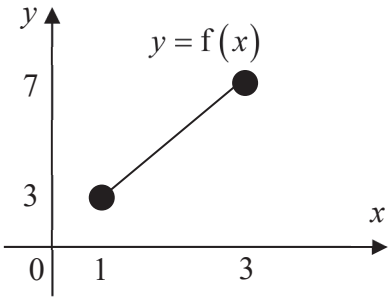
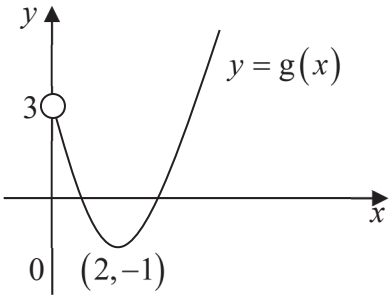
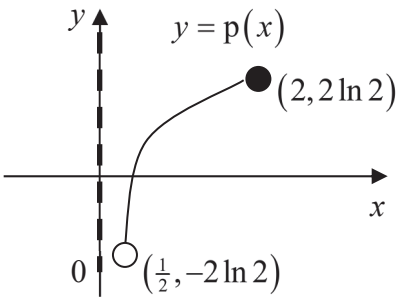
**Drill & Practice (1):**

State the rule, domain and range for each of the given functions:

| | |
|---|---|
| $\mathrm{f}: x \mapsto 2x+1, \quad x \in \mathbb{R}, \quad 1 \le x \le 3$<br>  <br> | $\mathrm{f}(x) = 2x+1$<br>$D_{\mathrm{f}} = [1,3]$<br>$R_{\mathrm{f}} = [3,7]$ |
| $\mathrm{g}: x \mapsto x^2 - 4x + 3, \; x \in \mathbb{R}, \quad x > 0$<br>  <br> | $\mathrm{g}(x) = x^2 - 4x + 3,$<br>$D_{\mathrm{g}} = (0, \infty)$<br>$R_{\mathrm{g}} = [-1, \infty)$ |
| $\mathrm{p}: x \mapsto 2\ln x, \quad x \in \mathbb{R}, \quad \frac{1}{2} < x \le 2$<br>  <br> | $\mathrm{p}(x) = 2\ln x$<br>$D_{\mathrm{p}} = \left(\frac{1}{2}, 2\right]$<br>$R_{\mathrm{p}} = (-2\ln 2, 2\ln 2]$ |

**Drill & Practice (2):**
Determine, with reason, if the following relations are functions.

| Relation | Is $y = \mathrm{f}(x)$ a function? | Reason |
|---|---|---|
| $y = \frac{1}{x}, \quad x \in \mathbb{R}$ | **No** | When $x = 0$, $y$ is undefined. $x = 0$ does not have an image. |
| $y = \frac{1}{x}, \quad x \in \mathbb{R}^+$ | **Yes** | Using the **vertical line test**, this relation is a function because every vertical line $x = k, k \in \mathbb{R}^+$ intersects the curve exactly once. |
| $y^2 = 1 - x, \quad x \in \mathbb{R}, \ x \le 1$ | **No** | This relation is not a function since using the **vertical line test**, $x = 0$ intersects the graph at more than one point. |
| $y = \sqrt{1-x}, \ x \in \mathbb{R}, \ x \le 1$ | **Yes** | Using the **vertical line test**, this relation is a function because every vertical line $x = k, k \in \mathbb{R}, k \le 1$ intersects the curve exactly once. |

Learning points ✍

### 1.3.2 Equivalence of Functions

Two functions are the same if and only if they have the same rule and domain.

Compare these two functions:

$$\mathrm{f} : x \mapsto x^2, \quad x \in \mathbb{R}^+$$
$$\mathrm{g} : x \mapsto x^2, \quad x \in \mathbb{R}$$

Although f and g share the same rule, they have <u>different</u> domains.
Thus they are different functions altogether.