrt{-\ln\left(\frac{x-1}{2}\right)}$ OR $-\sqrt{\ln\left(\frac{2}{x-1}\right)}$	First part of the question was quite well done; students know that they have to make x the subject of the formula. Some students were not familiar with the laws of logarithms as well. Many students however missed out the fact that $x \le 0$ and hence were not able to identify the correct sign required for the logarithmic function.
(iv)	$\mathrm{R_f} = (1,\ 3]$ (from graph) $\mathrm{D_g} = \mathrm{R_{g^{-1}}} = (1,\ 3]$ Since $\mathrm{R_f} \subseteq \mathrm{D_g}$, gf exists.	Very well done.

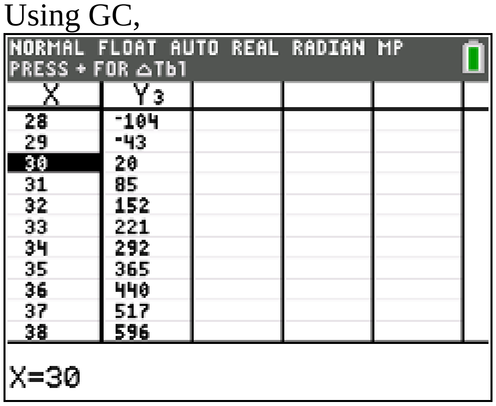
(v)	Domain of gf = Domain of f = $(-\infty,\ 0]$ Range of gf = $(-\infty,\ 0]$	Very badly done. Most students seem to have the misconception that Range of gf = Range of g or f, both are incorrect. They are required to draw a graph or use mapping method wherever necessary.

10.	Solution [10] Transformation	
(i)		Very badly done. Many students missed out the horizontal asymptote and they did not connect the graphs of the excluded point at x = -1. Many students also left this part empty.
(ii)	$y=\mathrm{f}'(x)$	Very badly done. Many were not able to obtain the correct shape of the graph as they were unable to obtain $y=-1$ as the asymptote.

		
(iii)	$y=2\mathrm{f}\left(x-k\right),k>2$ (1): Translate graph by *k* unit in the positive *x*-direction (2): Scaling parallel to *y*-axis by scale factor 2	Most students were able to obtain the shape of the graph but were unable to indicate the oblique asymptote correctly. Many were also unable to indicate the correct steps to obtain the coordinates of the turning points.



y
x = k − 1
y = −2x + 2k
y = 2f(x − k)
(k − 2, 6)
x
O
(k, −2)

11	Solution [12] AP GP	
(i)	$S_n = n^2 + 4n$ Let T_n be the amount Peter receives from Grandma each year. $T_n = (n^2+4n) - \left((n-1)^2 + 4(n-1)\right)$ $= n^2 - (n-1)^2 + 4n - 4(n-1)$ $= \left[n-n+1\right]\left[n+n-1\right] + 4$ $= 2n+3$ Consider $T_n - T_{n-1}$ $= (2n+3) - \left(2(n-1)+3\right)$ $= 2,$ a constant. Thus T_n is an AP	Generally well done. Some candidates successfully show that $T_n = S_n - S_{n-1} = 2n+3$. They went on to show that $T_{n-1} = S_{n-1} - S_{n-2} = 2n+1$, which can be quite tedious.
(ii)	$S_n > 1000$ $n^2 + 4n > 1000$ $n^2 + 4n - 1000 > 0$ Using GC,  n = 29, $n^2+4n-1000 = -43 < 0$ n = 30, $n^2+4n-1000 = 20 > 0$ n =31, $n^2+4n-1000 = 85 > 0$ Thus least n = 30. Thus, Peter will receive a total of more than \$1000 in the 30th year. Alternatively $S_n > 1000$ $n^2 + 4n > 1000$ $n^2 + 4n - 1000 > 0$ $n < -33.7$ (Reject since $n \geq 0$) or $n > 29.7$	For students who worked with the equality $n^2 + 4n = 1000$, they need to justify the least value of n is 30, with use of a table. Some candidates try to work out the first term and common difference based on the AP sequence found in (i). Thereafter, they set up an expression for S_n. It is advisable for them to use the $S_n = n^2 + 4n$ given in the question instead.

	Thus least n = 30. Thus, Peter will receive a total of more than \$1000 in the 30th year.	
(iii)	The amount that Peter will receives from his Grandpa in the long term is not finite as the GP common ratio, $\|r\| = 1.1 > 1$, S_∞ does not exist.	
	Let W_n be the total sum of money given by Grandpa $W_n = \frac{5\left(1.1^n - 1\right)}{1.1 - 1} > 1000$ $5\left(1.1^n - 1\right) > 100$ $1.1^n > 21$ $n \ln 1.1 > \ln 21$ $n > 31.9$ Least n = 32 Peter receives a total of more than \$1000 from his Grandpa on the 32nd year. Thus, peter will receive a sum of more than \$1000 from Grandma first.	A number of candidates assigned $r = 0.1$.